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YÖS

YABANCI ÖĞRENCİ SINAVLARI
FOREIGN STUDENT EXAMINATIONS

MATEMATİK
MATHEMATICS 1

KONU ANLATIMLI
SUBJECT EXPLANATIONS

759 Çözümlü Soru
Questions with Solutions

Çözümlü YÖS Sorusu 177
YÖS Questions with Solutions

1150 Test Sorusu
Test Questions

Türkiye'de ve Dünyada!
In Turkey and Around the World!

METROPOL YAYINLARI

8. Baskı - 8th Edition

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ÖNSÖZ

Değerli öğrenciler,

Metropol Eğitim Kurumları, 1995'den bugüne YÖS, SAT ve TÖMER sınavlarına hazırlanan öğrencilere kurs, yayın ve rehberlik hizmetleri sunan, Türkiye'de ve Dünyanın farklı ülkelerinde şube ve temsilcilikleri bulunan uluslararası bir eğitim kurumları zinciridir.

Kurumumuz, Türkiye üniversitelerinin gerçekleştirdikleri yurt dışından Öğrenci Kabul Sınavları ya da bilinen adıyla Yabancı Uyruklu Öğrenci Sınavlarına (YÖS) hazırlanan ve bu üniversitelerde yüksek öğrenim görmeyi hedefleyen yabancı uyruklu ve T.C. vatandaşı öğrencilere profesyonel olarak hizmet veren ilk ve tek eğitim kurumudur.

Türkiye'nin önde gelen üniversitelerinin düzenledikleri bu sınavlarda (YÖS), Temel Öğrenme Becerileri Testi ve Türkçe Yeterlilik Testi olmak üzere iki ayrı test yer almaktadır. Temel Öğrenme Becerileri Testleri sınavların değerlendirmelerinde ve üniversitelere kabul aşamalarında dikkate alınmakta olup yerleştirmeler bu testlerin sonuçlarına göre yapılmaktadır. Türkçe testleri ise, sınava katılan öğrencilerin Türkçe dil yeterliliklerini ölçmek için uygulanmakta olup, yerleştirme puanlarını etkilememektedir.

Yabancı Öğrenci Sınavlarında (YÖS) başarılı olabilmek ve üniversitede hedeflediğiniz bölümde okuyabilmek için öncelikle Temel Öğrenme Becerileri Testlerinde baraj olarak belirlenen puanları aşmanız ve daha yüksek puanlar almanız gerekmektedir.

Sınavlardaki Temel Öğrenme Becerileri Testlerinde Genel Yetenek (IQ), Matematik ve Geometri soruları yer almakta olup; bu soruları doğru yanıtlamanız sınavların başarı sıralamalarında öne çıkmanızı sağlayacaktır.

Türkiye'nin seçkin üniversitelerinde öğrenim görebilme şansını yakalayabilmek için YÖS hazırlık kurslarımıza katılmanızı ve yayınlarımızı hazırlamış olan, alanında uzman ve deneyimli öğretmenlerimizden destek almanızı öneriyoruz.

Tüm YÖS adaylarına başarılar dileriz!

PREFACE

Dear students,

Metropol Educational Institutions is a group of international educational institutions with branches and representatives in Turkey and different countries around the world, offering courses, publications and consultancy services to students studying for the YÖS, SAT and TÖMER exams since 1995.

Our institution is the first and the only educational institution that provides professional services to the international students and Turkish citizens preparing for the Foreign Students Acceptance Exams known as Foreign Students Exams (YÖS) and planning to continue their higher education at these universities in Turkey.

In these (YÖS) exams held by the renowned universities in Turkey, there are two separate tests; Basic Learning Skills Test and Turkish Language Proficiency Test. The Basic Learning Skills Test is taken into consideration in the evaluation processes of the exams and in the acceptance processes by the universities; the placements are performed according to the results of these tests. The Turkish Language Proficiency Tests, on the other hand, are intended to measure the Turkish language level of the students who take the exam and it does not affect the placement scores.

In order to be successful in the YÖS exams and to be able to enrol in the department in which you want to study at the university, you need to exceed the threshold score in the exam and get higher scores.

The Basic Learning Skills Tests include Intelligent Quotient (IQ), Mathematics and Geometry questions. You will be able to stand out in the rankings of the exam scores by answering these questions correctly.

To capture the opportunity to study at elite universities of Turkey, we recommend that you participate in our YÖS preparation courses and get support from our qualified and experienced teachers who have prepared our publications.

We wish great success to all YÖS candidates!

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BÖLÜM CHAPTER 1

TARİHSEL NOT / HISTORICAL NOTE

Johann Heinrich Lambert [1728 – 1777]

Johann Lambert, ilk defa π nin bir irrasyonel sayı olduğunu kesin olarak ispatlayan ünlü Fransız matematikçisidir.

Johann Lambert was the first to provide a rigorous proof that π is an irrational number.

SAYILAR / NUMBERS

Bu bölüm, 52 çözümlü örnek, 12 çözümlü test sorusu, 16 çözümlü YÖS sorusu, 128 test sorusu içermektedir.

This chapter includes 52 examples with solutions, 12 tests questions with solutions, 16 YÖS questions with solutions, 128 test questions.

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BÖLÜM
01
CHAPTER

SAYILAR
NUMBERS

ÖNEMLİ / IMPORTANT

• Aritmetikte kullanılan çeşitli işlemleri iyi anlayabilmek için, ortak sayı sistemine gereksinim vardır.

• Sayı sisteminde, rakam olarak adlandırılan 10 sembol kullanılır. Bunlar; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 dur. Bu on sembolü kullanarak sayıları yazabiliriz.

• Bu sayıların her biri, nesnelerin miktarlarını belirtmek için kullanılır. Semboller ilk olarak, sayma amacıyla kullanılmıştır.

• In order to get a good understanding of the various operations used in arithmetic, one should have a clear idea of the common number system.

• The system of numbers makes use of only ten symbols called digits: 0,1,2,3,4,5,6,7,8,9. By means of these ten symbols we can write a number of any size.

• Each of these symbols is used to denote the number of objects. The symbols were first used for counting.

TEMEL KAVRAMLAR / BASIC CONCEPTS

Sayı: Çoklukları gösterebilmek için kullanılan sembollere ***sayı*** denir.

Rakam: Sayıları yazmak için kullanılan sembollerden her birine ***rakam*** denir.

Rakamlar kümesi; { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

Örneğin; 716 sayısının rakamları 7, 1 ve 6 dır.

Number: Symbols used in order to being able to express quantity.

Digit: Each symbol used in expression of the numbers is called a *digit*.

Set of digits: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

For example; digits of the number 716 are 7, 1, and 6.

Etkinlikler / Activities

x, y, z farklı rakamlar olmak üzere / x, y, z are each a different digit

a. $\max(2\cdot x + 3\cdot y + 7\cdot z) = ?$

$$\left.\begin{matrix} z = 9 \\ y = 8 \\ x = 7 \end{matrix}\right\} \Rightarrow 2\cdot 7 + 3\cdot 8 + 7\cdot 9 = 14 + 24 + 63 = 101$$

b. $\min(2\cdot x + 3\cdot y + 7\cdot z) = ?$

$$\left.\begin{matrix} z = 0 \\ y = 1 \\ x = 2 \end{matrix}\right\} \Rightarrow 2\cdot 2 + 3\cdot 1 + 7\cdot 0 = 4 + 3 + 0 = 7$$

c. x, y ve z sıfırdan farklı rakamlar olmak üzere.
x, y, and z are each a different non-zero digit.

$x + 3y + 5z = 45 \Rightarrow \max(z) = ?$

$$\left.\begin{matrix} x = 2 \\ y = 1 \end{matrix}\right\} \Rightarrow \begin{aligned} 2 + 3\cdot 1 + 5z &= 45 \\ 5z &= 40 \\ z &= 8 \end{aligned}$$

$\max(z) = 8$

İŞLEM ÖNCELİĞİ / THE ORDER of OPERATIONS

Herhangi bir sayı kümesinde toplama, çıkarma, çarpma ve bölme işlemleri (yani, dört işlem) yapılırken aşağıdaki sıra uygulanır:

- Parantez, köşeli parantez, mutlak değer ve kesir çizgisi gibi grup sembolleri içeren işlemler yapılır.
- Üslü ifadeler sadeleştirilir.
- Soldan sağa doğru çarpma ve bölme yapılır.
- Soldan sağa doğru toplama ve çıkarma yapılır.

In any set of numbers addition, subtraction, multiplication and division must be done as the following order.

- Perform operations inside grouping symbols. like parantheses ( ), brackets [ ], braces { }, absolute value symbols | |, and the fraction bar.
- Simplify exponential expressions.
- Do multiplication and division as they occur from left to right.
- Do addition and subtraction as they occur from left to right.

Örnek / Example 1

(4 + 3) ·2 – 5 = ?

A) 8 B) 9 C) 10 D) 11 E) 12

Çözüm / Solution

Yanıt / Answer **B**

$(4 + 3) \cdot 2 - 5 = 7 \cdot 2 - 5$
$= 14 - 5$
$= 9$

Örnek / Example 2

–8 + 6 : (–5 + 2) – (4 – 16) = ?

A) 2 B) 3 C) 4 D) 5 E) 6

Çözüm / Solution

Yanıt / Answer **A**

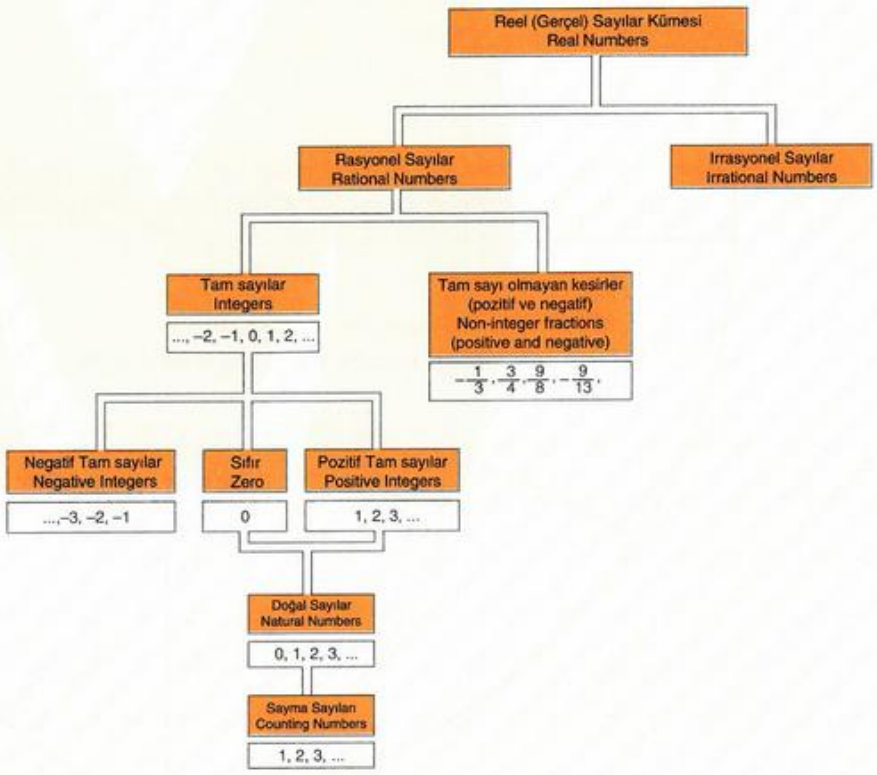
$-8 + 6 : (-3) - (-12) = -8 - 2 + 12 = 2$

SAYI KÜMELERİ / NUMBER SETS

- **Sayma sayıları kümesi /** Set of counting numbers: $C = \{1, 2, 3, 4, \dots\}$
- **Doğal sayılar kümesi /** Set of natural number: $N = \{0, 1, 2, 3, 4, \dots\}$
- **Tam sayılar kümesi /** Set of integers: $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- **Negatif tam sayılar kümesi /** Set of negative integers: $Z^- = \{\dots, -3, -2, -1\}$
- **Pozitif tam sayılar kümesi /** Set of positive integers: $Z^+ = \{1, 2, 3, \dots\}$
- **Rasyonel sayılar kümesi /** Set of rational integers: $Q = \{\frac{a}{b} : a, b \in Z, b \neq 0\}$
- **İrrasyonel sayılar kümesi /** Set of irrational numbers: Rasyonel olmayan sayıların kümesine irrasyonel sayılar kümesi denir. Q' ile gösterilir.
  The set of all the numbers which are non-rational is called the set of irrational numbers, and is shown by Q'.
- **Reel sayılar kümesi /** Set of real numbers: $R = Q \cup Q'$

ÖNEMLİ / IMPORTANT

- $C \subset N \subset Z \subset Q \subset R$
- $Z = Z^- \cup \{0\} \cup Z^+$



DOĞAL SAYILAR / NATURAL NUMBERS

$N = \{0, 1, 2, 3, ...\}$ kümesine doğal sayılar kümesi, $N^+ = \{1, 2, 3, ...\}$ kümesine de pozitif doğal sayılar kümesi denir.

$123 \in N$, $25 \in N$, $4572 \in N$

Doğal sayılarda basamak kavramı: Bir doğal sayıyı oluşturan rakamların yazıldıkları yere **basamak** denir.

Rakamın Sayı Değeri: Bir veya birden çok basamaklı bir sayıdaki, herhangi bir rakamın bulunduğu basamağa bağlı olmadan tek başına gösterdiği sayıya, bu rakamın **sayı değeri** denir. Rakamın sayı değeri hiç değişmez.

Rakamın Basamak Değeri: Bir veya birden çok basamaklı bir sayıdaki herhangi bir rakamın bulunduğu basamağa göre aldığı değere, bu rakamın **basamak değeri** denir.

$N = \{0, 1, 2, 3, ...\}$ is called set of natural numbers, and $N^+ = \{1, 2, 3, ...\}$ is called set of pozitive natural numbers.

$123 \in N$, $25 \in N$, $4572 \in N$

The concept of place in natural numbers: The position of each digit written in a natural number is called the place or position of that digit.

The number value of a digit: In a number consisting of one or more than one digits, each one of these digits, regardless of it's place in the number, has a value; this value is called the **number value** of that digit. The number value of a digit never changes.

The place value of a digit: In a number consisting of one or more than one digits, the value of each of these digits considering its place in the number is called its **place value**.



	Sayı değeri Number Value	Basamak değeri Place Value
Birler basamağı (Ones place)	e	1.e
Onlar basamağı (Tens place)	d	10.d
Yüzler basamağı (Hundreds place)	c	100.c
Binler basamağı (Thousands place)	b	1000.b
On binler basamağı (Ten thousands place)	a	10000.a

ÖNEMLİ / IMPORTANT

- *Doğal sayılar onluk sayı sisteminde yazıldıkları için basamak isimleri 10'un kuvvetleri olarak isimlendirilir.*
- *As the natural numbers are written in the decimal number system, the names of the places or positions in these numbers are named using powers of ten.*

ÖNEMLİ / IMPORTANT

• *Toplamları sabit olan iki doğal sayının çarpımlarının en büyük olabilmesi için sayılar arasındaki farkın en küçük olması gerekir.*

• *Two natural numbers with a constant sum should have minimum difference between them in order to maximize their product.*

Örnek / Example 3

$a, b \in N^+$

$a + b = 22 \Rightarrow$ **max(a·b) = ?**

A) 81 B) 100 C) 121 D) 144 E) 169

Çözüm / Solution

Yanıt / Answer **C**

$$\left.\begin{matrix} a+b=22 \\ \max(a\cdot b) \end{matrix}\right\} \Rightarrow \begin{matrix} a=11,\ b=11 \\ \max(a\cdot b)=121 \end{matrix}$$

ÖNEMLİ / IMPORTANT

• *Toplamları sabit olan iki doğal sayının çarpımlarının en küçük olabilmesi için sayılar arasındaki farkın en büyük olması gerekir.*

• *Two natural numbers with a constant sum should have maximum difference between them in order to minimize their product.*

Örnek / Example 4

$a, b \in N,\ a \neq b$

$a + b = 18 \Rightarrow$ **max(a·b) + min(a·b) = ?**

A) 72 B) 80 C) 88 D) 96 E) 104

Çözüm / Solution

Yanıt / Answer **B**

$$\left.\begin{matrix} a+b=18 \\ \min(a\cdot b) \end{matrix}\right\} \Rightarrow \begin{matrix} a=0,\ b=18 \\ \min(a\cdot b)=0 \end{matrix}$$

$$\left.\begin{matrix} a+b=18 \\ \max(a\cdot b) \\ a\neq b \end{matrix}\right\} \Rightarrow \begin{matrix} a=8,\ b=10 \\ \max(a\cdot b)=80 \end{matrix}$$

$\max(a\cdot b)+\min(a\cdot b)=80+0=80$

Örnek / Example 5

$a, b \in N$

$a \cdot b = 36 \Rightarrow$ **max(a + b) = ?**

A) 12 B) 16 C) 18 D) 28 E) 37

Çözüm / Solution

Yanıt / Answer **E**

$$\left.\begin{matrix} a \cdot b = 36 \\ \max(a+b) \end{matrix}\right\} \Rightarrow \begin{matrix} a = 1,\ b = 36 \\ \max(a+b) = 37 \end{matrix}$$

Örnek / Example 6

$a, b \in N$

$a \cdot b = 48 \Rightarrow$ **max(a + b) + min(a + b) = ?**

A) 57 B) 58 C) 59 D) 60 E) 63

Çözüm / Solution

Yanıt / Answer **E**

$$\left.\begin{matrix} a \cdot b = 48 \\ \max(a+b) \end{matrix}\right\} \Rightarrow \begin{matrix} a = 48,\ b = 1 \\ \max(a+b) = 49 \end{matrix}$$

$$\left.\begin{matrix} a \cdot b = 48 \\ \min(a+b) \end{matrix}\right\} \Rightarrow \begin{matrix} a = 6,\ b = 8 \\ \min(a+b) = 14 \end{matrix}$$

$\Rightarrow \max(a+b) + \min(a+b) = 63$

ÖNEMLİ / IMPORTANT

• ab ve ba iki basamaklı birer doğal sayı olmak üzere aşağıdaki eşitlikler vardır:

• If ab and ba are two-place natural numbers, the following equalities exist:

I. $ab + ba = (10 \cdot a + b) + (10 \cdot b + a)$
$= 11 \cdot a + 11 \cdot b = 11 \cdot (a + b)$

II. $ab - ba = (10 \cdot a + b) - (10 \cdot b + a)$
$= 9 \cdot a - 9 \cdot b$
$= 9 \cdot (a - b)$

DOĞAL SAYILARDA ÇÖZÜMLEME
ANALYSIS of the NATURAL NUMBERS

Doğal sayının, rakamlarının basamak değerlerinin toplamı şeklinde yazılmasına bu sayının **çözümlenmesi** denir.

The application of writing a natural number as summation of the place values of its digits is called the **analysis** of that natural number.

▼ 34 sayısı çözümlenirse,
$34 = 30 + 4 = 10 \cdot 3 + 4$ bulunur.
O halde, ab sayısının çözümlenişi aşağıdaki gibi tanımlanır:
$\mathbf{ab = 10 \cdot a + b}$

▼ 235 sayısı çözümlenirse,
$235 = 200 + 30 + 5 = 100 \cdot 2 + 10 \cdot 3 + 5$ bulunur.
O halde, abc sayısının çözümlenişi aşağıdaki gibi tanımlanır:
$\mathbf{abc = 100 \cdot a + 10 \cdot b + c}$

▼ If we analyze 34,
$34 = 30 + 4 = 10 \cdot 3 + 4$ is obtained.
Therefore, analysis of the number ab is defined as the following.
$\mathbf{ab = 10 \cdot a + b}$

▼ If we analyze 235,
$235 = 200 + 30 + 5 = 100 \cdot 2 + 10 \cdot 3 + 5$ is obtained.
Therefore, analysis of the number abc is defined as the following.
$\mathbf{abc = 100 \cdot a + 10 \cdot b + c}$

Örnek / Example 7

$ab, ba \in N$
$ab + ba = 132 \Rightarrow$ **max(ab) = ?**

A) 93 B) 84 C) 56 D) 48 E) 40

Çözüm / Solution

Yanıt / Answer **A**

$ab + ba = 132$
$10 \cdot a + b + 10 \cdot b + a = 132$
$11(a + b) = 132$
$a + b = 12$

a ve b nin alabileceği değerler: (The values that a and b can take:)

a + b = 12		ab sayısı (The number ab)
9 3	⟶	93
8 4	⟶	84
7 5	⟶	75
6 6	⟶	66
5 7	⟶	57
4 8	⟶	48
3 9	⟶	39

$\Rightarrow$ max(ab) = 93

Örnek / Example 8

AB, BA, BB ∈ N

$\frac{AB+BA}{BB}=4 \Rightarrow \max(A+B)=?$

A) 8 B) 9 C) 10 D) 11 E) 12

Çözüm / Solution

Yanıt / Answer **E**

$$\frac{AB+BA}{BB}=\frac{11A+11B}{11B}=4$$

$$\frac{11(A+B)}{11B}=4$$

$$A+B=4B$$

$$A=3B$$

$B=3 \Rightarrow A=9$

$\Rightarrow \max(A+B)=12$

Örnek / Example 9

ACB, BCA ∈ N

BCA − ACB = 594 ⇒ **B − A = ?**

A) 5 B) 6 C) 7 D) 8 E) 9

Çözüm / Solution

Yanıt / Answer **B**

BCA − ACB = 594

100B + 10C + A − 100A − 10C − B = 594

99(B − A) = 594

B − A = 6

TABAN ARİTMETİĞİ / BASE ARITHMETICS

Tanım: Sayının tanımlandığı sayma sistemi o sayının tabanını oluşturur. Bu tabanda yapılan işlemlere de **taban aritmetiği** adı verilir.

Definition: The countiong system on which the number is defined forms the base. Operations that are being done in this base are called **base arithmetics.**

FARKLI BİR TABANDA VERİLEN SAYININ ONLUK TABANDA YAZILMASI
WRITING a GIVEN NUMBER in a BASE other than TEN to the BASE of TEN

a, b, c, d birer rakam / are digits

$t > a, t > b, t > c, t > d$

$t \in N^+, t > 1$ olmak üzere

If $t \in N^+, t > 1$,

$(abcd)_t = x$

$(a\,b\,c\,d)_t = a.t^3 + b.t^2 + c.t + d$

$d.t^0$
$c.t^1$
$b.t^2$
$a.t^3$

$x = d\cdot t^0 + c\cdot t^1 + b\cdot t^2 + a\cdot t^3$

$(ab,cd)_t = x$

$(a\,b,c\,d)_t = a.t^1 + b.t^0 + c.t^{-1} + d.t^{-2}$

$d.t^{-2}$
$c.t^{-1}$
$b.t^0$
$a.t^1$

$x = d\cdot t^{-2} + c\cdot t^{-1} + b\cdot t^0 + a\cdot t^1$

PEKİŞTİRME / REINFORCE

✓ $(101)_3 = (x)_{10} \Rightarrow x = ?$

..... [10]

✓ $(203)_5 = (x)_{10} \Rightarrow x = ?$

..... [53]

✓ $(124)_6 = (x)_{10} \Rightarrow x = ?$

..... [52]

Örnek / Example 10

$(11011)_2 = (x)_{10} \Rightarrow x = ?$

A) 27 B) 29 C) 31 D) 33 E) 35

Çözüm / Solution

Yanıt / Answer **A**

$(1\,1\,0\,1\,1)_2 = 1 + 2 + 0 + 8 + 16 = 27$

1.2^0
1.2^1
0.2^2
1.2^3
1.2^4

Örnek / Example 11

$(3x4)_5 = 89 \Rightarrow x = ?$

A) 0 B) 1 C) 2 D) 3 E) 4

Çözüm / Solution

Yanıt / Answer **C**

$(3x4)_5 = 89$

$4\cdot 5^0 + x\cdot 5^1 + 3\cdot 5^2 = 89$

$79 + 5x = 89$

$5x = 10$

$x = 2$

10'LUK TABANDA VERİLEN BİR SAYININ BAŞKA BİR TABANDA YAZILMASI
WRITING a GIVEN NUMBER in the BASE of TEN to a NUMBER in ANOTHER BASE

10 tabanında verilen sayı, hangi tabanda yazılacaksa o tabana bölünür. Sıfır dahil olmak üzere tüm kalanlar yazılır. Son kalandan başlanarak yazılan rakamlar yeni sayıyı oluşturur.
Divide the number given in the base of 10 by the new base as many times as possible and write down all remainders, including zeros. The remainders starting from the last remainder are the digits of the number written in the desired base.

Örnek / Example 12

$63 = (x)_5 \Rightarrow x = ?$

A) 220 B) 221 C) 222 D) 223 E) 224

Çözüm / Solution

Yanıt / Answer **D**

63 | 5
60 | 12 | 5
③ 10 | ②
②

$63 = (223)_5 \Rightarrow x = 223$

Örnek / Example 13

$(abc)_9 - (321)_4 = (432)_5 \Rightarrow 2\cdot a + 3\cdot c = ?$

A) 13 B) 15 C) 17 D) 21 E) 23

Çözüm / Solution

Yanıt / Answer **A**

$(abc)_9 = (432)_5 + (321)_4 = 4 \cdot 5^2 + 3 \cdot 5 + 2 + 3 \cdot 4^2 + 2 \cdot 4 + 1$

$= 100 + 15 + 2 + 48 + 9 = 174$

174 | 9
171 | 19 | 9
③ 18 | ②
①

$(abc)_9 = (213)_9$

$a = 2, b = 1, c = 3$

$\Rightarrow 2 \cdot a + 3 \cdot c = 2 \cdot 2 + 3 \cdot 3 = 4 + 9 = 13$

PEKİŞTİRME / REINFORCE

✓ $27 = (x)_4 \Rightarrow x = ?$ ..... [123]

✓ $33 = (x)_5 \Rightarrow x = ?$ ..... [113]

✓ $(24)_6 = (x)_5 \Rightarrow x = ?$ ..... [31]

TABAN ARİTMETİĞİNDE İŞLEMLER
OPERATIONS in the BASE ARITHMETICS

TOPLAMA / ADDITION

Taban aritmetiğinde toplama işlemi yapılırken rakamların toplamı verilen tabandan büyük veya eşit olduğunda toplamın sonucu tabana bölünür, kalan sonuç kısmına yazılır ve bölüm bir sonraki basamağa eklenir.

As doing the operation of addition in base arithmetics, whenever the value of the added digits becomes bigger than or equal to the base, the result of the addition needs to be devided to the base, remainder becomes the new result and the quotient will add to one column to the left.

Örnek / Example 14

$(456)_7 + (345)_7 = (x)_7 \Rightarrow x = ?$

A) 1134 B) 1324 C) 1451 D) 1001 E) 1501

Çözüm / Solution Yanıt / Answer **A**

$6 + 5 = 11$ $5 + 4 + 1 = 10$ $4 + 3 + 1 = 8$

$11 \mid 7$ → 4 $\quad 10 \mid 7$ → 3 $\quad 8 \mid 7$ → 1

$\Rightarrow (456)_7 + (345)_7 = (1134)_7$

PEKİŞTİRME / REINFORCE

✓ $(12)_3 + (21)_3 = ?$

..... [$(110)_3$]

✓ $(214)_5 + (312)_5 = ?$

..... [$(1031)_5$]

ÇIKARMA / SUBTRACTION

Taban aritmetiğinde çıkarma işlemi yapılırken 10'luk sistemdekine benzer biçimde bir soldaki basamaktan 1(bir) almak gerektiğinde aktarıldığı basamağa katkısı tabanın sayı değeri kadardır. Fakat alındığı basamak 1 azalır.

When doing the operation of subtraction in base arithmetics, whenever we need to take a 1 (one) from one column to the left as we do in the base of ten, the added value to the right column will equal to the base and the digit-value of the left column will decrease by 1 unit.

Örnek / Example 15

$(421)_7 - (256)_7 = (x)_7 \Rightarrow x = ?$

A) 105 B) 132 C) 146 D) 163 E) 232

Çözüm / Solution Yanıt / Answer **B**

$$\begin{array}{r} (421)_7 \\ -\ (256)_7 \\ \hline (132)_7 \end{array}$$

PEKİŞTİRME / REINFORCE

✓ $(244)_5 - (123)_5 = ?$

..... [$(121)_5$]

✓ $(124)_6 - (25)_6 = ?$

..... [$(55)_6$]

ÇARPMA / MULTIPLICATION

Taban aritmetiğinde çarpma işlemi yapılırken rakamların çarpımı tabanın sayı değerinden büyük ise sonuç tabandaki sayıya bölünür.

When doing the operation of multiplication in base arithmetics, if the product of integers exeeds the number value of the base, the result will be devided to the base

Örnek / Example 16

$(13)_5 \cdot (4)_5 = (x)_5 \Rightarrow x = ?$

A) 110 B) 111 C) 112 D) 113 E) 114

Çözüm / Solution

Yanıt / Answer **C**

$$\begin{array}{r} (13)_5 \\ \times\ (4)_5 \\ \hline (112)_5 \end{array}$$

$4 \cdot 3 = 12$

12 | 5 → **taban** (base)
10 | 2 → **elde** (adding number)
2 → **kalan** (remainder)

$4 \cdot 1 + 2 = 6$

6 | 5 → **taban** (base)
5 | 1 → **elde** (adding number)
1 → **kalan** (remainder)

Örnek / Example 17

$(432)_6 \cdot (24)_6 = (x)_6 \Rightarrow x = ?$

A) 20052 B) 20144 C) 20232 D) 20321 E) 20332

Çözüm / Solution

Yanıt / Answer **A**

$$\begin{array}{r} (432)_6 \\ \times\ (24)_6 \\ \hline (3012)_6 \\ +\ (1304)_6 \\ \hline (20052)_6 \end{array}$$

ÖNEMLİ / IMPORTANT

• *0 sayısı ne pozitif ne de negatiftir. Bu yüzden, pozitif tam sayılardan ya da 0 dan bahsederken, negatif olmayan tam sayılar ifadesi kullanılabilir.*

• *The integer zero is neither positive nor negative. Thus, when you talk about integers that are positive or zero, you can use the term nonnegative integers.*

TAM SAYILAR / INTEGERS

Tanım: {..., –3, –2, –1, 0, 1, 2, 3, ...} kümesinin her elemanına **tam sayı** denir. Tam sayılar kümesi Z ile gösterilir.

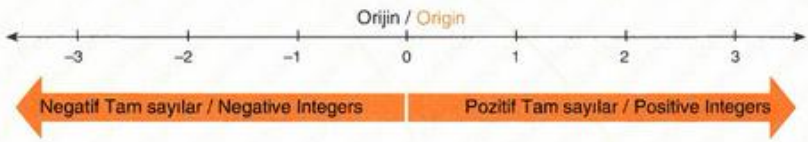
$Z = Z^- \cup \{0\} \cup Z^+$ veya $Z = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$

0 sayısı, ne negatif ne de pozitif tam sayıdır. 0 sayısı işaretsizdir.

Definition: Each element of the set {..., –3, –2, –1, 0, 1, 2, 3, ...} is an integer. The set of integers is denoted by the symbol Z.

$Z = Z^- \cup \{0\} \cup Z^+$ or $Z = \{... -3, -2, -1, 0, 1, 2, 3, ...\}$

The integer zero is neither positive nor negative. Thus, the number zero is unsigned integer.



ÖNEMLİ / IMPORTANT

Bir a sayısı için, aşağıdakilerden biri doğru olmalıdır.

- *a negatiftir.*
- *a = 0.*
- *a pozitiftir.*

For any number a, exactly one of the following is true:

- *a is negative.*
- *a = 0.*
- *a is positive.*

▼ Sayı doğrusu / The number line.

Sayıları göstermek için kullanılan şekle **sayı doğrusu** denir. Sayı doğrusu, orijini 0 olarak işaretlenen yatay bir doğrudan oluşur.

Sayı doğrusu, herhangi iki tam sayıyı karşılaştırma olanağı sağlar. Bu nedenle, sayı doğrusu üzerinde iki nokta seçilirse, bir nokta diğerinin solunda kalmak zorundadır. Bu durumda, soldaki sayı, sağdakinden küçük veya sağdaki sayı soldakinden daha büyük olmalıdır.

The picture that is used to represent the numbers is called the **number line**. It consists of a horizontal line with a point (the origin) labeled as 0.

The number line provides you with a way of comparing any two integers. For instance, if you choose any two (different) numbers on the number line, one of the numbers must be to the left of the other number. The number to the left is **less than** the number to the right, and the number to the right is **greater than** the number to the left.

POZİTİF TAM SAYILAR / POSITIVE INTEGERS

{1, 2, 3, ...} kümesine **pozitif tam sayılar kümesi** denir ve Z^+ ile gösterilir. Pozitif tam sayılar kümesinin her elemanına **pozitif tam sayı** denir.

$$Z^+ = \{1, 2, 3, ...\}$$

The set {1, 2, 3, ...} is called the **positive integers set** and is denoted by the symbol Z^+. Each element of the positive integers set is called a **positive integer**.

$$Z^+ = \{1, 2, 3, ...\}$$

NEGATİF TAM SAYILAR / NEGATIVE INTEGERS

{..., –3, –2, –1} kümesine **negatif tam sayılar kümesi** denir ve Z^- ile gösterilir. Negatif tam sayılar kümesinin her elemanına **negatif tam sayı** denir.

$$Z^- = \{..., -3, -2, -1\}$$

The set {..., –3, –2, –1} is called the **negative integers set** and is denoted by the symbol Z^-. Each element of the a negative integers set is called a **negative integer**.

$$Z^- = \{..., -3, -2, -1\}$$

TEK ve ÇİFT TAM SAYILAR / ODD and EVEN INTEGERS

Tanım: {..., –5, –3, –1, 1, 3, 5,...} kümesinin her bir elemanına **tek tam sayı** denir.
n bir tam sayı iken, tek sayılar **2n – 1** ile gösterilir.
Definition: Each element of the set {..., –5, –3, –1, 1, 3, 5...} is an odd number. While n is integer, the odd numbers are represented by **2n – 1**.

Tanım: {..., –4, –2, 0, 2, 4,...} kümesinin her bir elemanına **çift tam sayı** denir.
n bir tam sayı iken, çift sayılar **2n** ile gösterilir.
Definition: Each element of the set {..., –4, –2, 0, 2, 4,...} is called an **even integer**. While n is integer, the even numbers are represented by **2n**.

ÖNEMLİ / IMPORTANT

- *Çift sayılar 2'nin tam katlarıdır. Tek sayılar 2'nin tam katı değildir.*
- *Even integers are the multiples of 2. Odd integers are the non-multiples of 2.*

TEK ve ÇİFT TAM SAYILARDA İŞLEMLER
OPERATIONS in ODD and EVEN INTEGER

Tek tam sayılar / Odd integers: T/O
Çift tam sayılar / Even integers: Ç/E

a. T ± T = Ç / O ± O = E
b. T · T = T / O · O = E
c. T ± Ç = T / O ± E = O
d. T · Ç = Ç / D · E = E
e. Ç ± Ç = Ç / E ± E = E
f. Ç · Ç = Ç / E · E = E

ÖNEMLİ / IMPORTANT

- *Pozitif bir çift sayının pozitif tam sayı kuvveti çift, tek bir sayının pozitif tam sayı kuvveti tektir.*

$$n \in Z^+ \Rightarrow T^n = T,\ Ç^n = Ç$$

- *The positive integer power of a positive even number is even, the positive integer power of a positive odd number is odd.*

$$n \in Z^+ \Rightarrow E^n = E,\ O^n = O$$

Örnek / Example 18

$a \in N^+$, $(a + 2)^7$ bir tek sayı ise, aşağıdakilerden hangisi tektir?
$(a + 2)^7$ is an odd number where $a \in N^+$, which of the following is odd?

A) $3a + 1$ B) $a^2 + a$ C) $2a + 4$ D) $a^3 + 1$ E) $5a + 6$

Çözüm / Solution Yanıt / Answer **E**

$(a + 2)^7$ tek ise a bir tek sayıdır.

$\underbrace{5a}_{T} + \underbrace{6}_{Ç} = T$

If $(a + 2)^7$ is an odd number, then a must be an odd number.

$\underbrace{5a}_{O} + \underbrace{6}_{E} = O$

Örnek / Example 19

a, b ∈ Z^+
$(2a + 1)\cdot(b - 2) = 24 \Rightarrow$ **a + b = ?**

A) 8 B) 10 C) 11 D) 14 E) 16

Çözüm / Solution Yanıt / Answer **C**

$\underbrace{(2a + 1)}_{T}\cdot\underbrace{(b - 2)}_{Ç} = 24$

1·24 veya / or 3·8

$2a + 1 = 1 \Rightarrow a = 0$

$a \in Z^+ \Rightarrow a \neq 0$

$2a + 1 = 3 \Rightarrow a = 1,\quad b - 2 = 8\ \ b = 10 \Rightarrow a + b = 1 + 10 = 11$

PEKİŞTİRME / REINFORCE

✓ $8^5 + 7^7$
..... [Tek / Odd]

✓ $2\cdot(5^{10} + 3^6)$
..... [Çift / Even]

✓ $3^3 \cdot 5^2 \cdot 7^4 \cdot 2^5$
..... [Çift / Even]

✓ $\frac{10^6}{2}$
..... [Çift / Even]

ARDIŞIK SAYILAR / CONSECUTIVE NUMBERS

Tanım: Belirli bir kurala göre sıralanmış sayılara **ardışık sayılar** denir.

Definition: A group of numbers going forward following a certain rule are called **consecative numbers.**

Örneğin / For example:

1, 2, 3, 4, ..., n, ... ardışık sayılar (consecutive numbers)

2, 4, 6, 8, ..., 2n, ... ardışık çift sayılar (consecutive even numbers)

1, 3, 5, 7, ..., 2n – 1, ... ardışık tek sayılar (consecutive odd numbers)

3, 8, 13, 18, ... 78, ... ardışık sayı dizisi (consecutive number sequence)

ARDIŞIK SAYILARIN TOPLANMASI
SUMMATION of CONSECUTIVE NUMBERS

▼ Genel Toplam Formülü / General Summation Formula

Dizide kaç tane eleman olduğunu ve bu elemanların toplamını bulmak için aşağıda verilen formüller kullanılır.

To find the number of elements and the sum of these elements in a sequence, these formulas are used:

a. Terim sayısı : $\mathbf{T = \frac{Son\ terim - İlk\ terim}{Ortak\ fark} + 1}$

a. Number of terms : $T = \frac{\text{Last term} - \text{First term}}{\text{Common diference}} + 1$

b. Terimlerin toplamı : $\mathbf{S = \frac{Son\ terim + İlk\ terim}{2} \cdot T}$

b. Sum of terms : $S = \frac{\text{Last term} + \text{First term}}{2} \cdot T$

I. $1 + 2 + 3 + ... + n = \mathbf{\frac{n.(n+1)}{2}}$ [Ardışık tam sayıların toplamı]

[Sum of consecutive integers]

II. $2 + 4 + 6 + ... + 2n = \mathbf{n \cdot (n+1)}$ [Ardışık çift tam sayıların toplamı]

[Sum of consecutive even integers]

III. $1 + 3 + 5 + ... + (2n - 1) = \mathbf{n^2}$ [Ardışık tek tam sayıların toplamı]

[Sum of consecutive odd integers]

Etkinlikler / Activities

a. $1 + 2 + 3 + \ldots + 19 = \frac{19 \cdot (19 + 1)}{2} = 190$

b. $2 + 4 + 6 + \ldots + 42 = 21 \cdot (21+1) = 462$

c. $1 + 3 + 5 + \ldots + 37 = 19^2 = 361$

PEKİŞTİRME / REINFORCE

✓ *1 + 2 + 3 +...+ 24 = ?*

..... [300]

✓ *2 + 4 + 6 +...+ 40 = ?*

..... [420]

✓ *1 + 3 + 5 +...+ 27 = ?*

..... [196]

Örnek / Example 20

4 + 10 + 16 + ... + 76 = ?

A) 510 B) 515 C) 520 D) 525 E) 530

Çözüm / Solution

Yanıt / Answer **C**

$\left(\frac{76-4}{6}+1\right)\cdot\left(\frac{76+4}{2}\right) = 520$

Örnek / Example 21

4 + 8 + 12 + ... + 48 – 3 – 6 – 9 – ... – 36 = ?

A) 36 B) 42 C) 78 D) 84 E) 90

Çözüm / Solution

Yanıt / Answer **C**

$$4 + 8 + 12 + \ldots + 48 - 3 - 6 - 9 - \ldots - 36 = 4(1 + 2 + \ldots + 12) - 3(1 + 2 + \ldots + 12)$$
$$= (1 + 2 + \ldots + 12)\cdot(4 - 3)$$
$$= \frac{12 \cdot 13}{2}$$
$$= 78$$

Örnek / Example 22

$A = 1 + 2 + \ldots + n$

$B = 15 + 16 + \ldots + n - 1,$

$A - B = 120 \Rightarrow$ **n = ?**

A) 5 B) 12 C) 15 D) 22 E) 25

Çözüm / Solution

Yanıt / Answer **C**

$$A = 1 + 2 + \ldots + n$$
$$-\ B = 15 + 16 + \ldots + n - 1$$
$$A - B = 1 + 2 + \ldots + 14 + n = 120$$
$$\frac{14 \cdot 15}{2} + n = 120$$
$$105 + n = 120$$
$$n = 15$$

ÖNEMLİ / IMPORTANT

- *1 asal sayı değildir.*
- *En küçük asal sayı 2'dir.*
- *Hem çift hem de asal olan tek sayı 2'dir.*
- *Ardışık iki tam sayı aralarında asaldır.*
- *1 is not a prime number*
- *The smallest prime number is 2.*
- *The only even prime number is 2.*
- *Two consecutive integers are relatively prime numbers.*

ASAL SAYILAR / PRIME NUMBERS

Tanım : Kendisinden ve 1 sayısından başka pozitif tam sayı böleni olmayan, 1'den büyük pozitif tam sayılardır.

{2, 3, 5, 7, 11, 13, 17, 19, 23, ...}

Definition: Any positive integer bigger than 1 which is divisible only by 1 and the number itself is called a prime number.

{2, 3, 5, 7, 11, 13, 17, 19, 23, ...}

ARALARINDA ASAL SAYILAR / RELATIVELY PRIME NUMBERS

Tanım: 1'den başka ortak pozitif tam sayı böleni olmayan pozitif tam sayılardır.

Örneğin; 25 ve 12 , 32 ve 15...

Definition: Two integers of which only common factor is 1 are called relatively prime numbers.

For example ; 25 and 12 , 32 and 15...

Örnek / Example 23

$x, y, z \in Z^+$

$x \cdot (y + z) = 19 \Rightarrow x + y + z = ?$

A) 10 B) 12 C) 14 D) 20 E) 24

Çözüm / Solution

Yanıt / Answer **D**

$x, y, z \in Z^+$ ise 19 asal sayı olduğundan çarpanlardan biri 1 diğeri 19 olacaktır.

$x = 1, y + z = 19 \Rightarrow x + y + z = 20$

Since 19 is a prime number, one of its factors is 1 and the other one is 19.

$x = 1, y + z = 19 \Rightarrow x + y + z = 20$

Örnek / Example 24

$x, y \in Z^+$

$x^2 - y^2 = 23 \Rightarrow x \cdot y = ?$

A) 130 B) 132 C) 134 D) 136 E) 138

Çözüm / Solution

Yanıt / Answer **B**

$x^2 - y^2 = 23$

$(x - y) \cdot (x + y) = 23$

$(x - y) \cdot (x + y) = 1 \cdot 23$

$\left.\begin{matrix} x - y = 1 \\ x + y = 23 \end{matrix}\right\} \Rightarrow \begin{matrix} x = 12 \\ y = 11 \end{matrix}$

$x \cdot y = 12 \cdot 11 = 132$

ASAL ÇARPANLARA AYIRMA / PRIME-NUMBER FACTORIZATION

Tanım: Bir doğal sayının hangi asal sayıların çarpımı olduğunun bulunmasıdır.
Definition: If we consider a natural number as the product of prime numbers, then finding those prime numbers is called prime factorization.

Etkinlikler / Activities

a.

36	2
18	2
9	3
3	3
1	

$\Rightarrow 36 = 2^2 \cdot 3^2$

b.

120	2
60	2
30	2
15	3
5	5
1	

$\Rightarrow 120 = 2^3 \cdot 3.5$

Örnek / Example 25

$x, y, z \in N^+$

$120 = 2^x \cdot 3^y \cdot 5^z \Rightarrow$ **x + y + z = ?**

A) 3 B) 4 C) 5 D) 6 E) 7

Çözüm / Solution

Yanıt / Answer **C**

120	2
60	2
30	2
15	3
5	5
1	

$\Rightarrow 120 = 2^3 \cdot 3^1 \cdot 5^1$

$x = 3,\ y = 1,\ z = 1$

$x + y + z = 5$

Örnek / Example 26

$a, b \in Z^+$

$45 \cdot a = b^2 \Rightarrow$ **min (a + b) = ?**

A) 15 B) 17 C) 20 D) 23 E) 27

Çözüm / Solution

Yanıt / Answer **C**

45	3
15	3
5	5
1	

$\Rightarrow 3^2 \cdot 5^1 \cdot a = b^2$

$\min(a) = 5^1$

$\min(b) = 3 \cdot 5 = 15$

$\min(a + b) = 15 + 5 = 20$

ÖNEMLİ / IMPORTANT

• *Bir sayının pozitif tam sayı bölenlerinin sayısı n ise tam sayı bölenlerinin sayısı 2n'dir.*

• *If the number of positive integer divisors of a certain number is n, the number of its integer divisors is 2n*

Özellik / Property

★ Bir tam sayının pozitif tam sayı bölenlerinin sayısı:

x, y, z ∈ Z⁺, a, b, c birbirinden farklı asal sayılar olmak üzere

$A = a^x \cdot b^y \cdot c^z$ ise A sayısının pozitif tam sayı bölenlerinin sayısı $(x + 1)\cdot(y + 1)\cdot(z + 1)$ şeklinde bulunur.

★ The number of positive integer divisors of an integer:

x, y, z ∈ Z⁺, a, b, c are different prime numbers;

if $A = a^x \cdot b^y \cdot c^z$, The number of positive integer divisors of A will be calculated by

$(x + 1)\cdot(y + 1)\cdot(z + 1)$

Örnek / Example 27

600 sayısının pozitif tam sayı bölenleri kaç tanedir?

How many positive integer divisors does the number 600 have?

A) 21 B) 22 C) 23 D) 24 E) 25

Çözüm / Solution Yanıt / Answer **D**

600	2
300	2
150	2
75	3
25	5
5	5
1	

$\Rightarrow 600 = 2^3 \cdot 3^1 \cdot 5^2$ olduğundan

600 sayısının pozitif tam sayı bölenlerinin sayısı

$(3 + 1)\cdot(1 + 1)\cdot(2 + 1) = 24$ tür.

The number of positive integer divisors the number 600 is $(3 + 1)\cdot(1 + 1)\cdot(2 + 1) = 24$

ÖNEMLİ / IMPORTANT

• *Bir sayının tam sayı bölenlerinin toplamı 0 dır.*

• *Sum of integer divisors of a number is 0.*

Özellik / Property

★ Bir tam sayının pozitif tam sayı bölenlerinin toplamı:

m, n ∈ Z⁺, a ve b birbirinden farklı asal sayılar olmak üzere;

$A = a^m \cdot b^n$ ise A sayısının pozitif tam sayı bölenlerinin toplamı $\left(\frac{a^{m+1}-1}{a-1}\right)\cdot\left(\frac{b^{n+1}-1}{b-1}\right)$ dir.

★ Sum of positive integer divisors of an iteger:

m, n ∈ Z⁺, a and b are two different prime numbers; if $A = a^m \cdot b^n$, sum of positive integer divisors of A is $\left(\frac{a^{m+1}-1}{a-1}\right)\cdot\left(\frac{b^{n+1}-1}{b-1}\right)$

Örnek / Example 28

54 sayısının pozitif tam sayı bölenlerinin toplamı kaçtır?

What is sum of positive integer divisors of the number 54?

A) 8 B) 24 C) 72 D) 96 E) 120

Çözüm / Solution Yanıt / Answer **E**

54	2
27	3
9	3
3	3
1	

$\Rightarrow 54 = 2^1 \cdot 3^3 \Rightarrow \frac{2^{1+1}-1}{2-1}\cdot\frac{3^{3+1}-1}{3-1}$

$= 3 \cdot 40$

$= 120$

FAKTÖRİYEL / FACTORIAL

Tanım: 1'den n'ye kadar olan ardışık doğal sayıların çarpımına **n faktöriyel** denir ve **n!** şeklinde gösterilir.

Definition: For any positive integer n, n factorial represents the product of the first n positive integers and is shown by n!.

$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

$1! = 1$

$2! = 1 \cdot 2 = 2$

$3! = 1 \cdot 2 \cdot 3 = 6$

$4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$

$5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$

$6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720$

ÖNEMLİ / IMPORTANT

- *0! sayısı 1'e eşittir.*
- *0! is equal to 1.*

$0! = 1$

Örnek / Example 29

$\frac{(n+2)!}{n!} = 30 \Rightarrow$ **n = ?**

A) 4 B) 5 C) 6 D) 7 E) 8

Çözüm / Solution

Yanıt / Answer **A**

$$\frac{(n+2)!}{n!} = \frac{(n+2)\cdot(n+1)\cdot n!}{n!} = 30$$

$(n+1)\cdot(n+2) = 30$

$(n+1)\cdot(n+2) = 5\cdot 6$

$n+1 = 5$

$n = 4$

Örnek / Example 30

$\frac{n!}{(n-3)!} = 60 \Rightarrow$ **n = ?**

A) 4 B) 5 C) 6 D) 7 E) 8

Çözüm / Solution

Yanıt / Answer **B**

$$\frac{n!}{(n-3)!} = 60$$

$$\frac{n\cdot(n-1)\cdot(n-2)\cdot(n-3)!}{(n-3)!} = 60$$

$n\cdot(n-1)\cdot(n-2) = 60 = 5\cdot 4\cdot 3$

$n = 5$

PEKİŞTİRME / REINFORCE

✓ $\frac{(2-n)! + (n-2)!}{3! + 0!} = ?$

..... $\left[\frac{2}{7}\right]$

PEKİŞTİRME / REINFORCE

✓ $\frac{7! + 8! + 9!}{7!} = ?$

..... [81]

✓ $\frac{12!}{n!} = 132 \Rightarrow n = ?$

..... [10]

✓ $(n - 4)! + (4 - n)! = ?$

..... [2]

Örnek / Example 31

$\frac{8! - 7!}{8!} = ?$

A) $\frac{3}{4}$ B) $\frac{5}{6}$ C) $\frac{7}{8}$ D) $\frac{8}{9}$ E) $\frac{9}{10}$

Çözüm / Solution

Yanıt / Answer **C**

$$\frac{8! - 7!}{8!} = \frac{8 \cdot 7! - 7!}{8 \cdot 7!}$$

$$= \frac{7!(8 - 1)}{8 \cdot 7!}$$

$$= \frac{7}{8}$$

Örnek / Example 32

$\frac{n!}{110} = 9! \Rightarrow n = ?$

A) 7 B) 8 C) 9 D) 10 E) 11

Çözüm / Solution

Yanıt / Answer **E**

$$\frac{n!}{110} = 9! \Rightarrow n! = 110 \cdot 9!$$

$$n! = 11 \cdot 10 \cdot 9!$$

$$n! = 11!$$

$$n = 11$$

Örnek / Example 33

$\frac{(n + 1)! + n!}{n!} = ?$

A) $(n - 1)!$ B) $n!$ C) $2n$ D) $n + 1$ E) $n + 2$

Çözüm / Solution

Yanıt / Answer **E**

$$\frac{(n + 1)! + n!}{n!} = \frac{(n + 1) \cdot n! + n!}{n!}$$

$$= \frac{n!\,(n + 2)}{n!}$$

$$= n + 2$$

Örnek / Example 34

A, x ∈ N,

$26! = A \cdot 3^x \Rightarrow$ **max(x) = ?**

A) 8 B) 10 C) 12 D) 14 E) 16

Çözüm / Solution

Yanıt / Answer **B**

26 | 3
8 | 3
2

$\Rightarrow$ max(x) = 8 + 2 = 10

Örnek / Example 35

A, x ∈ N,

$20! = A \cdot 4^x \Rightarrow$ **max(x) = ?**

A) 6 B) 7 C) 8 D) 9 E) 10

Çözüm / Solution

Yanıt / Answer **D**

$20! = A \cdot 4^x$

$20! = A \cdot 2^{2x}$

20 | 2
10 | 2
5 | 2
2 | 2
1

$\Rightarrow$ max(2x) = 10 + 5 + 2 + 1 = 18
$\Rightarrow$ max(x) = 9

Örnek / Example 36

A, x ∈ N,

$32! = A \cdot 10^x \Rightarrow$ **max(x) = ?**

A) 5 B) 6 C) 7 D) 8 E) 9

Çözüm / Solution

Yanıt / Answer **C**

$32! = A \cdot 10^x$

2 5

32 | 5
6 | 5
1

$\Rightarrow$ max(x) = 6 + 1 = 7

PEKİŞTİRME / REINFORCE

✓ $27! = A \cdot 6^x \Rightarrow$ ***max(x) = ?***

..... [13]

✓ $25! = A \cdot 12^x \Rightarrow$ ***max(x) = ?***

..... [10]

PEKİŞTİRME / REINFORCE

✓ $29! = A \cdot 4^x \Rightarrow$ ***max(x) = ?***

..... [12]

✓ $29! + 28! = A \cdot 10^x \Rightarrow$ ***max(x) = ?***

..... [7]

✓ $27! - 26! = A \cdot 5^x \Rightarrow$ ***max(x) = ?***

..... [6]

DOĞAL SAYILARDA BÖLME
DIVISION in the SET of NATURAL NUMBERS

Tanım / Definition: A, B, C, D ∈ N, B ≠ 0

$$\begin{array}{r|l} A & B \\ \hline - & C \\ \hline D & \end{array}$$

A → Bölünen (Dividend)
B → Bölen (Divisor)
C → Bölüm (Quotient)
D → Kalan (Remainder)

▼ $A = B \cdot C + D$

▼ $D < B$

Örneğin / For example:

$$\begin{array}{r|l} 421 & 12 \\ \hline -\ 36 & \\ \hline 61 & 35 \\ -\ 60 & \\ \hline 1 & \end{array}$$

Örnek / Example 37

$$\begin{array}{r|l} A & 8 \\ \hline - & 5 \\ \hline 4 & \end{array} \Rightarrow \begin{array}{r|l} A & 11 \\ \hline - & \\ \hline ? & \end{array}$$

A) 0 B) 1 C) 2 D) 3 E) 4

Çözüm / Solution

Yanıt / Answer **A**

$A = 8 \cdot 5 + 4 = 44$

$$\begin{array}{r|l} 44 & 11 \\ \hline -\ 44 & \\ \hline 0 & 4 \end{array}$$

Örnek / Example 38

$$\begin{array}{r|l} A & B \\ \hline - & 6 \\ \hline 5 & \end{array} \Rightarrow \textbf{min(A) = ?}$$

A) 35 B) 40 C) 41 D) 42 E) 43

Çözüm / Solution

Yanıt / Answer **C**

$$\left.\begin{array}{l} A = 6 \cdot B + 5 \\ B > 5 \end{array}\right\} \Rightarrow \min(A) = 6 \cdot 6 + 5 = 41$$

Örnek / Example 39

$A \in N$

$$\begin{array}{r|l} A & 6 \\ \hline - & 7 \\ \hline x & \end{array} \quad \Rightarrow \max(A) = ?$$

A) 44 B) 45 C) 46 D) 47 E) 48

Çözüm / Solution Yanıt / Answer **D**

$A = 6 \cdot 7 + x$

$x < 6 \Rightarrow \max(x) = 5$

$A = 6 \cdot 7 + 5 = 42 + 5 = 47 \Rightarrow \max(A) = 47$

TAM BÖLÜNEBİLME KURALLARI / DIVISIBILITY RULES

2 ile TAM BÖLÜNEBİLME / DIVISIBILITY by 2

Bir doğal sayının 2 ile tam bölünebilmesi için sayı çift olmalıdır. Birler basamağındaki rakam çift ise kalan 0, tek ise kalan 1'dir.

The number must be even to be divisible by 2. If the ones-place of the numbers is even, the remainder is 0, and if the ones-place of the number is odd, the remainder is 1.

Örnek / Example 40

$$\begin{array}{r|l} 342x & 2 \\ \hline - & \\ \hline 0 & \end{array} \quad \Rightarrow \sum x = ?$$

A) 16 B) 18 C) 20 D) 22 E) 24

Çözüm / Solution Yanıt / Answer **C**

$\sum x = 0 + 2 + 4 + 6 + 8 = 20$

ÖNEMLİ / IMPORTANT

• *Bir sayının rakamları toplamının 3 ile bölümünden kalan, o sayının 3 ile bölümünden kalana eşittir.*

• *The remainder of the sum of digits of a number when it is divided by 3 is equal to the remainder of that number, when it is divided by 3.*

3 ile TAM BÖLÜNEBİLME / DIVISIBILITY by 3

Bir doğal sayının 3 ile tam bölünebilmesi için bu sayının rakamları toplamı 3'ün katları olmalıdır.

The sum of the digits of a number must be multiple of 3 in order to the number itself be divisible by 3.

Örnek / Example 41

$$\begin{array}{r|l} 124x & 3 \\ \hline - & \\ \hline 0 & \end{array} \Rightarrow \sum x = ?$$

A) 6 B) 9 C) 12 D) 15 E) 20

Çözüm / Solution Yanıt / Answer **D**

$1 + 2 + 4 + x = 3k \quad (k \in Z)$

$7 + x = 3k$

$x = 2, 5, 8 \Rightarrow x = 2 + 5 + 8 = 15$

ÖNEMLİ / IMPORTANT

• *Bir sayının 4 ile bölümünden kalan, sayının son iki basamağının 4 ile bölümünden kalana eşittir.*

• *The remainder of the number when it is divided by 4 is equal to the number when the last two digits are divided by 4.*

4 ile TAM BÖLÜNEBİLME / DIVISIBILITY by 4

Bir doğal sayının 4 ile tam bölünebilmesi için, sayının son iki basamağının 4 ile tam bölünebilmesi gerekir.

The last two digits of the number must be divisible by 4 in order to the number itself be divisible by 4.

Örnek / Example 42

$$\begin{array}{r|l} 123x4 & 4 \\ \hline - & \\ \hline 0 & \end{array} \Rightarrow \sum x = ?$$

A) 18 B) 20 C) 22 D) 24 E) 26

Çözüm / Solution Yanıt / Answer **B**

123x4

0 2 4 6 8

$\Rightarrow \sum x = 0 + 2 + 4 + 6 + 8 = 20$

5 ile TAM BÖLÜNEBİLME / DIVISIBILITY by 5

Bir doğal sayının 5 ile tam bölünebilmesi için sayının birler basamağı, 0 ya da 5 olmalıdır.

The units digit of the number must be 0 or 5 to be divisible by 5.

ÖNEMLİ / IMPORTANT

- *Bir doğal sayının 5 ile bölümünden kalan, sayının birler basamağındaki rakamın 5 ile bölümünden kalana eşittir.*
- *The remainder of the number when it is divided by 5 is equal to the remainder, when the units digit of the number is divided by 5.*

Örnek / Example 43

$$\begin{array}{r|l} 973x & 5 \\ \hline - & \\ \hline 3 & \end{array} \Rightarrow \sum x = ?$$

A) 3 B) 6 C) 8 D) 11 E) 13

Çözüm / Solution

Yanıt / Answer **D**

$x = 5k + 3$

$k = 0 \Rightarrow x = 3$

$k = 1 \Rightarrow x = 8$

$\Rightarrow \sum x = 3 + 8 = 11$

9 ile TAM BÖLÜNEBİLME / DIVISIBILITY by 9

Bir doğal sayının 9 ile tam bölünebilmesi için, bu sayının rakamlarının toplamı 9'un katları olmalıdır.

The sum of the digits of the number must be multiple of 9 in order to the number itself be divisible by 9.

ÖNEMLİ / IMPORTANT

- *Bir sayının 9 ile bölümünden kalan, o sayının rakamları toplamının 9 ile bölümünden kalana eşittir.*
- *The remainder of the number when it is divided by 9 is equal to the remainder, when the sum of the digits of the number is divided by 9.*

Örnek / Example 44

$$\begin{array}{r|l} 34x7 & 9 \\ \hline - & \\ \hline 3 & \end{array} \Rightarrow x = ?$$

A) 3 B) 5 C) 6 D) 7 E) 9

Çözüm / Solution

Yanıt / Answer **D**

$3 + 4 + x + 7 = 9k + 3$

$11 + x = 9k \Rightarrow x = 7$

ÖNEMLİ / IMPORTANT

Verilen bir doğal sayının;

- *6 ile bölünebilmesi için 2 ve 3'e*
- *15 ile bölünebilmesi için 3 ve 5'e*
- *36 ile bölünebilmesi için 4 ve 9'a*
- *45 ile bölünebilmesi için 5 ve 9'a tam bölünebilmesi gerekir.*

A natural number;

- *must be divisible by 2 and 3 to be divisible by 6*
- *must be divisible by 3 and 5 to be divisible by 15*
- *must be divisible by 4 and 9 to be divisible by 36*
- *must be divisible by 5 and 9 to be divisible by 45*

11 ile TAM BÖLÜNEBİLME / DIVISIBILITY by 11

Bir doğal sayının 11 ile tam bölünebilmesi için sayının birler basamağından başlayarak her bir rakam sağdan sola doğru sırasıyla "+ – + – + – ...." işaretleriyle işaretlenir. Daha sonra (+) işaretliler ile (–) işaretliler toplanarak farkları alınır ve fark 11'in katı ise bu sayı 11 ile tam bölünür.

In order to a number be divisible by 11, the digits of the number, starting from the units digit, from left to right get signs "+ – + – + – ...." respectively; afterwars, the positive and negative numbers sum up to find their difference, if the difference is a multiple of 11, then this number is divisible by 11.

Örnek / Example 45

$$\begin{array}{r|l} 9458761 & 11 \\ \hline - & \\ \hline ? & \end{array}$$

A) 4 B) 6 C) 8 D) 11 E) 13

Çözüm / Solution Yanıt / Answer **A**

9 4 5 8 7 6 1
↓ ↓ ↓ ↓ ↓ ↓ ↓
+ – + – + – +

$(1 + 7 + 5 + 9) - (6 + 8 + 4) = 22 - 18 = 4$

Örnek / Example 46

$$\begin{array}{r|l} 3572x41 & 11 \\ \hline - & \\ \hline 3 & \end{array} \quad \Rightarrow \quad x = ?$$

A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution Yanıt / Answer **C**

3 5 7 2 x 4 1
↓ ↓ ↓ ↓ ↓ ↓ ↓
+ – + – + – +

$(3 + 7 + x + 1) - (5 + 2 + 4) = 11k + 3$

$11 + x - 11 = 11k + 3$

$x = 11k + 3$

$x = 3$

OBEB (ORTAK BÖLENLERİN EN BÜYÜĞÜ)
GCF (GREATEST COMMON FACTOR)

Tanım: İki ya da daha fazla doğal sayıyı ortak olarak tam bölebilen, en büyük doğal sayıya bu sayıların **ortak bölenlerinin en büyüğü** (OBEB) denir.

Definition: The greatest number, which divides two or more than two numbers commonly, is called GCF(Greatest Common Factor).

ÖNEMLİ / IMPORTANT

- *a ve b aralarında asal ise,*
  OBEB(a, b) = 1'dir.
- *x, a, b, c, d ∈ Z*
  x · a = c , x · b = d
  ⇒ max(x) = OBEB(c, d)
- *If a and b are relatively prime, then GCF(a,b) = 1.*
- *x, a, b, c, d ∈ Z*
  x · a = c , x · b = d
  ⇒ max(x) = GCF(c, d)

Örnek / Example 47

OBEB / GCF **(18, 24) = ?**

A) 6 B) 8 C) 9 D) 12 E) 16

Çözüm / Solution

Yanıt / Answer **A**

18	24	② → Her iki sayıyı da böldü (Divides both numbers.)
9	12	2
9	6	2
9	3	③ → Her iki sayıyı da böldü (Divides both numbers.)
3	1	3
1		

⇒ OBEB/GCF(18, 24) = 2·3 = 6

Örnek / Example 48

a, b, c ∈ N^+,

$\left.\begin{matrix} a \cdot b = 96 \\ a \cdot c = 72 \end{matrix}\right\}$ ⇒ **max(a) = ?**

A) 6 B) 9 C) 12 D) 18 E) 24

Çözüm / Solution

Yanıt / Answer **E**

$\left.\begin{matrix} a \cdot b = 96 \\ a \cdot c = 72 \end{matrix}\right\}$ ⇒ max(a) = OBEB / GCF (72,96)

72	96	②
36	48	②
18	24	②
9	12	2
9	6	2
9	3	③
3	1	3
1		

⇒ max(a) = OBEB/GCF(72, 96) = 2·2·2·3 = 24

ÖNEMLİ / IMPORTANT

- *a ve b aralarında asal ise,*
  OKEK(a, b) = a · b'dir.
- *a, b ∈ N⁺*
  OKEK(a, b) · OBEB(a, b) = a · b
- *If a and b are relatively prime, then*
  LCM(a, b) = a · b
- *a, b ∈ N⁺*
  LCM(a, b) · GCF(a, b) = a · b

OKEK (ORTAK KATLARIN EN KÜÇÜĞÜ) LCM (LEAST COMMON MULTIPLE)

Tanım: İki ya da daha fazla doğal sayının ortak katlarının en küçüğüne o sayıların **ortak katlarının en küçüğü (OKEK)** denir.

Definition: The least common multiple of two or more than two numbers is called **LCM(Least Common Multiple)** of those numbers.

Örnek / Example 49

OKEK / LCM (18, 24) = ?

A) 72 B) 64 C) 56 D) 48 E) 36

Çözüm / Solution

Yanıt / Answer **A**

18	24	2
9	12	2
9	6	2
9	3	3
3	1	3
1		

Bu sayıların çarpımı OKEK'i verir. (Multiplication of those numbers gives LCM.)

⇒ OKEK/LCM (18, 24) = 2·2·2·3·3 = 72

Örnek / Example 50

$A \in Z^+$,

$A = 3a + 1 = 4b + 2 = 5c + 3 \Rightarrow$ **min(A) = ?**

A) 56 B) 58 C) 60 D) 62 E) 64

Çözüm / Solution

Yanıt / Answer **B**

$A = 3a + 1 = 4b + 2 = 5c + 3$

$A + 2 = 3a + 3 = 4b + 4 = 5c + 5$

$A + 2 = 3(a + 1) = 4(b + 1) = 5(c + 1)$

$\Rightarrow$ min(A + 2) = OKEK/LCM(3, 4, 5)

$A + 2 = 60$

$A = 58$

Örnek / Example 51

$\frac{4}{9}$, $\frac{2}{7}$ ve $\frac{9}{11}$ kesirlerine tam bölünebilen en küçük doğal sayı kaçtır?

What is the smallest natural nimber which can be divided by the factors $\frac{4}{9}$, $\frac{2}{7}$ and $\frac{9}{11}$?

A) 18 B) 20 C) 26 D) 32 E) 36

Çözüm / Solution

Yanıt / Answer **E**

$$\text{OKEK / LCM}\left(\frac{4}{9}, \frac{2}{7}, \frac{9}{11}\right) = \frac{\text{OKEK / LCM}(4, 2, 9)}{\text{OBEB / GCF}(9, 7, 11)} = \frac{36}{1} = 36$$

Örnek / Example 52

$a, b \in N$,

OKEK / LCM$(a, b) = 72$

OBEB / GCF$(a, b) = 6$

$\Rightarrow$ **a + b = ?**

A) 40 B) 42 C) 44 D) 46 E) 48

Çözüm / Solution

Yanıt / Answer **B**

$a = 6x$, $b = 6y$

$a \cdot b = 6x \cdot 6y = 6 \cdot 72$

$6 \cdot x \cdot y = 72 \Rightarrow x \cdot y = 12$

$\Rightarrow x = 3, y = 4$

$\left.\begin{matrix} a = 6 \cdot 3 = 18 \\ b = 6 \cdot 4 = 24 \end{matrix}\right\} \Rightarrow a + b = 18 + 24 = 42$

1. $(a - c + b) - (a + b - c) = ?$

A) 2a B) b C) 0

D) 1 E) c

2. $a - (b + 1) - [2 - (a + 1)] + b - 2a = ?$

A) – 2 B) – 1 C) 0

D) 1 E) 2

3. $AB + BA = 66$

$AB - BA = 36 \Rightarrow A \cdot B = ?$

A) 3 B) 4 C) 5

D) 6 E) 7

4. $(342)_5 = (x)_{10} \Rightarrow x = ?$

A) 95 B) 97 C) 99

D) 101 E) 103

5. $(63) = (x)_5 \Rightarrow x = ?$

A) 221 B) 224 C) 216

D) 223 E) 213

6. $1 + 2 + 3 + 4 + ... + 35 = ?$

A) 630 B) 650 C) 670

D) 680 E) 690

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7. $2 + 4 + 6 + \ldots + 24 = ?$

A) 144 B) 156 C) 158 D) 160 E) 162

8. $15! - a \cdot 13! = 105 \cdot 13! \Rightarrow a = ?$

A) 82 B) 90 C) 100 D) 105 E) 130

9. $\frac{10! - 5 \cdot 9!}{45 \cdot 8!} = ?$

A) 15 B) 5 C) 3 D) 2 E) 1

10. $4x17 \mid 3$, $= __ \mid$, 1 $\Rightarrow \sum x = ?$

A) 10 B) 11 C) 12 D) 13 E) 14

11. **[OKEK(LCM) (12, 15)] · [OBEB(GCF) (12,15)] = ?**

A) 120 B) 160 C) 180 D) 200 E) 240

12. $A = 4m + 1 = 5n + 1 = 7k + 1$,

$250 < A < 300 \Rightarrow A = ?$

A) 279 B) 280 C) 281 D) 287 E) 293

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
C	A	C	B	D	A	B	D	E	C	C	C

1. $(a - c + b) - (a + b - c)$

$= a - c + b - a - b + c$

$= 0$

Yanıt / The answer is C

---

2. $a-(b+1)-[2-(a+1)]+b-2a$

$= a - b - 1 - [2 - a - 1] + b - 2a$

$= a - 1 - (1 - a) - 2a$

$= a - 1 - 1 + a - 2a$

$= -2$

Yanıt / The answer is A

---

3. $AB + BA = 66$

$+\ AB - BA = 36$

$2 \cdot (AB) = 102$

$AB = 51$

$\Rightarrow A = 5, B = 1 \Rightarrow A \cdot B = 5$

Yanıt / The answer is C

---

4. $(342)_5 = (x)_{10} \Rightarrow x = 2 \cdot 5^0 + 4 \cdot 5^1 + 3 \cdot 5^2$

$x = 2 + 20 + 75 = 97$

Yanıt / The answer is B

5.

63 | 5		
12 | 5		$\Rightarrow x = 223$
③	②	
②		

Yanıt / The answer is D

---

6. $1 + 2 + 3 + 4 + \ldots + 35 = \dfrac{35 \cdot 36}{2}$

$= 35 \cdot 18$

$= 630$

Yanıt / The answer is A

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7. $2 + 4 + 6 + \ldots + 24 \Rightarrow 2n = 24$

$n = 12$

$2 + 4 + 6 + \ldots + 24 = n \cdot (n + 1)$

$= 12 \cdot 13$

$= 156$

Yanıt / The answer is B

8. $15! - a \cdot 13! = 105 \cdot 13!$

$15 \cdot 14 \cdot 13! - a \cdot 13! = 105 \cdot 13!$

$13 \cdot (14 \cdot 15 - a) = 105 \cdot 13!$

$210 - a = 105$

$a = 105$

Yanıt / The answer is D

9. $\frac{10! - 5 \cdot 9!}{45 \cdot 8!} = \frac{10 \cdot 9 \cdot 8! - 5 \cdot 9 \cdot 8!}{45 \cdot 8!}$

$= \frac{8! \cdot (9 \cdot 10 - 5 \cdot 9)}{8! \cdot 45}$

$= \frac{45}{45}$

$= 1$

Yanıt / The answer is E

10. $4 + x + 1 + 7 = 3 \cdot k + 1$

$12 + x = 3k + 1$

$\Rightarrow x = 1 + 4 + 7 = 12$

Yanıt / The answer is C

11. $[\text{OKEK(LCM) } (12, 15)] \cdot [\text{OBEB(GCF) } (12,15)] = 12 \cdot 15$

$= 180$

Yanıt / The answer is C

12. $\min(A) = \text{OKEK(LCM) } (4, 5, 7) + 1$

$= 140 + 1$

$250 < A < 300 \Rightarrow A = 140 \cdot 2 + 1$

$= 281$

Yanıt / The answer is C

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $a, b, c \in Z^+$

$4a + 5b + 3c = 87$

$\Rightarrow$ **max(a) = ?**

A) 22 B) 19 C) 21 D) 18 E) 20

[GAZİANTEP ÜNİVERSİTESİ – YÖS 2020]

---

2. **$2020 - 2018 + 2016 - 2014 + \ldots - 14 = ?$**

A) 1006 B) 1004 C) 1002 D) 1000 E) 998

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2020]

---

3. $\frac{n! + (n+1)!}{(n+2)!} = \frac{1}{7}$

$\Rightarrow$ **n = ?**

A) 3 B) 4 C) 5 D) 6 E) 7

[ULUDAĞ ÜNİVERSİTESİ – YÖS 2019]

---

4. AB, BA $\in$ N,

$AB - BA = 54 \Rightarrow$ **$2 \cdot (A - B) = ?$**

A) 3 B) 6 C) 9 D) 12 E) 18

[DÜZCE ÜNİVERSİTESİ – YÖS 2019]

---

5. **Aşağıdakilerden hangisi tek sayıdır?**

Which of the following is an odd number?

A) $2^{24} + 7!$ B) $63! - 3^{10}$ C) $0! + 5^{20}$

D) $4^7 \cdot 3^5 - 8^{14}$ E) $11! - 9!$

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2018]

---

6. $(301)_4 = (122)_4 + (1ab)_4$ $\Rightarrow$ **a + b = ?**

A) 2 B) 3 C) 4 D) 5 E) 6

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2018]

---

7. $\frac{(n+1)!}{(n-1)!} = 420 \Rightarrow$ **n = ?**

A) 16 B) 20 C) 24 D) 28 E) 32

[İNÖNÜ ÜNİVERSİTESİ – YÖS 2018]

---

8. A, B, C birer rakam / A, B and C are digits

$$\begin{array}{r} BAC \\ -\ ACB \\ \hline 279 \end{array} \Rightarrow \textbf{min(A + B + C) = ?}$$

A) 11 B) 7 C) 9 D) 13 E) 8

[FIRAT ÜNİVERSİTESİ – YÖS 2018]

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $a, b, c \in Z$

$$\left.\begin{array}{l} a^3 \cdot b^2 < 0 \\ b^3 \cdot c^3 > 0 \\ c^5 \cdot a^4 < 0 \end{array}\right\} \Rightarrow \textbf{a,b,c = ?,?,?}$$

A) −,−,− B) −,−,+ C) +,+,+

D) +,+,− E) +,−,−

[SELÇUK ÜNİVERSİTESİ – YÖS 2017]

10.

$$\begin{array}{r} KLM \\ \times\ \ 8N \\ \hline \bullet\bullet\bullet \\ +\ 1968 \\ \hline 20418 \end{array} \Rightarrow \textbf{N = ?}$$

A) 2 B) 3 C) 4 D) 6 E) 8

[ANADOLU ÜNİVERSİTESİ – YÖS 2017]

---

11. Aşağıdaki bölüm işleminde A sayısı en çok kaçtır?

What is the greatest A number in division below?

$$\begin{array}{r|l} A & 20 \\ \hline - & x \\ \hline x^2 & \end{array}$$

A) 96 B) 98 C) 100 D) 110 E) 120

[EGE ÜNİVERSİTESİ – YÖS 2016]

---

12. $\dfrac{16! + 17!}{18!} = ?$

A) $\frac{1}{9}$ B) $\frac{1}{14}$ C) $\frac{1}{15}$ D) $\frac{1}{16}$ E) $\frac{1}{17}$

[EGE ÜNİVERSİTESİ – YÖS 2016]

METROPOL YAYINLARI • METROPOL PUBLICATIONS

13. $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$

$$\frac{n! \cdot (n-3)!}{(n-2)! \cdot (n+1)!} = ?$$

A) $\frac{n-1}{n+2}$ B) $\frac{n+1}{n-2}$ C) $\frac{n}{(n+1)}$

D) $\frac{1}{(n+1)\cdot(n-2)}$ E) $\frac{n-3}{n^2-2}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2015]

---

14. $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$

$$\frac{n! + (n+1)!}{(n+2)!} = ?$$

A) $\frac{n-1}{n+1}$ B) $\frac{n}{n+2}$ C) $\frac{1}{(n+1)}$

D) $\frac{n!}{(n+1)}$ E) $\frac{n+1}{n}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2014]

---

15. $n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots \cdot (n-1) \cdot n$

$$\frac{5! - 4!}{5! + 4!} = ?$$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) $1\frac{1}{3}$ E) $1\frac{1}{2}$

[ONSEKİZ MART ÜNİVERSİTESİ – YÖS 2014]

---

16. $\dfrac{(n+1)! + n!}{(n-1)!} = 35 \Rightarrow \textbf{n = ?}$

A) 3 B) 5 C) 7

D) 9 E) 11

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2014]

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	B	D	D	B	C	B	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	B	A	E	D	C	B	B

1. $\left.\begin{matrix} c = 2 \\ b = 1 \end{matrix}\right\} \Rightarrow 4a + 5 + 6 = 87$

$4a = 76$

$a = 19$

Yanıt / The Answer is B

---

2. $2020 - 2018 + 2016 - 2014 + \ldots - 14 = x$

$\underbrace{(2020 - 2018)}_{2} + \underbrace{(2016 - 2014)}_{2} + \ldots + \underbrace{(16 - 14)}_{2} = x$

$\left(\frac{2020 - 16}{4} + 1\right) \cdot 2 = x$

$502 \cdot 2 = x$

$1004 = x$

Yanıt / The Answer is B

---

3. $\frac{n! + (n+1)!}{(n+2)!} = \frac{1}{7} \Rightarrow \frac{n! + (n+1) \cdot n!}{(n+2) \cdot (n+1) \cdot n!} = \frac{1}{7}$

$\frac{n! \cdot (n+2)}{(n+2) \cdot (n+1) \cdot n!} = \frac{1}{7}$

$\frac{1}{n+1} = \frac{1}{7}$

$n = 6$

Yanıt / The Answer is D

---

4. $AB - BA = 54$

$10A + B - 10B - A = 54$

$9A - 9B = 54$

$A - B = 6 \Rightarrow 2 \cdot (A - B) = 12$

Yanıt / The Answer is D

---

5. $\underbrace{63!}_{Ç} - \underbrace{3^{10}}_{T} = T$

$E - O = O$

Yanıt / The Answer is B

---

6. $\begin{matrix} (301)_4 \\ -\ (122)_4 \\ \hline (113)_4 \end{matrix}$

$(113)_4 = (1ab)_4$

$a = 1,\ b = 3 \Rightarrow a + b = 4$

Yanıt / The Answer is C

---

7. $\frac{(n+1)!}{(n-1)!} = 420$

$\frac{(n+1) \cdot n \cdot (n-1)!}{(n-1)!} = 420 \Rightarrow n \cdot (n+1) = 420$

$\downarrow$

20

Yanıt / The Answer is B

---

8. $BAC - ACB = 279$

$99B - 90A - 9C = 279$

$11B - 10A - C = 31$

$\downarrow \quad \downarrow \quad \downarrow$

$6 + 3 + 5 = 14$

$5 + 2 + 4 = 11$

$4 + 1 + 3 = 8$

$\Rightarrow \min(A + B + C) = 8$

Yanıt / The Answer is E

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9. $a^3 \cdot b^2 < 0, \quad b^2 > 0 \Rightarrow a < 0$

$c^5 \cdot a^4 < 0, \quad a^4 > 0 \Rightarrow c < 0$

$b^3 \cdot c^3 > 0, \quad c^3 < 0 \Rightarrow b < 0$

$\Rightarrow$ a, b, c = –, –, –

Yanıt / The Answer is **A**

10.

$$\begin{array}{r} KLM \\ \times \quad 8N \\ \hline \bullet\bullet\bullet \\ +\,1968 \\ \hline 20418 \end{array}$$

$$\begin{array}{r|l} 1968 & 8 \\ \hline 16 & 246 = KLM \\ \hline 36 & M = 6,\ N = 3 \\ 32 & \\ \hline 48 & \\ 48 & \\ \hline 0 & \end{array}$$

Yanıt / The Answer is **B**

---

11. $A = 20 \cdot x + x^2$

$x^2 < 20$

$\max(A) \Rightarrow x = 4$

$\max(A) = 20 \cdot 4 + 4^2$

$= 96$

Yanıt / The Answer is **A**

---

12. $\dfrac{16! \cdot (1 + 17)}{18 \cdot 17 \cdot 16!} =$

$\dfrac{18}{18 \cdot 17} = \dfrac{1}{17}$

Yanıt / The Answer is **E**

---

13. $\dfrac{n! \cdot (n-3)!}{(n-2)! \cdot (n+1)!} = \dfrac{n! \cdot (n-3)!}{(n-2) \cdot (n-3)! \cdot (n+1) \cdot n!}$

$= \dfrac{1}{(n-2) \cdot (n+1)}$

Yanıt / The Answer is **D**

---

14. $\dfrac{n! + (n+1)!}{(n+2)!} = \dfrac{n! + (n+1) \cdot n!}{(n+2) \cdot (n+1) \cdot n!}$

$= \dfrac{n! \cdot (1 + n + 1)}{(n+2) \cdot (n+1) \cdot n!}$

$= \dfrac{n! \cdot (n+2)}{n! \cdot (n+2) \cdot (n+1)}$

$= \dfrac{1}{n+1}$

Yanıt / The Answer is **C**

---

15. $\dfrac{5! - 4!}{5! + 4!} = \dfrac{5 \cdot 4! - 4!}{5 \cdot 4! + 4!}$

$= \dfrac{4! \cdot (5-1)}{4! \cdot (5+1)}$

$= \dfrac{4}{6}$

$= \dfrac{2}{3}$

Yanıt / The Answer is **B**

---

16. $\dfrac{(n+1) \cdot n \cdot (n-1)! + n \cdot (n-1)!}{(n-1)!} = 35$

$\dfrac{(n-1)! \cdot [(n+1) \cdot n + n]}{(n-1)!} = 35$

$(n+1) \cdot n + n = 35$

$n(n+1+1) = 35$

$n(n+2) = 5 \cdot 7 \Rightarrow n = 5$

Yanıt / The Answer is **B**

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $a,b,c \in N^+$

$a - b = 5$

$a - c = 12 \Rightarrow$ **min(a + b + c) = ?**

A) 17 B) 18 C) 22 D) 24 E) 25

2. $x, y \in Z^+$

$3x + 2y = 18 \Rightarrow \sum x = ?$

A) 4 B) 5 C) 6 D) 7 E) 8

3. $a, x \in Z$

$ax - 5 = 4a - 13 \Rightarrow \sum x = ?$

A) 5 B) 7 C) 12 D) 14 E) 32

4. x, y birer rakam / x and y are digits

$x = 4 + \frac{10}{y} \Rightarrow \sum x = ?$

A) 10 B) 11 C) 13 D) 15 E) 17

5. x, y, z birer rakam / x, y and z are digits

$x \cdot y = 30$

$y \cdot z = 42$

$\Rightarrow$ **x + y + z = ?**

A) 16 B) 17 C) 18 D) 19 E) 20

6. **(–7 + 10 : 2 + 3) – (–5 + 8 · 2 – 1) = ?**

A) –11 B) –9 C) –5 D) 10 E) 5

7. **1 – [(16 : 2) · (–1) + 8 : 4 + 1)] = ?**

A) –6 B) –5 C) 1 D) 5 E) 6

8. **81 : 9 + 3 – 18 : 2 = ?**

A) 0 B) 1 C) 2 D) 3 E) 4

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9. $12 : 4 + 3 \cdot (6 : 2 + 1) - 1 = ?$

A) 10 B) 11 C) 12 D) 13 E) 14

10. $76 - 16 : 4 + 3 \cdot 6 = ?$

A) 86 B) 90 C) 94 D) 98 E) 100

11. $[12 \cdot (5 - 3)] - 12 : 2 + 1 = ?$

A) 17 B) 18 C) 19 D) 20 E) 21

12. $(56 : 8) \cdot 2 - 16 : (21 : 7 + 1) = ?$

A) 10 B) 12 C) 14 D) 16 E) 18

13. $-9 : 3 + 2 \cdot 5 + 16 : (-4) - 7 = ?$

A) 14 B) 3 C) 4 D) -3 E) -4

14. $A, B \in N^+$,

$A + B = 13 \Rightarrow \max(A \cdot B) + \min(A \cdot B) = ?$

A) 50 B) 51 C) 52 D) 53 E) 54

15. $A, B \in N$,

$A \cdot B = 72 \Rightarrow \max(A + B) + \min(A + B) = ?$

A) 60 B) 70 C) 80 D) 90 E) 100

16. $A, B \in N$,

$A + B = 17 \Rightarrow \max(A \cdot B) + \min(A \cdot B) = ?$

A) 27 B) 36 C) 48 D) 72 E) 81

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	C	E	D	C	B	E	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	B	C	A	E	E	D	D

1. $(abc) \in N^+$,

$a \neq b \neq c$

$a = b \cdot c \Rightarrow$ **max(abc) = ?**

A) 842 B) 933 C) 991

D) 632 E) 701

2. $AA, BB \in N$

$AA + BB = 121 \Rightarrow$ **A + B = ?**

A) 7 B) 8 C) 9

D) 10 E) 11

3. $AB, BA \in N$

$AB + BA = 77 \Rightarrow$ **A + B = ?**

A) 5 B) 6 C) 7

D) 8 E) 9

4. $AB, BA \in N$

$AB - BA = 36 \Rightarrow$ **A − B = ?**

A) 8 B) 7 C) 6

D) 5 E) 4

5. $ABC, BCA, CAB \in N$

$ABC + BCA + CAB = 1443 \Rightarrow$ **A + B + C = ?**

A) 12 B) 13 C) 14

D) 15 E) 16

6. $AB, BC, CA \in N$

$AB + BC + CA = 88 \Rightarrow$ **max(ABC) = ?**

A) 800 B) 710 C) 611

D) 116 E) 107

7. $ABC, CBA \in N$

$ABC - CBA = 198 \Rightarrow$ **max(ABC) = ?**

A) 997 B) 987 C) 977

D) 947 E) 957

8. $AB, BA \in N$

$AB - BA = 54 \Rightarrow$ **max(AB) + min(AB) = ?**

A) 160 B) 161 C) 162

D) 163 E) 164

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9. AB, BA ∈ N

$$\left.\begin{matrix} AB + CA = 21 \\ BA - CA = 23 \end{matrix}\right\} \Rightarrow \textbf{min(AB) = ?}$$

A) 10 B) 13 C) 22 D) 31 E) 40

10. AB, BC, CA ∈ N,

$$\frac{AB + BC + CA}{AA + BB + CC} = x \Rightarrow \textbf{x = ?}$$

A) 1 B) 2 C) 3 D) 4 E) 5

11. AB, BA ∈ N,

$$\frac{AB - BA}{AB + BA} = \frac{18}{55} \Rightarrow \textbf{BA = ?}$$

A) 17 B) 27 C) 37 D) 47 E) 57

12. XX, YY ∈ N,

(XX)·(YY) = 847 ⇒ **X + Y = ?**

A) 4 B) 5 C) 6 D) 7 E) 8

13. A, B ∈ N⁺,

(AA)·(BB) = 1452 ⇒ **max(A + B) − min(A + B) = ?**

A) 1 B) 4 C) 7 D) 11 E) 15

14. 5xy1, 1yx5 ∈ N,

max(5xy1 − 1yx5) = ?

A) 3806 B) 9906 C) 3916 D) 4806 E) 4716

15. ABCD ∈ N, A ≠ B ≠ C ≠ D

A·B = C·D ⇒ **max(ABCD) = ?**

A) 9466 B) 9263 C) 8142 D) 8364 E) 8244

16. ab, ba ∈ N,

ab + 2 = 3·(ba) ⇒ **ab = ?**

A) 71 B) 74 C) 78 D) 82 E) 86

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	E	C	E	B	C	A	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	A	C	E	A	D	B	D

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1. $(243)_5 = (x)_{10} \Rightarrow \mathbf{x = ?}$

A) 73 B) 82 C) 88 D) 93 E) 97

2. $(ba2)_5 = (233)_a \Rightarrow \mathbf{b = ?}$

A) 0 B) 1 C) 2 D) 3 E) 4

3. $(1a3)_4 = (102)_5 \Rightarrow \mathbf{a = ?}$

A) 0 B) 1 C) 2 D) 3 E) 4

4. $(a312)_4 = (182)_{10} \Rightarrow \mathbf{a = ?}$

A) 0 B) 1 C) 2 D) 3 E) 4

5. $(102)_n + (21n)_4 = (x)_{10} \Rightarrow \mathbf{x = ?}$

A) 30 B) 40 C) 48 D) 50 E) 112

6. $a > b > c$

$(abc)_3 = (x)_{10} \Rightarrow \mathbf{x = ?}$

A) 18 B) 21 C) 25 D) 27 E) 29

7. $(340)_{10} = (x)_7 \Rightarrow \mathbf{x = ?}$

A) 453 B) 466 C) 566 D) 656 E) 664

8. $(42)_a + (43)_b = 45 \Rightarrow \mathbf{b - a = ?}$

A) 0 B) 1 C) 2 D) 3 E) 4

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9. $3 \cdot 9^4 + 2 \cdot 9^2 + 6 \cdot 9 + 5 = (x)_9 \Rightarrow x = ?$

A) 3265 B) 3026 C) 30265

D) 300265 E) 32065

10. $(324)_5 \cdot (23)_5 = (x)_5 \Rightarrow x = ?$

A) 12312 B) 14212 C) 11342

D) 14112 E) 21342

11. $(425)_6 + (551)_6 = (x)_6 \Rightarrow x = ?$

A) 976 B) 1420 C) 1520

D) 1510 E) 1240

12. $(621)_7 - (145)_7 = (x)_7 \Rightarrow x = ?$

A) 440 B) 441 C) 442

D) 443 E) 444

13. $(42,13)_5 = x \Rightarrow x = ?$

A) $\frac{558}{25}$ B) $\frac{471}{25}$ C) $\frac{417}{25}$

D) $\frac{558}{5}$ E) $\frac{425}{127}$

14. $\left(\frac{47}{2}\right) = (x)_6 \Rightarrow x = ?$

A) 34,3 B) 35,2 C) 35,3

D) 35,4 E) 40,2

15. $2^{10} = (x)_4 \Rightarrow x = ?$

A) 10 B) 100 C) 1000

D) 10000 E) 100000

16. $(1332)_x = (A)_{x+1} \Rightarrow A = ?$

A) 1001 B) 2001 C) 1002

D) 3000 E) 133

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	B	C	C	D	B	E	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	D	B	D	A	C	E	A

1.
$$\begin{array}{r} ABC \\ -\,BAC \\ \hline 540 \end{array} \Rightarrow A - B = ?$$

A) 2 B) 3 C) 4 D) 5 E) 6

2. $x \neq y,\ x,y \in Z^+$

$$\frac{10}{y} = x - 3 \Rightarrow \sum x = ?$$

A) 22 B) 23 C) 24 D) 25 E) 26

3. $a,b,c \in Z^+$

$$\left.\begin{array}{l} 2a = 3b \\ 3a = 4c \end{array}\right\} \Rightarrow \min(a+b+c) = ?$$

A) 28 B) 29 C) 30 D) 31 E) 32

4.
$$\left.\begin{array}{l} x^2 \cdot y^3 > 0 \\ y^2 \cdot z > 0 \\ x \cdot y \cdot z < 0 \end{array}\right\} \Rightarrow x,y,z = ?,?,?$$

A) −,−,− B) −,−,+ C) −,+,+ D) +,+,+ E) +,+,−

5. $a, b, c \in Z^+$

a + b = 4c + 3 ise aşağıdakilerden hangisi daima tek sayıdır?

If a+b = 4c + 3 , which of the following is certainly an odd number?

A) a + b + c B) a − b C) a·c D) 2a + b E) a + c

6. **Aşağıdakilerden hangisi bir çift sayıdır?**

Which of the followings is an even number?

A) $21^5 \cdot 17^3 \cdot 15^0$ B) $5^{13} + 2^8$ C) $4^5 - 3^7$ D) $3^7 \cdot 2^5 + 1$ E) $5^{21} - 3^{15}$

7. **x ∈ Z ise aşağıdakilerden hangisi daima tektir?**

If x ∈ Z which of the followings is certainly an odd number?

A) $x^5 + x^3$ B) $x^2 + x$ C) $x^2 + 2$ D) $2x^3 + 3$ E) 8x

8. **a, b, c ∈ Z⁺**

2a − b = 6c + 4 ise aşağıdakilerden hangisi daima tek sayıdır?

If 2a − b = 6c + 4, which of the following is certainly an odd number?

A) b + 1 B) 3b C) $b^2 + b$ D) b^3 E) $b^3 + 3b$

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9. $x, y \in Z^+$

$(2x + 1)\cdot(y + 1) = 12 \Rightarrow$ **x + y = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

10. **x + 2 tek sayı ve y + 3 çift sayı olduğuna göre aşağıdakilerden hangisi tek sayıdır?**

As x+2 is an odd and x+3 is an even number, Which of the followings is an odd number?

A) x·y B) x·y + 3 C) x + y

D) (x + 2)·(y + 3) E) 8x·y

11. **a çift sayı ve b tek sayı ise aşağıdakilerden hangisi çift sayıdır?**

If a is an even and b is an odd number, which of the followings is an even number?

A) a·b + 2 B) 3b + a C) 9·(a + b)

D) a + b + 6 E) a·b + b

12. **a, b ve c ardışık sayılardır. a < b < c olmak üzere**

a, b, and c are consecutive numbers. If a < b < c

$\frac{(a-c)+(b-a)}{2} = ?$

A) $-\frac{1}{2}$ B) 2 C) $\frac{3}{2}$

D) $\frac{3}{4}$ E) 4

13. **x, y ve z ardışık tek sayıdır. x > y > z olmak üzere**

x, y, and z are consecutive odd numbers. If x > y > z

x + y – 2z = ?

A) 3 B) 9 C) 6

D) –6 E) –4

14. **x, y ve z ardışık çift sayıdır. x < y < z olmak üzere**

x, y, and z are consecutive even numbers. If x < y < z

$\frac{2x-(y+z)}{3} = ?$

A) –1 B) 0 C) –2

D) 4 E) 5

15. **a, b ve c ardışık sayılardır. a > b > c olmak üzere**

a, b, and c are consecutive numbers. If a > b > c

2a + 2 – (b + c) = ?

A) 5 B) 7 C) 12

D) 14 E) 32

16. **a, b ve c ardışık sayılardır. c < b < a olmak üzere**

a, b, and c are consecutive numbers. If c < b < a

$\frac{6c-3(a+b+3)}{9} = ?$

A) –3 B) –2 C) –1

D) 0 E) 1

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	D	B	C	B	E	D	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	A	A	A	C	C	A	B

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1. $5 + 6 + 7 + \ldots + 39 = ?$

A) 780 B) 770 C) 742

D) 700 E) 680

2. $1 + 3 + 5 + \ldots + 17 = ?$

A) 64 B) 81 C) 100

D) 121 E) 144

3. $7 + 13 + 19 + \ldots + 67 = ?$

A) 370 B) 376 C) 382

D) 394 E) 407

4. $1 + 3 + 5 + 7 + \ldots + 25 + 24 + 22 + \ldots + 6 + 4 + 2 = ?$

A) 300 B) 324 C) 325

D) 330 E) 332

5. $4 - 5 + 6 - 7 + 8 - 9 \ldots + 18 - 19 + 20 - 21 = ?$

A) –10 B) –9 C) –8

D) –7 E) –6

6. $5 - 7 + 9 - 11 + 13 - 15 + \ldots + 45 - 47 + 49 = ?$

A) –23 B) –22 C) 0

D) 5 E) 27

7. $11 + 13 + 15 + \ldots + 2n - 1 = 416 \Rightarrow n = ?$

A) 18 B) 19 C) 20

D) 21 E) 22

8. $3 + 6 + 9 + 12 + \ldots + 3n = 513 \Rightarrow n = ?$

A) 15 B) 16 C) 17

D) 18 E) 20

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $\frac{2+4+6+\ldots+24}{6a} = 13 \Rightarrow$ **a = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

10. $2n + (2n + 2) + (2n + 4) + \ldots + (2n + 32) = 544$

$\Rightarrow$ **n = ?**

A) 4 B) 6 C) 8

D) 10 E) 12

11. $13 + 15 + 17 + \ldots + 2n - 1 = 540 \Rightarrow$ **n = ?**

A) 21 B) 22 C) 23

D) 24 E) 25

12. $14 + 16 + 18 + \ldots + K = 378 \Rightarrow$ **K = ?**

A) 38 B) 40 C) 42

D) 44 E) 46

13. $A = 19 + 20 + 21 + \ldots + n$

$B = 1 + 2 + 3 + \ldots + n$

$\Rightarrow$ **B – A = ?**

A) 171 B) 172 C) 173

D) 174 E) 175

14. $1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + 4 \cdot 9 + \ldots + 10 \cdot 21 = A$

$1 \cdot 6 + 2 \cdot 8 + 3 \cdot 10 + 4 \cdot 12 + \ldots + 10 \cdot 24 = ?$

A) A + 80 B) A + 75 C) A + 105

D) A + 165 E) A + 180

15. $1 + 2 + 3 + \ldots + n = A$

$15 + 16 + 17 + \ldots + n = B$

$A + B = 315 \Rightarrow$ **A = ?**

A) 180 B) 200 C) 210

D) 240 E) 250

16. $A = 3 \cdot 5 + 4 \cdot 6 + 5 \cdot 7 + \ldots + 12 \cdot 14$

$\Rightarrow$ **$17 + 26 + 37 + \ldots + 170 = ?$**

A) A + 18 B) A + 20 C) A + 24

D) A + 30 E) A + 33

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	B	E	C	B	E	D	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	C	D	B	A	D	C	B

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $x, y \in N$

$135 = 3^x \cdot 5^y$

$\Rightarrow$ **x + y = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

2. $x, y \in N$

$392 = 2^x \cdot 7^y$

$\Rightarrow$ **x · y = ?**

A) 4 B) 6 C) 8

D) 10 E) 12

3. $x, y, z \in N$

$1500 = 2^x \cdot 3^y \cdot 5^z$ $\Rightarrow$ **2x + 3y − z = ?**

A) 2 B) 3 C) 4

D) 5 E) 6

4. $x, y, z \in N$

$882 = 2^x \cdot 3^y \cdot 7^z$ $\Rightarrow$ **x·y·z = ?**

A) 4 B) 6 C) 9

D) 12 E) 15

5. $A = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot 9$

$A = x \cdot 2^a \cdot 3^b$ $\Rightarrow$ **min(x) = ?**

A) 30 B) 32 C) 35

D) 38 E) 40

6. $A = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots \cdot 12$

$A = x \cdot 2^a \cdot 3^b \cdot 5^c$ $\Rightarrow$ **min(x) = ?**

A) 42 B) 49 C) 56

D) 63 E) 77

7. $x, y \in N^+$

$30 \cdot x = y^2$ $\Rightarrow$ **min(x) = ?**

A) 30 B) 45 C) 50

D) 60 E) 75

8. $x, y \in N^+$

$60 \cdot x = y^3$ $\Rightarrow$ **min(x) = ?**

A) 350 B) 400 C) 450

D) 500 E) 600

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9. $x, y \in N^+$

$240 \cdot x = y^3 \Rightarrow$ **min(x) = ?**

A) 900 B) 450 C) 225

D) 125 E) 64

10. 150 sayısının kaç tane pozitif tam sayı böleni vardır?

How many positive integer divisors does the number 150 have?

A) 10 B) 12 C) 14

D) 15 E) 18

11. 540 sayısının kaç tane tam sayı böleni vardır?

How many positive integer divisors does the number 540 have?

A) 24 B) 30 C) 36

D) 42 E) 48

12. 462 sayısının asal sayı bölenlerinin toplamı kaçtır?

What is the sum of prime number divisors of the number 462?

A) 20 B) 21 C) 22

D) 23 E) 24

METROPOL YAYINLARI • METROPOL PUBLICATIONS

13. $8^4 - 1$ **sayısının en büyük asal sayı böleni kaçtır?**

What is the biggest prime divisor of the number $8^4 - 1$?

A) 2 B) 3 C) 7

D) 13 E) 17

14. $A = 12 \cdot 15 \cdot 40$

A sayısının pozitif tam sayı bölenlerinden kaç tanesi 3'ün katıdır?

How many of the positive integer divisors of the number A are multiples of 3?

A) 30 B) 32 C) 36

D) 40 E) 48

15. 240 sayısının asal olmayan pozitif tam sayı bölenlerinin sayısı kaçtır?

What is the number of non-prime positive divisors of the number 240?

A) 16 B) 17 C) 18

D) 19 E) 20

16. 72 sayısının pozitif tam sayı bölenlerinin toplamı kaçtır?

What is the sum of positive integer divisors of the number 72?

A) 195 B) 200 C) 205

D) 210 E) 215

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	B	C	A	C	E	A	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	B	E	D	D	C	B	A

1. $\frac{7! + 6!}{7! - 6!} = ?$

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{4}{3}$ D) $\frac{7}{6}$ E) $\frac{6}{5}$

2. $\frac{8! - 6!}{5 \cdot 6!} = ?$

A) 9 B) 10 C) 11 D) 12 E) 14

3. $\frac{26! \cdot 38!}{37! \cdot 25! \cdot 13} = ?$

A) 5! B) 76 C) 58 D) 38 E) 21

4. $\frac{(n-5)!}{2 \cdot (n-4)!} = \frac{1}{22} \Rightarrow n = ?$

A) 8 B) 10 C) 12 D) 14 E) 15

5. $\frac{(n+1)!}{n! \cdot (n-2)!} : \frac{(n+1)!}{(n-2)!} = ?$

A) 0 B) 1 C) $\frac{1}{(n+1)!}$ D) $\frac{1}{n!}$ E) n!

6. $\frac{n!}{(n-2)!} = 6 \Rightarrow n = ?$

A) 0 B) 1 C) 3 D) 6 E) 12

7. $\frac{n! \cdot (n-2)!}{(n-3)!} : \frac{n! \cdot (n-2)}{n} = 15 \Rightarrow n = ?$

A) 1 B) 12 C) 15 D) 17 E) 21

8. $\frac{(n+1)!}{n!} : 4 = 1 \Rightarrow n = ?$

A) 1 B) 3 C) 12 D) 16 E) 21

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9. $\frac{(n+1)! + n!}{n! \cdot (n+2)} = ?$

A) 0 B) 1 C) 2

D) 3 E) 4

10. $\frac{(2n-4)!}{(3n+1)!} : \frac{(2n-3)!}{(3n+2)!} = \frac{17}{7} \Rightarrow n = ?$

A) 3 B) 4 C) 5

D) 6 E) 7

11. $(n+1)! = 4! \cdot n! \Rightarrow n = ?$

A) 12 B) 21 C) 23

D) 27 E) 32

12. $n! = a \Rightarrow (n+1) \cdot n! - n!\,(n+2) = ?$

A) a^2 B) $-a$ C) $a + 2$

D) $a - 1$ E) $a + 1$

13. $A \cdot x \in N$

$42! = A \cdot 10^x \Rightarrow \max(x) = ?$

A) 7 B) 8 C) 9

D) 10 E) 11

14. $A \cdot x \in N$

$38! + 39! = A \cdot 10^x \Rightarrow \max(x) = ?$

A) 7 B) 8 C) 9

D) 10 E) 11

15. $A = 40!$

$A - 123 = \ldots abc \Rightarrow a+b+c = ?$

A) 18 B) 19 C) 20

D) 21 E) 22

16. $A = 24!$

$A - 1 = \ldots \underbrace{99\ldots9}_{\text{n tane/n times}} \Rightarrow \max(n) = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	C	B	E	D	C	C	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	C	C	B	C	C	E	D

1. $\begin{array}{c|c} A & B \\ \hline - & 2 \\ \hline 3 & \end{array}$ $\Rightarrow$ **min(A) = ?**

A) 5 B) 7 C) 9

D) 11 E) 13

2. $\begin{array}{c|c} A & B \\ \hline - & 4 \\ \hline 5 & \end{array}$ $\begin{array}{c|c} B & C \\ \hline - & 3 \\ \hline 1 & \end{array}$ $\Rightarrow$ $\begin{array}{c|c} \mathbf{A} & \mathbf{12} \\ \hline - & \\ \hline ? & \end{array}$

A) 6 B) 8 C) 9

D) 10 E) 11

3. $\begin{array}{c|c} A & 8 \\ \hline - & B \\ \hline 1 & \end{array}$ $\begin{array}{c|c} A & 3 \\ \hline - & B+5 \\ \hline 1 & \end{array}$ $\Rightarrow$ **A = ?**

A) 25 B) 28 C) 30

D) 42 E) 49

4. $\begin{array}{c|c} A & 9 \\ \hline - & \\ \hline 7 & \end{array}$ $\begin{array}{c|c} B & 9 \\ \hline - & \\ \hline 8 & \end{array}$ $\Rightarrow$ $\begin{array}{c|c} \mathbf{A+B} & \mathbf{9} \\ \hline - & \\ \hline ? & \end{array}$

A) 2 B) 3 C) 4

D) 5 E) 6

5. $\begin{array}{c|c} m+6 & 7 \\ \hline - & n \\ \hline 3 & \end{array}$ $\begin{array}{c|c} m-4 & n+1 \\ \hline - & 2 \\ \hline 1 & \end{array}$ $\Rightarrow$ **n = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

6. $\begin{array}{c|c} 23a & 4 \\ \hline - & \\ \hline 3 & \end{array}$ $\Rightarrow \sum a = ?$

A) 12 B) 13 C) 14

D) 15 E) 16

7. $\begin{array}{c|c} 2AB & 3 \\ \hline - & \\ \hline 2 & \end{array}$ $\begin{array}{c|c} 2AB & 5 \\ \hline - & \\ \hline 3 & \end{array}$ $\Rightarrow$ **max(A + B) = ?**

A) 11 B) 12 C) 13

D) 14 E) 15

8. $\begin{array}{c|c} 35x7 & 9 \\ \hline - & \\ \hline 3 & \end{array}$ $\Rightarrow$ **x = ?**

A) 4 B) 5 C) 6

D) 7 E) 8

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9. $A4A6 \in N$

$$\begin{array}{r|l} A4A6 & 12 \\ \hline - & \\ \hline 0 & \end{array} \Rightarrow \sum A = ?$$

A) 7 B) 8 C) 9

D) 10 E) 12

10.

$$\begin{array}{r|l} 56xy & 15 \\ \hline - & \\ \hline 0 & \end{array} \Rightarrow \max(x) = ?$$

A) 9 B) 8 C) 7

D) 6 E) 5

11. $\left.\begin{array}{l} OBEB(GCF)(42,126,140) = x \\ OKEK(LCM)(42,126,140) = y \end{array}\right\} \Rightarrow \frac{y}{x} = ?$

A) 40 B) 50 C) 60

D) 80 E) 90

12. $A = 5m + 2 = 6n + 3 = 7k + 11,$

$400 < A < 500 \Rightarrow A = ?$

A) 416 B) 417 C) 418

D) 419 E) 421

13. $x, y, z \in N^+$

$A = 3x + 5 = 2y + 3 = 4z - 1$

$\Rightarrow \min(x + y + z) = ?$

A) 9 B) 12 C) 15

D) 18 E) 21

14. Uzunlukları 48 cm ve 56 cm olan farklı iki tel çubuk kesilerek eş uzunlukta en az kaç parça elde edilir?

Two wire bars with lenghes of 48 cm and 56 cm are going to get cut to pieces with same lenghs. What is the minimum possible number of pieces obtained?

A) 9 B) 10 C) 11

D) 12 E) 13

15. 80, 112 ve A kg üç farklı pirinç poşeti, pirinçler karıştırılmadan ve hiç artmayacak şekilde eşit ağırlıkta 15 pakete konulacaktır. Buna göre A kaç kg dır?

Three different types of rice in bins with weights of 80, 112 and A kilograms are going to be packed in same-sized packs without mixing any two types of them and also without any rice being left unpacked in 15 bags. Accordingly, how many kilograms is A?

A) 28 B) 34 C) 42

D) 48 E) 52

16. Boyutları 10 cm, 15 cm ve 24 cm olan legolardan en az kaç tanesi ile bir küp elde edilir?

At least how many legos with dimentions of 10, 15, and 20 cm are needed to make a cube?

A) 300 B) 360 C) 480

D) 520 E) 600

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	C	A	E	B	D	E	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	B	E	B	A	E	D	C

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BÖLÜM CHAPTER 2

TARİHSEL NOT / HISTORICAL NOTE

Ali Kuşçu [1403 – 1474]

Ali Kuşçu asıl adı Ali Bin Muhammed Türk gök bilimci, matematikçi ve dilbilimcidir. Küçük yaştan itibaren matematik ve astronomiye ilgi duyan Ali Kuşçu, Uluğ Bey'in oğlu ve Bursalı Kadızade Rumi'nin öğrencisidir. Felsefeden bağımsız olarak astronomi ile de ilgilenmiştir.

Ali Bin Muhammed known as Ali Qushji was a persian astronomer, mathematician and physicist.He was the son of an official of Ulugh Beg, and also a student of Qadi Zadeh. He is best known for his development of an astronomical physics independent from natural philosophy.

RASYONEL ve ONDALIK SAYILAR
RATIONAL and DECIMAL NUMBERS

Bu bölüm, 23 çözümlü örnek, 12 çözümlü test sorusu, 16 çözümlü YÖS sorusu, 80 test sorusu içermektedir.

This chapter includes 23 examples with solutions, 12 tests questions with solutions, 16 YÖS questions with solutions and 80 test questions.

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BÖLÜM
02
CHAPTER

RASYONEL ve ONDALIK SAYILAR
RATIONAL and DECIMAL NUMBERS

ÖNEMLİ / IMPORTANT

• $\frac{a}{b}$ *kesrinde, a sayısına* ***pay****, b sayısına* ***payda****, aradaki çizgiye* ***kesir çizgisi*** *denir.*

kesir çizgisi ← $\frac{a}{b}$ → pay / payda

• *In the fraction* $\frac{a}{b}$*, a is called* ***numerator****, b is called* ***denominator*** *and the line in between is called* ***fraction bar.***

Fraction bar ← $\frac{a}{b}$ → Numerator / Denominator

ÖNEMLİ / IMPORTANT

• $a\frac{b}{c}$ *tamsayılı kesrinde,* $\frac{b}{c}$ *kesri basit kesir olmak zorundadır.*

• *In the mixed fraction* $a\frac{b}{c}$*, the fraction* $\frac{b}{c}$ *must be a proper fraction.*

ÖNEMLİ / IMPORTANT

• $a\frac{b}{c} = a + \frac{b}{c} = \frac{a \cdot c + b}{c}$

• $-a\frac{b}{c} = -\left(a + \frac{b}{c}\right) = -\frac{a \cdot c + b}{c}$

PEKİŞTİRME / REINFORCE

✓ $\frac{17}{5} = 3\frac{2}{5}$

✓ $2\frac{3}{4} = 2 + \frac{3}{4} = \frac{11}{4}$

RASYONEL SAYILAR / RATIONAL NUMBERS

KESİR KAVRAMI / FRACTION CONCEPT

Tanım: a ve b tamsayı ve $b \neq 0$ olmak üzere $\frac{a}{b}$ ifadesi bir ***kesir*** dir.

Definition: Provided that a and b are integers and $b \neq 0$, the expression $\frac{a}{b}$ is called a ***fraction.***

BASİT KESİR / PROPER FRACTION

Tanım: $\frac{a}{b}$ kesrinde $|a| < |b|$ ise $\frac{a}{b}$ kesrine ***basit kesir*** denir.

Definition: In the fraction $\frac{a}{b}$, if $|a| < |b|$, the fraction $\frac{a}{b}$ is called a proper fraction.

BİLEŞİK KESİR / IMPROPER FRACTION

Tanım: $\frac{a}{b}$ kesrinde $|a| \geq |b|$ ise $\frac{a}{b}$ kesrine ***bileşik kesir*** denir.

Definition: In the fraction $\frac{a}{b}$, if $|a| \geq |b|$, the fraction $\frac{a}{b}$ is called a improper fraction.

TAM SAYILI KESİR / MIXED FRACTION

Tanım: $a\frac{b}{c}$ kesrine ***tam sayılı kesir*** denir. a ya kesrin tam kısmı, $\frac{b}{c}$ ye kesir kısmı denir.

Definition: $a\frac{b}{c}$ is called a mixed fraction. The number a is called the whole part, and $\frac{b}{c}$ is called the fraction part.

KESİRLERDE SADELEŞTİRME ve GENİŞLETME
SIMPLIFICATION and EXPANSION of FRACTIONS

Sadeleştirme: Bir kesrin pay ve paydasının aynı sayıya bölünerek küçültülmesine sadeleştirme denir.

Simplification (or reduction): Dividing both the numerator and denominator of a fraction to the same number to form a new fraction with the same value of the original one is called reducing or simplifying the fraction.

Genişletme: Bir kesrin pay ve paydasının aynı sayı ile çarpılarak büyütülmesine genişletme denir.

expansion; Multiplying both the numerator and denominator of a fraction by the same number to form a new fraction with the same value of the original one is called expanding the fraction.

DENK KESİRLER / EQUIVALENT FRACTIONS

Tanım: İki veya daha fazla kesrin pay ve paydaları sadeleştirme veya genişletme işlemleri yaparak birbirlerine eşit hale getirilebiliyor ise bu kesirlere **denk kesir** denir.

Definition: Two or more than two fractions that look different but can be equal to each other by reducing or expanding are called **equivalent fractions**.

Etkinlikler / Activities

a. $\frac{4}{10} \equiv \frac{8}{20}$,

b. $\frac{3}{7} \equiv \frac{9}{21}$

PEKİŞTİRME / REINFORCE

✓ $\frac{1}{2} - \frac{3}{2} = ?$

.....[-1]

✓ $\frac{1}{3} + \frac{2}{3} = ?$

.....[1]

✓ $\frac{1}{2} + \frac{3}{4} = ?$

.....[$\frac{5}{4}$]

RASYONEL SAYILAR / RATIONAL NUMBERS

Tanım: Birbirine denk kesirlerin oluşturduğu her küme bir sayı belirtir ve bu sayıya rasyonel sayı denir. Rasyonel sayıların oluşturduğu kümeye **rasyonel sayılar kümesi** denir ve **Q** ile gösterilir.

Definition: The representative element of a set of equivalent fractions is called a rational number. The rational numbers form a set that is called **the set of rational numbers** and is shown by the letter **Q**.



PEKİŞTİRME / REINFORCE

✓ $\frac{1}{5} - \frac{3}{4} = ?$

.....[$-\frac{11}{20}$]

✓ $\frac{2}{3} - \frac{3}{2} = ?$

.....[$-\frac{5}{6}$]

✓ $\frac{3}{4} + \frac{4}{5} = ?$

.....[$\frac{31}{20}$]

▼ **0'ın sağındaki rasyonel sayılara pozitif rasyonel sayılar denir ve Q^+ ile gösterilir.**
The rational numbers to the right side of zero are called positive rational numbers and shown by Q^+.

$$Q^+ = \left\{..., \frac{1}{4}, \frac{1}{2}, ..., 1, ..., \frac{3}{2}, ..., 2, ..., 3, ...\right\}$$

▼ **0'ın solundaki rasyonel sayılara negatif rasyonel sayılar denir ve Q^- ile gösterilir.**
The rational numbers to the left side of zero are called negative rational numbers and shown by Q^-.

$$Q^- = \left\{..., -4, -3, -2, -\frac{3}{2}, -1, -\frac{1}{2}, ...\right\}$$

▼ **Her tam sayı bir rasyonel sayıdır.**
Every integer is a rational number.

$3 = \frac{3}{1}, 4 = \frac{4}{1}, ...$

O halde / Therefore, $N \subset Z \subset Q$

PEKİŞTİRME / REINFORCE

✓ $\frac{2}{7} + \frac{3}{2} = ?$

.....[$\frac{25}{14}$]

✓ $\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - 3\right) = ?$

.....[-2]

✓ $2 - \frac{3}{4} - \frac{3}{5} + 1 = ?$

.....[$\frac{33}{20}$]

RASYONEL SAYILAR KÜMESİNDE İŞLEMLER
OPERATIONS in the SET of RATIONAL NUMBERS

TOPLAMA ve ÇIKARMA İŞLEMİ
ADDITION and SUBTRACTION OPERATIONS

Paydaları eşit olan iki kesir toplanırken veya çıkarılırken payların toplamı paya, ortak payda da paydaya yazılır. Bunun için aşağıdaki genel kural yazılabilir.

To add or subtract rational numbers with the same denominators, add or subtract the numerators and write the sum or difference as the numerator. Then write the common denominator as the denominator. The following general rule can be written:

$$a,b,c,d, \in Z \text{ ve / and } b,d \neq 0 \Rightarrow \frac{a}{b} \mp \frac{c}{d} = \frac{a \cdot d \mp b \cdot c}{b \cdot d}$$

Paydaları farklı rasyonel sayılar toplanırken veya çıkarılırken, toplama veya çıkarma işleminden önce kesirler ortak paydaya sahip kesirler olarak yazılır.

To add or subtract rational numbers with the different denominators, we write the fractions as fractions with common denominators before adding or subtracting.

ÖNEMLİ / IMPORTANT

- *Tam sayılı kesirleri çarparken tam sayılı kesirler bileşik kesirlere dönüştürülür.*
- *İki sayının çarpımı 1 ise, bu sayılar çarpma işlemine göre birbirinin tersidir.*
- *Sıfırın başka bir rasyonel sayıya bölümü sıfırdır.*
- *Bir rasyonel sayının sıfıra bölümü tanımsızdır.*
- *To multiply mixed fractions we first write each mixed fraction as an improper one*
- *Two numbers whose product is 1 are called multiplicative inverses or reciprocals.*
- *The division of zero by any other rational number is zero.*
- *The division of a rational number by zero is undefined.*

Örnek / Example 1

$\frac{3}{5} - \frac{3}{5} + \left(\frac{1}{3} - \frac{1}{3} + 9\right) = ?$

A) 5 B) 9 C) 11 D) 20 E) 22

Çözüm / Solution Yanıt / Answer **B**

$$\frac{3}{5} - \frac{3}{5} + \left(\frac{1}{3} - \frac{1}{3} + 9\right) = \frac{3}{5} - \frac{3}{5} + \left(\frac{1}{3} - \frac{1}{3} + 9\right)$$
$$= \frac{3}{5} - \frac{3}{5} + 9$$
$$= 9$$

Örnek / Example 2

$\left(\frac{1}{3} - \frac{1}{2}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) = ?$

A) $\frac{1}{20}$ B) $\frac{1}{10}$ C) $\frac{1}{5}$ D) $\frac{1}{2}$ E) $\frac{1}{4}$

Çözüm / Solution Yanıt / Answer **A**

$$\left(\frac{1}{3} - \frac{1}{2}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{3} - \frac{1}{2} + \frac{1}{4} - \frac{1}{5} + \frac{1}{2} - \frac{1}{3}$$
$$= \underset{(5)}{\frac{1}{4}} - \underset{(4)}{\frac{1}{5}}$$
$$= \frac{5-4}{20}$$
$$= \frac{1}{20}$$

ÖNEMLİ / IMPORTANT

- $\frac{\frac{a}{b}}{c} = \frac{a}{1} \cdot \frac{c}{b} = \frac{a \cdot c}{b}$
- $\frac{\frac{a}{b}}{c} = \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{b \cdot c}$
- $a^{-1} = \frac{1}{a}$
- $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

ÇARPMA ve BÖLME İŞLEMİ / MULTIPLICATION and DIVISION OPERATION

Çarpma işleminde iki rasyonel sayının çarpımı bulunurken, payların çarpımı paya, paydaların çarpımı ise paydaya yazılır.

Bölmede ise birinci kesrin aynısı yazılıp, ikinci kesir ters çevrilir ve işlem çarpmaya dönüştürülür.

To multiply two fractions multiply their numerators and write the result to the numerator and multiply their denominators and write the result to the denominator.

To divide any fraction by another fraction, multiply the first fraction by the reciprocal of the second fraction.

▼ $\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}$ ▼ $\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}$

▼ $\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}$ ▼ $\frac{1}{\frac{a}{b}}=\frac{b}{a}$

Etkinlikler / Activities

a. $\frac{7}{3}\cdot\frac{4}{5}=\frac{7\cdot4}{3\cdot5}=\frac{28}{15}$

b. $3\frac{1}{2}\cdot4\frac{1}{5}=\frac{7}{2}\cdot\frac{21}{5}=\frac{7\cdot21}{2\cdot5}=\frac{147}{10}$

c. $3\cdot\frac{2}{5}=\frac{3\cdot2}{5}=\frac{6}{5}$

d. $\frac{3}{5}:\frac{2}{9}=\frac{3}{5}\cdot\frac{9}{2}=\frac{3\cdot9}{5\cdot2}=\frac{27}{10}$

e. $\frac{\frac{3}{5}}{2}=3\cdot\frac{2}{5}=\frac{3\cdot2}{5}=\frac{6}{5}$

f. $\frac{\frac{1}{4}}{7}=\frac{1}{4}\cdot\frac{1}{7}=\frac{1}{28}$

Örnek / Example 3

$\left(\frac{3}{2-\frac{1}{2}}\right)\cdot\left(1-\frac{7}{2}\right)=?$

A) $-\frac{1}{30}$ B) $-\frac{5}{6}$ C) $-\frac{1}{5}$ D) -6 E) -5

Çözüm / Solution

Yanıt / Answer **E**

$\left(\frac{3}{2-\frac{1}{2}}\right)\cdot\left(1-\frac{7}{2}\right)=\frac{3}{\frac{4-1}{2}}\cdot\frac{2-7}{2}=\frac{6}{3}\cdot\frac{-5}{2}=-5$

PEKİŞTİRME / REINFORCE

✓ $\frac{1}{2}\cdot\frac{3}{4}=?$ ..... $\left[\frac{3}{8}\right]$

✓ $\frac{1}{5}\cdot\frac{6}{7}=?$ ..... $\left[\frac{6}{35}\right]$

✓ $\frac{1}{4}\cdot\frac{8}{5}=?$ ..... $\left[\frac{2}{5}\right]$

✓ $\frac{2}{9}\cdot\frac{3}{4}=?$ ..... $\left[\frac{1}{6}\right]$

✓ $\frac{1}{2}:\frac{1}{3}=?$ ..... $\left[\frac{3}{2}\right]$

PEKİŞTİRME / REINFORCE

✓ $\frac{1}{7}\div\frac{6}{7}=?$ ..... $\left[\frac{1}{6}\right]$

✓ $\frac{1}{\frac{1}{3}}=?$ ..... $[3]$

✓ $\frac{\frac{1}{2}}{\frac{3}{2}}=?$ ..... $\left[\frac{1}{3}\right]$

✓ $\frac{\frac{1}{6}}{2}=?$ ..... $\left[\frac{1}{12}\right]$

✓ $\frac{\frac{1}{5}}{10}=?$ ..... $\left[\frac{1}{50}\right]$

PEKİŞTİRME / REINFORCE

✓ $1+\frac{1}{\frac{1}{2}}=?$

.....[3]

✓ $1-\frac{1}{\frac{1}{3}}=?$

.....[-2]

✓ $2-\frac{1}{2-\frac{1}{2}}=?$

.....$\left[\frac{4}{3}\right]$

✓ $1+\frac{1}{1+\frac{1}{2}}=?$

.....$\left[\frac{5}{3}\right]$

✓ $1+\frac{1}{1-\frac{1}{3}}=?$

.....$\left[\frac{5}{2}\right]$

Örnek / Example 4

$$\frac{\left(\frac{1}{2}-5\right)+\left(\frac{1}{3}-3\right)}{\left(2-\frac{5}{6}\right)\cdot\left(\frac{3}{2}-3\right)}=?$$

A) $\frac{43}{10}$ B) $\frac{86}{21}$ C) $\frac{43}{11}$ D) $\frac{86}{23}$ E) $\frac{43}{12}$

Çözüm / Solution Yanıt / Answer **B**

$$\frac{\left(\frac{1}{2}-5\right)+\left(\frac{1}{3}-3\right)}{\left(2-\frac{5}{6}\right)\cdot\left(\frac{3}{2}-3\right)}=\frac{\frac{1-10}{2}+\frac{1-9}{3}}{\frac{12-5}{6}\cdot\frac{3-6}{2}}$$

$$=\frac{-\frac{9}{2}-\frac{8}{3}}{\frac{7}{6}\cdot\left(-\frac{3}{2}\right)}$$

$$=\frac{-27-16}{6}\cdot\left(-\frac{12}{21}\right)$$

$$=\frac{43}{6}\cdot\frac{12}{21}=\frac{86}{21}$$

Örnek / Example 5

$$\left[\frac{5}{2}-\frac{1}{1-\frac{1}{2}}\right]:\left[\frac{1}{2}-\frac{\frac{1}{2}}{1+\frac{1}{2}}\right]=?$$

A) 1 B) $\frac{3}{2}$ C) 2 D) 3 E) $\frac{9}{2}$

Çözüm / Solution Yanıt / Answer **D**

$$\left[\frac{5}{2}-\frac{1}{1-\frac{1}{2}}\right]:\left[\frac{1}{2}-\frac{\frac{1}{2}}{1+\frac{1}{2}}\right]=\left[\frac{5}{2}-2\right]:\left[\frac{1}{2}-\frac{\frac{1}{2}}{\frac{3}{2}}\right]$$

$$=\left(\frac{5}{2}-2\right):\left(\frac{1}{2}-\frac{1}{3}\right)$$

$$=\frac{1}{2}:\frac{1}{6}=\frac{1}{2}\cdot 6=3$$

ZİNCİRLEME KESİRLER / CONTINUED FRACTIONS

a ≠ 0, a bir tamsayı olmak üzere,

a is an integer and a ≠ 0. The fractions defined as:

$$x = a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d + \cfrac{1}{e}}}}$$

biçiminde tanımlanan kesirlere ***sonlu zincirleme kesirler*** denir.

are called *finite continued fractions.*

Örnek / Example 6

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3}}}} = ?$$

A) $\frac{17}{3}$ B) $\frac{18}{11}$ C) $\frac{11}{8}$ D) $\frac{11}{7}$ E) $\frac{7}{11}$

Çözüm / Solution

Yanıt / Answer **B**

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3}}}} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\frac{4}{3}}}} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{3}{4}}} = 1 + \cfrac{1}{1 + \cfrac{1}{\frac{7}{4}}}$$

$$= 1 + \cfrac{1}{1 + \frac{4}{7}}$$

$$= 1 + \cfrac{1}{\frac{11}{7}} = 1 + \frac{7}{11} = \frac{18}{11}$$

Örnek / Example 7

$$1 + \cfrac{x}{1 + \cfrac{x}{1 + \cfrac{x}{\vdots}}} = 5 \Rightarrow x = ?$$

A) 20 B) 25 C) 30 D) 35 E) 40

Çözüm / Solution

Yanıt / Answer **A**

$$1 + \cfrac{x}{1 + \cfrac{x}{1 + \cfrac{x}{\vdots}}} = 5 \Rightarrow 1 + \frac{x}{5} = 5$$

$$\frac{x}{5} = 4$$

$$x = 20$$

ÖNEMLİ / IMPORTANT

• *a, b ∈ Z, a ≠ 0 olmak üzere*

$$x = a + \cfrac{b}{a + \cfrac{b}{\vdots}}$$

biçiminde tanımlanan kesirlere ***sonsuza giden zincirleme kesirler*** *denir.*

• *a, b ∈ Z, a ≠ 0*

Fractions defined as $x = a + \cfrac{b}{a + \cfrac{b}{\vdots}}$ *are called* **infinite continued fractions.**

PEKİŞTİRME / REINFORCE

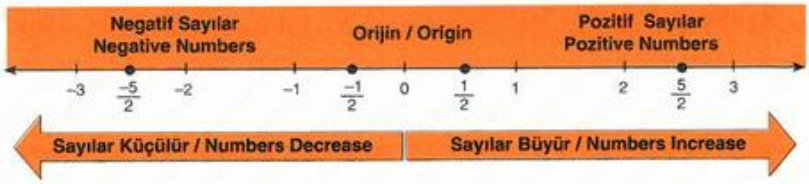
✓ $2 + \cfrac{3}{2 + \cfrac{3}{2 + \cfrac{3}{\vdots}}} = ?$

.....[3]

✓ $4 + \cfrac{4 + \cfrac{5}{5}}{4 + \cfrac{5}{5}} = ?$

.....[5]

RASYONEL SAYILARIN SIRALANMASI
THE ARRANGEMENT of RATIONAL NUMBERS



Sayı doğrusu üzerinde, 0 dan sağa doğru uzaklaştıkça sayıların değeri büyür ve 0 dan sola doğru uzaklaştıkça sayıların değeri küçülür.

In the number line the values of numbers increase by going to the right from zero and decrease by going to the left from zero.

Etkinlikler / Activities

a. $\frac{5}{2} > \frac{1}{2}$

b. $\frac{-5}{2} > -3$

c. $\frac{1}{2} > -2$

d. $1 > -3$

Sıralamanın Özellikleri / Properties of Arrangement

★ Negatif bir rasyonel sayı, pozitif bir rasyonel sayıdan daha küçüktür.

A negative rational number is smaller than a positive one.

$-\frac{2}{5} < \frac{2}{5}, -\frac{4}{7} < \frac{4}{7}, \cdots$

★ Paydaları eşit olan iki pozitif rasyonel sayıdan, payı küçük olan daha küçüktür.

If the fractions have the same positive denominator, the fraction with the smaller numerator is smaller.

$\frac{3}{8} < \frac{5}{8}, \frac{1}{7} < \frac{2}{7}, \cdots$

★ Payları eşit olan pozitif iki rasyonel sayıdan, paydası küçük olan sayı daha büyüktür.

If the fractions have the same numerator and the denominators are positive, the fraction with the smaller denominator is bigger.

$\frac{6}{5} < \frac{6}{4}, \frac{7}{8} < \frac{7}{4}, \cdots$

★ Pozitif her rasyonel sayı sıfırdan büyük, negatif her rasyonel sayı sıfırdan küçüktür.

Any positive rational number is bigger than zero and any negative rational number is smaller than zero.

$\frac{1}{4} > 0, -\frac{1}{4} < 0, \cdots$

ÖNEMLİ / IMPORTANT

• Negatif rasyonel sayılar kendi aralarında sıralanırken, pozitif rasyonel sayılardaki gibi sıralama yapılır sonra sıralamanın tam tersi alınır.

• When arronging negative numbers, the arrangement will be done as in positive numbers, and then the order of numbers will be reversed.

ÖNEMLİ / IMPORTANT

$\frac{a}{b}, \frac{c}{d}, \frac{e}{f} \in Q,$

- $\frac{a}{b} > \frac{c}{d}$ ve $\frac{c}{d} > \frac{e}{f} \Rightarrow \frac{a}{b} > \frac{e}{f}$
- $\frac{a}{b} < \frac{c}{d}$ ve $\frac{c}{d} < \frac{e}{f} \Rightarrow \frac{a}{b} < \frac{e}{f}$

★ $\frac{a}{b}, \frac{c}{d} \in Q$

$\frac{a}{b} < \frac{c}{d} \Rightarrow \frac{a}{b} < \frac{1}{2}\left(\frac{a}{b}+\frac{c}{d}\right) < \frac{c}{d}$

★ Payı ile paydası arasındaki fark aynı olan pozitif basit kesirlerden payı küçük olan daha küçüktür.

In positive proper fractions whose differences between numerators and denominators are equal, the fraction with smaller numerator is the smaller fraction.

$\frac{23}{27} < \frac{25}{29} < \frac{37}{41}$,

$\frac{1}{2} < \frac{15}{16} < \frac{51}{52}$.

★ Payı ile paydası arasındaki fark aynı olan, pozitif bileşik kesirlerden payı büyük olan daha küçüktür.

In positive improper fractions whose differences between numerators and denominators are equal, the fraction with bigger numerator is the smaller fraction.

$\frac{36}{31} < \frac{24}{19} < \frac{12}{7}$,

$\frac{50}{49} < \frac{25}{24} < \frac{3}{2}$.

★ Pozitif basit kesirlerin kuvvetleri büyüdükçe kesrin değeri küçülür.

Increasing the powers of the positive proper fractions, the values of fractions decrease.

$\left(\frac{1}{3}\right)^2 > \left(\frac{1}{3}\right)^3 > \left(\frac{1}{3}\right)^4 > \dots$

★ İki rasyonel sayının ortasındaki sayıyı bulmak için bu sayılar, toplanıp ikiye bölünür.

$\frac{3}{4}$ ile $\frac{11}{6}$ sayılarının ortasındaki sayı,

To find the number in the middle of two rational numbers, the sum of them is divided by 2.

The number in the middle of $\frac{3}{4}$ and $\frac{11}{6}$ is

$\frac{1}{2}\cdot\left(\frac{3}{4}+\frac{11}{6}\right) = \frac{1}{2}\cdot\frac{31}{12} = \frac{31}{24}$

PEKİŞTİRME / REINFORCE

✓ $a = \frac{73}{75}$, $b = \frac{75}{77}$, $c = \frac{77}{79}$ $\Rightarrow$? > ? > ?

.....[a < b < c]

✓ $a = -\frac{85}{87}$, $b = -\frac{87}{89}$, $c = -\frac{89}{91}$ $\Rightarrow$? > ? > ?

.....[c < b < a]

PEKİŞTİRME / REINFORCE

✓ $\frac{3}{10} = ?$
.....[0,3]

✓ $\frac{2}{10} = ?$
.....[0,2]

✓ $\frac{11}{1000} = ?$
.....[0,011]

✓ $\frac{13}{100} = ?$
.....[0,13]

ONDALIK SAYILAR / DECIMAL NUMBERS

Tanım: Paydası 10, 100, 1000 gibi 10 un kuvveti olan kesirlere *ondalık kesir,* bu kesrin belirttiği sayıya da **ondalık sayı** denir.

Definition: The fractions whose denominators are powers of 10 like 10, 100, 1000 are called *decimal fractions* and the values of these fractions are called *decimal numbers.*

Etkinlikler / Activities

a. $\frac{2}{10} = 0,2$

b. $\frac{11}{100} = 0,11$

c. $5\frac{15}{1000} = 5,015$

d. $4\frac{2}{100} = 4,02$

PEKİŞTİRME / REINFORCE

✓ $\frac{2}{3} = ?$
.....[0,666...]

✓ $\frac{3}{4} = ?$
.....[0,75]

✓ $\frac{1}{5} = ?$
.....[0,2]

✓ $\frac{2}{25} = ?$
.....[0,08]

BİR RASYONEL SAYIYI ONDALIK SAYIYA ÇEVİRME
CONVERTING a RATIONAL NUMBER into a DECIMAL NUMBER

Bir rasyonel sayıyı ondalık sayıya çevirirken, aşağıdaki iki kuraldan biri uygulanır:

- Paydaki sayı, paydadaki sayıya bölünür.
- Paydadaki sayı, 10 un kuvveti olarak yazıldıktan sonra ondalık sayıya çevrilir.

To convert a rational number into a decimal number we use either of the two following rules:

- Divide the numerator into the denominator.
- After writing denominator as a power of 10 convert it into decimal number.

Etkinlikler / Activities

a. $\frac{1}{3} = 0,333....$

b. $\frac{3}{5} = \frac{3 \cdot 2}{5 \cdot 2} = \frac{6}{10} = 0,6$

c. $\frac{4}{25} = \frac{4}{25} \cdot \frac{4}{4} = \frac{16}{100} = 0,16$

d. $\frac{3}{125} = \frac{3}{125} \cdot \frac{8}{8} = \frac{24}{1000} = 0,024$

e. $\frac{250}{200} = \frac{250:2}{200:2} = \frac{125}{100} = 1,25$

ONDALIK SAYILARDA İŞLEMLER
OPERATIONS in DECIMAL NUMBERS

TOPLAMA / ADDITION

- Ondalık sayılar alt alta yazılır.
- Ondalık virgülünün sağındaki her bir sütunda yer alan her sayı için ondalık virgülünün sağına sıfır eklenir.
- Tam sayılardaki gibi toplama yapılır.
- Sayılar toplandıktan sonra ondalık virgülü yerleştirilir.
- Decimals are written in a column vertically.
- Add zeros to the right of the decimal comma for all of the numbers having an entry in each column on the right of the decimal comma.
- Add the numbers in the same way with the whole numbers.
- After adding the numbers a decimal comma is placed.

PEKİŞTİRME / REINFORCE

✓ 0,4 + 0,6 = ?
..... [1]

✓ 0,5 + 0,3 = ?
..... [0,8]

✓ 0,2 + 0,7 = ?
..... [0,9]

✓ 0,5 + 0,11 = ?
..... [0,61]

Örnek / Example 8

0,5 + 0,9 = ?

A) 1 B) 1,4 C) 2,2 D) 2,4 E) 2,6

Çözüm / Solution Yanıt / Answer **B**

$$\begin{array}{r} 0{,}5 \\ +\ 0{,}9 \\ \hline 1{,}4 \end{array}$$

PEKİŞTİRME / REINFORCE

✓ 2,113 + 2,109 = ?
..... [4,222]

✓ 1,12 + 0,15 = ?
..... [1,27]

✓ 0,09 + 0,19 = ?
..... [0,28]

✓ 2,13 + 0,19 = ?
..... [2,32]

Örnek / Example 9

2,013 + 13,378 + 3,615 = ?

A) 19,006 B) 20,006 C) 22,006 D) 23,006 E) 24,006

Çözüm / Solution Yanıt / Answer **A**

$$\begin{array}{r} 2{,}013 \\ 13{,}378 \\ +\ 3{,}615 \\ \hline 19{,}006 \end{array}$$

PEKİŞTİRME / REINFORCE

✓ 0,2 – 0,3 = ?

..... [–0,1]

✓ 0,12 – 0,08 = ?

..... [0,04]

✓ 0,134 – 0,104 = ?

..... [0,03]

✓ 0,315 – 0,009 = ?

..... [0,306]

ÇIKARMA / SUBTRACTION

- Virgüller ve aynı adlı basamaklar, alt alta gelecek şekilde yazılır.
- Eksilen sayının, çıkan ondalıklı kesre göre eksik basamağı varsa, yerine sıfır yazılır.
- Sayılar tam sayılardaki gibi çıkarılır.
- Bulunan fark, virgüllerin hizasından yine virgülle ayrılır.
- Commas and places in the same name are written in a column vertically.
- If the minuend number has deficient places according to the subtrahend decimal fraction, add zeros instead.
- Subtract the numbers as you would with the whole numbers.
- The difference is seperated with a decimal comma.

Örnek / Example 10

0,54 – 0,12 = ?

A) 0,12 B) 0,18 C) 0,24 D) 0,32 E) 0,42

Çözüm / Solution

Yanıt / Answer **E**

$$\begin{array}{r} 0{,}54 \\ -\ 0{,}12 \\ \hline 0{,}42 \end{array}$$

PEKİŞTİRME / REINFORCE

✓ 0,139 – 0,019 = ?

..... [0,12]

✓ 1,23 – 0,03 = ?

..... [1,2]

✓ 1,13 – 0,09 = ?

..... [1,04]

✓ 0,07 – 0,02 = ?

..... [0,05]

Örnek / Example 11

325,04 – 4,21 = ?

A) 320,78 B) 320,79 C) 320,83 D) 320,96 E) 320,98

Çözüm / Solution

Yanıt / Answer **C**

$$\begin{array}{r} 325{,}04 \\ -\ 4{,}21 \\ \hline 320{,}83 \end{array}$$

ÇARPMA / MULTIPLICATION

Çarpma işlemi virgül yokmuş gibi yapılır. İşlemin sonunda, çarpılan sayıların virgülden sonraki basamak sayılarının toplamı kadar, sağdan sola doğru virgül ile ayrılır.

In the multiplication of decimal numbers, do the multiplication like there is no comma. At the end of the operation, the result is seperated with a comma from right to left as much as sum of the numbers of the digits after comma in multiplicand numbers.

Örnek / Example 12

4,33 · 3,5 = ?

A) 11,135 B) 12,155 C) 13,155 D) 14,145 E) 15,155

Çözüm / Solution

Yanıt / Answer **E**

4,33 ⟶ virgülden sonra iki sayı / two digits after comma
x 3,5 ⟶ virgülden sonra bir sayı / one digit after comma
2165
+ 1299
15,155 → (1 + 2 = 3 sayı sayılıp virgül yazılır / count and place the comma)

PEKİŞTİRME / REINFORCE

✓ **0,12 × 0,13 = ?**
..... [0,0156]

✓ **0,01 × 0,1 = ?**
..... [0,001]

✓ **1,13 × 1,4 = ?**
..... [1,582]

✓ **1,7 × 2,2 = ?**
..... [3,74]

BÖLME / DIVISION

- ▼ Bölen sayı ondalık sayıdan kurtuluncaya kadar, bölendeki ondalık virgülü sağa taşınır.
- ▼ Adım 1'de taşınan ondalık virgül kadar bölünen sayıda da sağa taşınır.
- ▼ Bölünen ve bölen tam sayı ise 2. adımdaki sonuç 1.ye bölünür.
- ▼ Bölümün ondalık kısmının sayısı, 2. adımdaki sonucun ondalık kısmının sayısına eşittir.
- ▼ Bölünen sayının sağına sıfır ekleyerek elde edilebilir ve 3. adım tekrar edilir. Bölünene eklenen her bir sıfır için bölümde birden fazla ondalık kısım gerekir.
- ▼ Move the decimal comma in the divisor to the right until there is no decimal part in the divisor (this is the same as multiplying the divisor by a power of 10.)
- ▼ Move the decimal comma in the dividend the same number of places to the right as you moved the decimal comma in step 1.
- ▼ Divide the result of step (2) by the result of step (1) if they were whole numbers.
- ▼ The number of decimal places in the result (quotient) is equal to the number of decimal places in the result of step (2).
- ▼ You may obtain as many decimal places as you wish in the quotient by adding zeros to the right of the dividend and then repeating step (3). For each zero you add to the dividend, you need one more decimal place in the quotient.

PEKİŞTİRME / REINFORCE

✓ **0,13 : 0,01 = ?**
..... [13]

✓ **0,09 : 0,3 = ?**
..... [0,3]

✓ **0,06 : 0,02 = ?**
..... [3]

✓ **−1,14 : 0,03 = ?**
..... [−38]

Örnek / Example 13

(36) : (1,8) = ?

A) 15 B) 18 C) 20 D) 22 E) 24

Çözüm / Solution Yanıt / Answer **C**

$$36 \mid 1{,}8 \rightarrow \begin{array}{r|l} 360 & 18 \\ 36 & 20 \\ \hline 00 & \end{array} \Rightarrow 36 : 1{,}8 = 20$$

Örnek / Example 14

(51,8) : (1,4) = ?

A) 37 B) 39 C) 41 D) 43 E) 45

Çözüm / Solution Yanıt / Answer **A**

$$51{,}8 \mid 1{,}4 \rightarrow \begin{array}{r|l} 518 & 14 \\ 42 & 37 \\ \hline 98 & \\ 98 & \\ \hline 00 & \end{array}$$

$\Rightarrow 51{,}8 : 1{,}4 = 37$

Örnek / Example 15

$\left(\frac{3}{5} - \frac{2}{5}\cdot 0{,}05\right) : 0{,}29 = ?$

A) $\frac{1}{4}$ B) 2 C) 4 D) 1 E) $\frac{1}{145}$

Çözüm / Solution Yanıt / Answer **B**

$$\left(\frac{3}{5} - \frac{2}{5}\cdot 0{,}05\right) : 0{,}29 = \left(\frac{3}{5} - \frac{2}{5}\cdot\frac{5}{100}\right) : \frac{29}{100}$$

$$= \left(\frac{3}{5} - \frac{2}{100}\right) : \frac{29}{100}$$

$$= \frac{60-2}{100} : \frac{29}{100}$$

$$= \frac{58}{100}\cdot\frac{100}{29} = \frac{58}{29} = 2$$

ONDALIK BİR SAYIYI RASYONEL BİR SAYIYA ÇEVİRME
CONVERTING a DECIMAL NUMBER into a RATIONAL NUMBER

Ondalık bir sayıyı rasyonel sayıya çevirirken;

- Tam kısmı varsa yazılır.
- Virgülden sonra kaç tane sıfır varsa, o sayı 10 un kuvveti olarak paydaya yazılır.
- Virgülden sonraki rakamlar olduğu gibi paya yazılır.
- Varsa sadeleştirme yapılır.

To convert a decimal number into a rational number;

- If there is a whole part present, write it down.
- The number of zeros after comma is written as the power of 10 to denominator.
- The digits after comma are written to the numerator as they are.
- If simplification is possible, it's done.

PEKİŞTİRME / REINFORCE

✓ 0,04 = ? ..... $\left[\frac{1}{25}\right]$

✓ 0,375 = ? ..... $\left[\frac{3}{8}\right]$

✓ 4,2 = ? ..... $\left[\frac{21}{5}\right]$

✓ 1,02 = ? ..... $\left[\frac{51}{50}\right]$

Etkinlikler / Activities

a. $0,25 = \frac{25}{100} = \frac{1}{4}$

b. $1,025 = 1 + \frac{25}{1000} = 1 + \frac{1}{40} = 1\frac{1}{40}$

c. $3,0050 = 3 + \frac{50}{10000} = 3 + \frac{1}{200} = 3\frac{1}{200}$

d. $4,025 = 4 + \frac{25}{1000} = 4\frac{1}{40}$

e. $12,004 = 12 + \frac{4}{1000} = 12\frac{1}{250}$

f. $0,0125 = \frac{125}{10000} = \frac{1}{80}$

Örnek / Example 16

$$\frac{x,0\overline{x} - y,0\overline{y}}{x,\overline{x} - y,\overline{y}} = ?$$

A) $\frac{91}{100}$ B) $\frac{9}{10}$ C) $\frac{100}{91}$ D) $\frac{100}{91}$ E) $\frac{11}{9}$

Çözüm / Solution

Yanıt / Answer **A**

$$\frac{\frac{(x0x - x0) - (y0y - y0)}{90}}{\frac{(xx - x) - (yy - y)}{9}} = \frac{\frac{91(x-y)}{90}}{\frac{10(x-y)}{9}} = \frac{91}{100}$$

Örnek / Example 17

$\frac{0,144}{0,12} + \frac{1,21}{1,1} = ?$

A) 2,9 B) 2,7 C) 2,5 D) 2,3 E) 2,1

Çözüm / Solution Yanıt / Answer **D**

$$\frac{0,144}{0,12} + \frac{1,21}{1,1} = \frac{144}{120} + \frac{121}{110}$$
$$= \frac{12}{10} + \frac{11}{10} = \frac{23}{10} = 2,3$$

Örnek / Example 18

$$\frac{\frac{6,5}{0,13} - \frac{0,42}{0,21}}{\frac{1,9}{0,76}} = ?$$

A) 19 B) 19,2 C) 19,4 D) 19,6 E) 19,8

Çözüm / Solution Yanıt / Answer **B**

$$\frac{\frac{650}{13} - \frac{42}{21}}{\frac{190}{76}} = \frac{50-2}{\frac{5}{2}}$$
$$= \frac{48 \cdot 2}{5}$$
$$= \frac{96}{5}$$
$$= 19,2$$

PEKİŞTİRME / REINFORCE

✓ **35,244...= ?**

..... $35,24\overline{}$

✓ **1,555... = ?**

..... $1,\overline{5}$

✓ **3,123123... = ?**

..... $3,\overline{123}$

✓ **0, 2222... = ?**

..... $0,\overline{2}$

DEVİRLİ ONDALIK SAYILAR / RECURRING DECIMALS

Tanım: Bir rasyonel sayı ondalık sayı biçiminde yazıldığında virgülden sonra gelen kısımdaki sayılar belli bir rakamdan sonra tekrar ediyorsa bu sayılara ***devirli ondalık sayı*** denir.

Definition: When a rational number is written in the decimal form, if the numbers coming after the comma repeats infinitely afterwards a certain number, then these numbers are called *recurring (repeating) or periodic decimals.*

Etkinlikler / Activities

a. $\frac{3}{10} = 0,3333.....= 0,\overline{3}$

b. $0,42222.....= 0,4\overline{2}$

c. $5,6575757.....= 5,6\overline{57}$

DEVİRLİ ONDALIK SAYININ RASYONEL SAYIYA DÖNÜŞTÜRÜLMESİ
CONVERTIONY a RECCURING DECIMAL NUMBER to a RATIONAL NUMBER

$$ab,cd\overline{efg} = \frac{abcdefg - abcd}{99900}$$

Etkinlikler / Activities

a. $0,\overline{4} = \frac{4-0}{9} = \frac{4}{9}$

b. $2,\overline{1} = \frac{21-2}{9} = \frac{19}{9}$

c. $12,\overline{3} = \frac{123-12}{9} = \frac{111}{9} = \frac{37}{3}$

d. $0,2\overline{5} = \frac{25-2}{90} = \frac{23}{90}$

e. $5,1\overline{8} = \frac{518-51}{90} = \frac{467}{90}$

f. $32,1\overline{54} = \frac{32154-321}{990} = \frac{31833}{990} = \frac{3537}{110}$

PEKİŞTİRME / REINFORCE

✓ $0,0\overline{3} = ?$ ..... $\left[\frac{1}{30}\right]$

✓ $0,\overline{1} = ?$ ..... $\left[\frac{1}{9}\right]$

✓ $0,05\overline{2} = ?$ ..... $\left[\frac{47}{900}\right]$

✓ $0,1\overline{3} = ?$ ..... $\left[\frac{2}{15}\right]$

✓ $2,3444... = ?$ ..... $\left[\frac{211}{90}\right]$

Örnek / Example 19

$$\frac{0,2\overline{1}+0,1\overline{2}}{0,2\overline{1}-0,1\overline{6}} = ?$$

A) 5,5 B) 6 C) 6,5 D) 7 E) 7,5

Çözüm / Solution

Yanıt / Answer **E**

$$\frac{0,2\overline{1}+0,1\overline{2}}{0,2\overline{1}-0,1\overline{6}} = \frac{\frac{21-2}{90}+\frac{12-1}{90}}{\frac{21-2}{90}-\frac{16-1}{90}}$$

$$= \frac{\frac{19+11}{90}}{\frac{19-15}{90}} = \frac{30}{90}\cdot\frac{90}{4} = \frac{30}{4} = \frac{15}{2} = 7,5$$

ÖNEMLİ / IMPORTANT

• *Devirli ondalık sayılarda toplama işlemi yapılırken aynı basamakta devreden sayıların toplamı 10 dan küçükse toplama işlemi tam sayılarda olduğu gibi yapılır.*

• *In reccuring decimals, when doing the addition, if the reccuring digits in same place add up to a number less than 10, the addition is done like when in integer numbers.*

Örnek / Example 20

$1,\overline{1}+2,\overline{3}+3,\overline{4}-4,\overline{6}=?$

A) $\frac{10}{9}$ B) $\frac{5}{3}$ C) $\frac{20}{9}$ D) $\frac{25}{9}$ E) $\frac{10}{3}$

Çözüm / Solution Yanıt / Answer **C**

$$1,\overline{1}+2,\overline{3}+3,\overline{4}-4,\overline{6}=6,\overline{8}-4,\overline{6}$$
$$=2,\overline{2}$$
$$=\frac{22-2}{9}=\frac{20}{9}$$

ÖNEMLİ / IMPORTANT

• Devirli ondalık sayılarda devreden kısım 9 ise sayı bir üst basamağa yuvarlanır.

$4,1\overline{9}=4,2$

$3,\overline{9}=4$

• In reccuring numbers, if the repeating part is 9, the number will round-up to one bigger place

$4,1\overline{9}=4,2$

$3,\overline{9}=4$

Örnek / Example 21

$x=3,4\overline{57}$
$y=3,\overline{457}$ $\Rightarrow$? > ? > ?
$z=3,45\overline{7}$

A) x > z > y B) z > y > x C) z > x > y D) y > z > x E) x > y > z

Çözüm / Solution Yanıt / Answer **C**

$x=3,4\overline{57}=3,45757...$
$y=3,\overline{457}=3,457457...$
$z=3,45\overline{7}=3,457777...$
$\Rightarrow z>x>y$

Örnek / Example 22

a, b, c ve d birer rakam

a, b, c and d are numbers

$5,\overline{15} < a,bcd < 5,1\overline{5}$

$\Rightarrow \sum d = ?$

A) 3 B) 5 C) 7 D) 9 E) 11

Çözüm / Solution

Yanıt / Answer **D**

$5,\overline{15} = 5,151515...$

$5,1\overline{5} = 5,1555...$

$5,151 < a,bcd < 5,155$

$\sum d = 2 + 3 + 4 = 9$

Örnek / Example 23

$\frac{3}{27} + \frac{5}{11} + \frac{28}{99} = 0,\overline{ab} \Rightarrow a + b = ?$

A) 12 B) 11 C) 10 D) 9 E) 8

Çözüm / Solution

Yanıt / Answer **A**

$\frac{3}{27} = \frac{1}{9} = 0,\overline{1} = 0,1111...$

$\frac{5}{11} = \frac{45}{99} = 0,\overline{45} = 014545...$

$\frac{28}{99} = 0,\overline{28} = 0,282828$

$$\begin{array}{r} 0,111111... \\ 0,454545... \\ +\ 0,282828... \\ \hline 0,848484 \end{array}$$

$\frac{3}{27} + \frac{5}{11} + \frac{28}{99} = 0,8484... = 0,\overline{84}$

$0,\overline{ab} = 0,\overline{84}$

$\Rightarrow a + b = 12$

1. $\dfrac{1}{1+\dfrac{1}{1-\dfrac{1}{2}}} = ?$

A) $\frac{3}{4}$ B) $\frac{4}{3}$ C) $\frac{1}{3}$ D) 1 E) 3

2. $\frac{6}{7}\left[\frac{1}{3}-\left(\frac{1}{2}-\left(\frac{1}{4}+\frac{1}{2}\right)\right)\right] = ?$

A) $\frac{1}{2}$ B) $\frac{1}{7}$ C) $\frac{1}{12}$ D) $\frac{7}{12}$ E) $\frac{2}{7}$

3. $\dfrac{\left(8-\frac{1}{3}\right)+\left(\frac{1}{3}+4\right)}{\left(7+\frac{5}{6}\right)+\left(6+\frac{1}{6}\right)} = ?$

A) $\frac{7}{5}$ B) $\frac{6}{7}$ C) $\frac{5}{8}$ D) $\frac{8}{5}$ E) -1

4. $\dfrac{1+a}{1-\dfrac{1-a}{1-\dfrac{1}{a}}} = ?$

A) $-a$ B) -1 C) 0 D) 1 E) a

5. $\dfrac{3x-6}{3} - \dfrac{2x+4}{2} = ?$

A) 2 B) x C) $x-2$ D) 4 E) -4

6. $2+\dfrac{x}{2+\dfrac{x}{2+\frac{x}{\vdots}}} = 3 \Rightarrow x = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

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7. $5 \cdot (0{,}03 + 0{,}02 - 0{,}04) = ?$

A) 0,01 B) 0,02 C) 0,03 D) 0,04 E) 0,05

8. $\frac{1}{0{,}001} + \frac{2}{0{,}02} + \frac{3}{0{,}3} = ?$

A) 111 B) 123 C) 1110 D) 1111 E) 1230

9. $(2{,}397 + 0{,}3 \cdot 0{,}01) : 0{,}001 - 400 = ?$

A) 2000 B) 2380 C) 2390 D) 2397 E) 2400

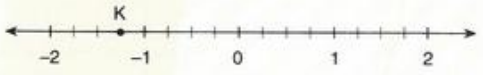
10. $\frac{0{,}3}{x} = \frac{0{,}9}{0{,}03} \Rightarrow x = ?$

A) 0,01 B) 0,1 C) 1 D) 1,1 E) 10

11. $0,\overline{3} + \frac{1}{2 + \frac{3}{2}} = ?$

A) $\frac{13}{21}$ B) $\frac{3}{7}$ C) $\frac{11}{21}$ D) $\frac{3}{4}$ E) $\frac{8}{23}$

12.



Yukarıdaki sayı doğrusunda –2 ile 2 arasında 16 eşit aralık vardır. Buna göre, aşağıdakilerden hangisi K noktasının koordinatıdır?

On the number line above there are twelve equal sixteen intervals between –2 and 2, what number is the coordinate of point K?

A) $-\frac{3}{4}$ B) $-1\frac{1}{4}$ C) $-1\frac{1}{3}$ D) $-1\frac{1}{2}$ E) $-1\frac{3}{4}$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
C	A	B	D	E	C	E	C	A	A	A	B

1.
$$\frac{1}{1+\frac{1}{1-\frac{1}{2}}}=\frac{1}{1+\frac{1}{\frac{1}{2}}}=\frac{1}{1+2}=\frac{1}{3}$$

Yanıt / The Answer is **C**

2.
$$\frac{6}{7}\left[\frac{1}{3}-\left(\frac{1}{2}-\left(\frac{1}{4}+\frac{1}{2}\right)\right)\right]=\frac{6}{7}\left[\frac{1}{3}-\left(\frac{1}{2}-\left(\frac{1+2}{4}\right)\right)\right]$$

$$=\frac{6}{7}\cdot\left[\frac{1}{3}-\left[\frac{1}{2}-\frac{3}{4}\right]\right]$$

$$=\frac{6}{7}\cdot\left[\frac{1}{3}-\left(\frac{2-3}{4}\right)\right]$$

$$=\frac{6}{7}\cdot\left[\frac{1}{3}+\frac{1}{4}\right]$$

$$=\frac{6}{7}\cdot\frac{4+3}{12}=\frac{6}{7}\cdot\frac{7}{12}=\frac{1}{2}$$

Yanıt / The Answer is **A**

3.
$$\frac{\left(8-\frac{1}{3}\right)+\left(\frac{1}{3}+4\right)}{\left(7+\frac{5}{6}\right)+\left(6+\frac{1}{6}\right)}=\frac{\frac{24-1}{3}+\frac{1+12}{3}}{\frac{42+5}{6}+\frac{36+1}{6}}$$

$$=\frac{\frac{23+13}{3}}{\frac{47+37}{6}}$$

$$=\frac{\frac{36}{3}}{\frac{84}{6}}=\frac{12}{14}=\frac{6}{7}$$

Yanıt / The Answer is **B**

4.
$$\frac{1+a}{1-\frac{1-a}{1-\frac{1}{a}}}=\frac{1+a}{1-\frac{1-a}{\frac{a-1}{a}}}$$

$$=\frac{1+a}{1+\frac{a(a-1)}{a-1}}$$

$$=\frac{1+a}{1+a}=1$$

Yanıt / The Answer is **D**

5.
$$\frac{3x-6}{3}-\frac{2x+4}{2}=\frac{2(3x-6)-3(2x+4)}{6}$$

$$=\frac{6x-12-6x-12}{6}=\frac{-24}{6}=-4$$

Yanıt / The Answer is **E**

6.
$$2+\frac{x}{2+\frac{x}{2+\frac{x}{\vdots}}}\to 3$$

$$2+\frac{x}{3}=3\Rightarrow\frac{6+x}{3}=3$$

$$6+x=9$$

$$x=3$$

Yanıt / The Answer is **C**

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7. $5\cdot(0,03 + 0,02 - 0,04) = 5\cdot(0,05 - 0,04)$

$= 5\cdot(0,01)$

$= 0,05$

Yanıt / The Answer is E

8. $\frac{1}{0,001} + \frac{2}{0,02} + \frac{3}{0,3} = \frac{1}{\frac{1}{1000}} + \frac{2}{\frac{2}{100}} + \frac{3}{\frac{3}{10}}$

$= 1000 + 100 + 10$

$= 1110$

Yanıt / The Answer is C

9. $(2,397 + 0,3\cdot0,01) : 0,001 - 400$

$= (2,397 + 0,003) : 0,001 - 400$

$= 2,4 : 0,001 - 400 = 2400 - 400 = 2000$

Yanıt / The Answer is A

10. $\frac{0,3}{x} = \frac{0,9}{0,03}$

$\frac{3}{10}\cdot\frac{1}{x} = \frac{9}{10}\cdot\frac{100}{3}$

$\frac{1}{10\cdot x} = 10$

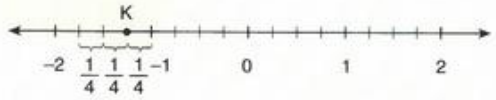
$x = \frac{1}{100} = 0,01$

Yanıt / The Answer is A

11. $0,\overline{3} + \frac{1}{2+\frac{3}{2}} = \frac{3}{9} + \frac{1}{2+\frac{3}{2}} = \frac{1}{3} + \frac{2}{7} = \frac{13}{21}$

Yanıt / The Answer is A

12.



$\Rightarrow K = -1 - \frac{1}{4} = -1\frac{1}{4}$

Yanıt / The Answer is B

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1. $B = \frac{17}{9} + \frac{19}{11} - \frac{3}{13}$

$\Rightarrow \frac{8}{11} - \frac{1}{9} - \frac{16}{13} = ?$

A) B + 2 B) 2B + 2 C) B – 2
D) 2B + 4 E) B – 4

[GAZİANTEP ÜNİVERSİTESİ – YÖS 2020]

2. **Aşağıdaki sayılardan hangisi, $\frac{7}{2}$ ve $\frac{21}{5}$ sayılarına eşit uzaklıktadır?**

Which of the following numbers is in the same distance with the numbers $\frac{7}{2}$ and $\frac{21}{5}$?

A) $\frac{77}{20}$ B) $\frac{18}{5}$ C) $\frac{96}{25}$
D) $\frac{83}{20}$ E) 4

[KARABÜK ÜNİVERSİTESİ – YÖS 2020]

3. $\frac{\frac{5}{1}}{0,5} - \frac{0,5}{\frac{1}{5}} = ?$

A) 25 B) $\frac{5}{2}$ C) $\frac{1}{2}$
D) 5 E) 0

[KARABÜK ÜNİVERSİTESİ – YÖS 2020]

4. **$2,\overline{1897}$ sayısının virgülden sonraki 153. rakamı aşağıdakilerden hangisidir?**

What is the 153rd digit of $2,\overline{1897}$ after comma?

A) 1 B) 2 C) 7
D) 8 E) 9

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2020]

5. $\left(\frac{3}{7} + \frac{4}{11} - \frac{5}{14}\right) - \left(\frac{4}{11} + \frac{9}{14} + \frac{3}{7}\right) = ?$

A) –2 B) –1 C) 0
D) $-\frac{1}{2}$ E) $-\frac{3}{2}$

[AKDENİZ ÜNİVERSİTESİ – YÖS 2019]

6. $A = \frac{1}{9} + \frac{2}{11} + \frac{3}{13}$

$B = \frac{29}{13} + \frac{19}{9} + \frac{24}{11}$

$\Rightarrow$ **B = ?**

A) 9 + A B) 3 + A C) –3 + A
D) –6 + A E) 6 + A

[DOKUZ EYLÜL ÜNİVERSİTESİ – YÖS 2019]

7. $A = \frac{3}{29} + \frac{5}{32} + \frac{7}{39}$

$B = \frac{61}{29} + \frac{101}{32} + \frac{202}{39} \Rightarrow$ **B = ?**

A) 10A B) 10 + A C) 100 + A
D) 100A E) 9A

[DUMLUPINAR ÜNİVERSİTESİ – YÖS 2018]

8. $\frac{\frac{5,7}{0,19} - \frac{0,51}{0,17}}{\frac{1,4}{0,56}} = ?$

A) 6,8 B) 8,4 C) 8,6
D) 9,5 E) 10,8

[İSTANBUL ÜNİVERSİTESİ – YÖS 2018]

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9. $\left(\frac{3}{2}-\frac{5}{4}\right)-\left[\left(2:\frac{1}{4}\right)\cdot\frac{1}{4}-1\right]=?$

A) $\frac{-4}{5}$ B) $\frac{-3}{4}$ C) $\frac{-1}{4}$ D) $\frac{1}{4}$ E) $\frac{3}{4}$

[İNÖNÜ ÜNİVERSİTESİ – YÖS 2018]

10. $100\frac{1}{4}-200\frac{1}{3}+101\frac{1}{12}=?$

A) 1 B) 2 C) $\frac{7}{3}$ D) $\frac{5}{2}$ E) 3

[BALIKESİR ÜNİVERSİTESİ – YÖS 2017]

11. $\frac{3}{1-\frac{1}{3}}-3-\frac{\frac{1}{3}}{3}=?$

A) $\frac{-10}{9}$ B) $\frac{25}{18}$ C) $\frac{10}{9}$ D) $\frac{-19}{9}$ E) $\frac{17}{18}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2017]

12. $\frac{1}{4}+\left[\frac{1}{2}:\left(\frac{5}{6}-\frac{2}{3}\right)\right]=?$

A) $\frac{5}{2}$ B) $\frac{7}{4}$ C) $\frac{13}{4}$ D) $\frac{15}{4}$ E) 5

[SELÇUK ÜNİVERSİTESİ – YÖS 2017]

13. $a=5,\ b=2 \Rightarrow \frac{0,03}{0,002}+\frac{0,1}{0,02}+\frac{0,03}{0,005}=?$

A) a + b B) 2a + b C) 3a – b D) 4a + 3b E) 5a + 2b

[İSTANBUL ÜNİVERSİTESİ – YÖS 2017]

14. $\frac{0,7}{0,07}+\frac{3,6}{6}+\frac{3}{20}=?$

A) 1,05 B) 1,75 C) 10,5 D) 10,75 E) 100,7

[KARADENİZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

15. $\frac{2-\frac{2}{3}-3}{\frac{1-\frac{2}{3}}{4}}=?$

A) –2 B) $-\frac{1}{6}$ C) $-\frac{4}{3}$ D) –10 E) –20

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2015]

16. $5:\frac{5}{4}-2+3\cdot4+3:\frac{1}{2}=?$

A) $\frac{31}{2}$ B) $\frac{3}{2}$ C) $\frac{71}{4}$ D) 20 E) 26

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2015]

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	A	E	A	B	E	B	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	A	B	C	D	D	E	D

1. $A = \frac{8}{11} - \frac{1}{9} - \frac{16}{3}$

$B = \frac{17}{9} + \frac{19}{11} - \frac{3}{13}$

$\Rightarrow B - A = \frac{17}{9} + \frac{1}{9} + \frac{19}{11} - \frac{8}{11} - \frac{3}{13} + \frac{16}{13}$

$B - A = 2 + 1 + 1$

$A = B - 4$

Yanıt / The Answer is E

---

2. $\frac{\frac{7}{2} + \frac{21}{5}}{2} = \frac{\frac{77}{10}}{2} = \frac{77}{20}$

Yanıt / The Answer is A

---

3. $\frac{5}{\frac{1}{0,5}} - \frac{0,5}{\frac{1}{5}} = \frac{5}{2} - \frac{5}{2} = 0$

Yanıt / The Answer is E

---

4. $2,\overline{1897} = 2,18971897\ldots$

$153 \mid 4$

$-\ \ \ \ \mid$

1

$\Rightarrow$ 153. rakam 1 dir. / The 153rd digit is 1.

Yanıt / The Answer is A

---

5. $\frac{3}{7} + \frac{4}{11} - \frac{5}{14} - \frac{4}{11} - \frac{9}{14} - \frac{3}{7} = -\frac{5}{14} - \frac{9}{14}$

$= -\frac{14}{14}$

$= -1$

Yanıt / The Answer is B

---

6. $-1/A = \frac{1}{9} + \frac{2}{11} + \frac{3}{13}$

$+ \quad B = \frac{29}{13} + \frac{19}{9} + \frac{24}{11}$

$B - A = 2 + 2 + 2$

$= 6$

$\Rightarrow B = A + 6$

Yanıt / The Answer is E

---

7. $A = \frac{3}{29} + \frac{5}{32} + \frac{7}{39}$

$-1/B = \frac{61}{29} + \frac{101}{32} + \frac{202}{39}$

$+$

$A - B = -2 - 3 - 5$

$A + 10 = B$

Yanıt / The Answer is B

---

8. $\frac{\frac{570}{19} - \frac{51}{17}}{\frac{140}{56}} = \frac{30 - 3}{\frac{5}{2}} = \frac{27 \cdot 2}{5} = 10,8$

Yanıt / The Answer is E

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9. $\left(\frac{3}{2}-\frac{5}{4}\right)-\left[\left(2:\frac{1}{4}\right)\cdot\frac{1}{4}-1\right]=$

$\frac{6-5}{4}-\left[2\cdot\frac{4}{1}\cdot\frac{1}{4}-1\right]=$

$\frac{1}{4}-[2-1]=\frac{1-4}{4}=\frac{-3}{4}$

Yanıt / The Answer is B

---

10. $100\frac{1}{4}-200\frac{1}{3}+101\frac{1}{12}$

$(100-200+101)+\left(\frac{1}{4}-\frac{1}{3}+\frac{1}{12}\right)=$

$1+\frac{3-4+1}{12}=1$

Yanıt / The Answer is A

---

11. $\frac{\frac{3}{1-\frac{1}{3}}-3-\frac{1}{3}}{3}$

$\frac{\frac{3}{3-1}}{3}-3-\frac{1}{3}\cdot\frac{1}{3}=$

$\frac{9}{2}-\frac{3}{1}-\frac{1}{9}=\frac{81-54-2}{18}=\frac{25}{18}$

$(9)\quad(18)\quad(2)$

Yanıt / The Answer is B

---

12. $\frac{1}{4}+\left[\frac{1}{2}:\left(\frac{5}{6}-\frac{2}{3}\right)\right]=$

$(1)\quad(2)$

$\frac{1}{4}+\left[\frac{1}{2}:\frac{5-4}{6}\right]=\frac{1}{4}+\frac{1}{2}\cdot\frac{6}{1}=\frac{13}{4}$

Yanıt / The Answer is C

---

13. $a=5,\ b=2$

$\frac{0,03}{0,002}+\frac{0,1}{0,02}+\frac{0,03}{0,005}=$

$\frac{30}{2}+\frac{10}{2}+\frac{30}{5}=$

$15+5+6=4\cdot5+3\cdot2=4a+3b$

Yanıt / The Answer is D

---

14. $\frac{0,7}{0,07}+\frac{3,6}{6}+\frac{3}{20}=\frac{70}{7}+\frac{36}{60}+\frac{3}{20}$

$=10+\frac{3}{5}+\frac{3}{20}$

$=\frac{215}{20}=\frac{1075}{100}=10,75$

Yanıt / The Answer is D

---

15. $\frac{\frac{2-\frac{2}{3}-3}{1-\frac{2}{3}}}{4}=\frac{\frac{6-2-9}{3}}{\frac{1}{3}}$

$=\frac{\frac{-5}{3}}{\frac{1}{12}}$

$=-\frac{5}{3}\cdot\frac{12}{1}=-20$

Yanıt / The Answer is E

---

16. $5:\frac{5}{4}-2+3\cdot4+3:\frac{1}{2}=5\cdot\frac{4}{5}-2+12+3\cdot2$

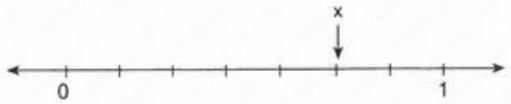
$=4-2+12+6$

$=20$

Yanıt / The Answer is D

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1.



Yukarıdaki sayı doğrusunda 0 ile 1 arasında 7 eşit aralık vardır.

Buna göre x'in değeri kaçtır?

On the number line above, there are 7 equal spaced intervals between 0 and 1. **What is the value of x ?**

A) $\frac{1}{7}$ B) $\frac{3}{7}$ C) $\frac{4}{7}$

D) $\frac{5}{7}$ E) $\frac{6}{7}$

2. $\frac{1}{x}-\left(\frac{1}{y}-\frac{1}{t}\right)-\left(\frac{1}{x}-\frac{1}{y}\right)-\frac{1}{t}=?$

A) 0 B) $\frac{2}{x}$ C) $\frac{2}{y}$

D) $\frac{2}{t}$ E) $-\frac{2}{y}$

3. $1^{-1}-2^{-1}-3^{-1}-6^{-1}=?$

A) 1 B) 0 C) $\frac{1}{2}$

D) $\frac{1}{3}$ E) 10

4. $\frac{1}{2}+\frac{1}{2}:\frac{1}{3}-1=?$

A) $-\frac{1}{3}$ B) $\frac{1}{3}$ C) 1

D) 2 E) 3

5. $\dfrac{1-\frac{2}{3}}{1-\frac{1}{1+\frac{2}{5}}}=?$

A) $\frac{22}{15}$ B) $\frac{14}{15}$ C) $\frac{15}{7}$

D) $\frac{7}{6}$ E) $\frac{13}{6}$

6. $\frac{\frac{1}{2}}{3}+\frac{1}{\frac{2}{3}}+\frac{\frac{1}{3}}{2}+\frac{1}{6}=?$

A) 1 B) 2 C) 3

D) 4 E) 5

7. $\left(1+\frac{1}{1-\frac{1}{3}}\right)\cdot\left(1+\frac{1+\frac{1}{3}}{2}\right)\cdot 6=?$

A) 36 B) 30 C) 25

D) 20 E16

8. $3-\dfrac{2-\frac{1}{2}}{3-\frac{2}{1-\frac{3}{2}}}=?$

A) $\frac{19}{7}$ B) $\frac{9}{2}$ C) $\frac{37}{9}$

D) $\frac{39}{14}$ E) $\frac{3}{2}$

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9. $\dfrac{1-\frac{1}{3}}{1+\frac{1}{3}}+\dfrac{x}{3}=2 \Rightarrow x=?$

A) $-\frac{1}{3}$ B) $\frac{1}{3}$ C) $\frac{3}{4}$ D) $\frac{9}{2}$ E) $\frac{15}{2}$

10. $2\frac{1}{2} : \dfrac{\frac{1}{3}}{1+\frac{1}{9}} = ?$

A) $\frac{3}{4}$ B) $\frac{1}{2}$ C) $\frac{25}{3}$ D) $\frac{5}{2}$ E) $\frac{51}{2}$

11. $\dfrac{\frac{2}{3}}{\frac{2}{3}-1} : \dfrac{1+\frac{2}{3}}{2+\frac{1}{2}} = ?$

A) -1 B) -2 C) 2 D) -3 E) 3

12. $a=\frac{3}{4},\ b=\frac{4}{5},\ c=\frac{3}{5} \Rightarrow ? > ? > ?$

A) $b > a > c$ B) $b > c > a$ C) $a > b = c$ D) $a > b > c$ E) $c > a > b$

13. $\frac{2}{3} : \frac{3}{2} - \frac{1}{2}\cdot\left(\frac{3}{4}-\frac{1}{3}\right) = ?$

A) $\frac{17}{72}$ B) $\frac{13}{36}$ C) $\frac{13}{18}$ D) $\frac{17}{18}$ E) $\frac{7}{36}$

14. $\dfrac{\frac{1}{2} : \frac{2}{3}\cdot\frac{3}{4}}{2-\frac{5}{4}} = ?$

A) $\frac{-1}{9}$ B) $\frac{-9}{16}$ C) $\frac{-3}{16}$ D) $\frac{3}{4}$ E) $\frac{-1}{4}$

15. $\dfrac{2}{1-\frac{4}{5}} - \dfrac{2}{\frac{1}{3}-1} = ?$

A) $\frac{10}{3}$ B) $\frac{12}{5}$ C) $\frac{20}{3}$ D) 13 E) 15

16. $\dfrac{2}{2-\frac{2}{3}} + \dfrac{1+\frac{1}{3}}{2} = ?$

A) $\frac{5}{2}$ B) $\frac{7}{3}$ C) $\frac{13}{6}$ D) $\frac{9}{4}$ E) $\frac{11}{3}$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	A	B	C	D	B	C	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	C	D	A	A	D	D	C

1. $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{6}+\frac{5}{12}}-1=?$

A) $\frac{6}{7}$ B) 1 C) $\frac{12}{7}$ D) $\frac{13}{7}$ E) 2

2. $\left(-\frac{3}{5}\right)^{0}\cdot\left(-\frac{1}{3}\right)^{-1}-\frac{1}{1-\frac{1}{2}}=?$

A) –5 B) –4 C) –3 D) –1 E) 0

3. $\left[\dfrac{1-\frac{2}{3}}{2-\frac{1}{3}}\right]^{-1}:5=?$

A) $\frac{1}{5}$ B) $\frac{1}{2}$ C) 1 D) 3 E) 5

4. $\frac{1}{5}+\frac{3}{5}+\frac{5}{5}+\ldots+\frac{19}{5}=?$

A) 18 B) 19 C) 20 D) 21 E) 22

5. $4+\dfrac{12}{2+\dfrac{4}{2-\frac{1}{x}}}=6 \Rightarrow x=?$

A) –1 B) 0 C) 1 D) 2 E) 3

6. $\dfrac{2+\dfrac{3}{1+\frac{1}{3}}}{1+\dfrac{2}{1+\frac{1}{2}}}=?$

A) $\frac{51}{28}$ B) $\frac{50}{49}$ C) $\frac{46}{25}$ D) $\frac{45}{14}$ E) $\frac{40}{7}$

7. $\left(1+\frac{1}{n}\right)\cdot\left(1+\frac{1}{n+1}\right)\cdot\left(1+\frac{1}{n+2}\right)\cdot\ldots\cdot\left(1+\frac{1}{n+10}\right)=12$
$\Rightarrow n=?$

A) 1 B) 2 C) 3 D) 4 E) 5

8. $6+\dfrac{a}{6+\dfrac{a}{6+\frac{a}{\vdots}}}=10 \Rightarrow a=?$

A) 30 B) 32 C) 34 D) 36 E) 40

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9. $\dfrac{\left(\frac{1}{3}\right)^{-1}-(-3)^{-1}}{3-\frac{1}{3}}=?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{3}{4}$ D) 1 E) $\frac{5}{4}$

10. $a+\frac{a}{10}+\frac{a}{100}+\ldots=\frac{40}{9} \Rightarrow a=?$

A) 1 B) 2 C) 3 D) 4 E) 5

11. $a=(2b)^{-1} \Rightarrow \left(b+\frac{1}{a}\right)\left(a+\frac{1}{b}\right)=?$

A) 9 B) $\frac{9}{2}$ C) 3 D) $\frac{9}{4}$ E) $\frac{9}{5}$

12. $\dfrac{\frac{8}{9}}{2}+\dfrac{3}{\frac{18}{7}}=?$

A) $\frac{3}{2}$ B) $\frac{14}{9}$ C) $\frac{29}{18}$ D) $\frac{5}{3}$ E) $\frac{31}{18}$

13. $\frac{1}{2}+\frac{2}{2}+\frac{3}{2}+\ldots+\frac{20}{2}=?$

A) $\frac{101}{2}$ B) 51 C) $\frac{103}{2}$ D) 52 E) $\frac{105}{2}$

14. $\dfrac{3\frac{1}{2}-2\frac{3}{4}}{4\frac{1}{3}-1\frac{1}{4}}=?$

A) $\frac{7}{37}$ B) $\frac{8}{37}$ C) $\frac{9}{37}$ D) $\frac{10}{37}$ E) $\frac{11}{37}$

15. $\dfrac{105}{22}=a+\dfrac{1}{b+\dfrac{1}{c+\frac{2}{5}}} \Rightarrow a+b+c=?$

A) 2 B) 4 C) 6 D) 8 E) 10

16. $\left(1\frac{1}{2}\right)\cdot\left(1\frac{1}{3}\right)\cdot\left(1\frac{1}{4}\right)\ldots\left(1\frac{1}{20}\right)=A \Rightarrow A=?$

A) $2\frac{1}{2}$ B) $4\frac{1}{2}$ C) $5\frac{1}{2}$ D) $8\frac{1}{2}$ E) $10\frac{1}{2}$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	A	C	C	C	A	A	E
9	10	11	12	13	14	15	16
E	D	B	C	E	C	D	E

1. $3{,}075 = ?$

A) $\frac{123}{40}$ B) $\frac{40}{9}$ C) $\frac{15}{4}$

D) $\frac{1}{8}$ E) $\frac{33}{10}$

2. $\frac{5{,}1}{0{,}017} + \frac{0{,}09}{0{,}003} + \frac{1}{0{,}1} = ?$

A) 610 B) 601 C) 340

D) 331 E)304

3. $\frac{0{,}25}{2{,}5} + \frac{1{,}01}{0{,}1} + \frac{15}{0{,}02} = ?$

A) 77,1 B) 95,1 C) 186

D) 760,2 E) 861

4. $(0{,}734 + 0{,}266)\,(1 - 0{,}99) = ?$

A) 0,01 B) 0,1 C) 0,101

D) 0,2 E) 0,02

5. $(0{,}842 - 0{,}342) \cdot (0{,}118 + 0{,}082) = ?$

A) 0,02 B) 0,4 C) 0,2

D) 0,01 E) 0,1

6. $\frac{0{,}012 : 0{,}03}{0{,}12 + 0{,}06} : \frac{5}{9} = ?$

A) $\frac{10}{9}$ B) $\frac{18}{5}$ C) $\frac{2}{5}$

D) 4 E) 6

7. $\left[(0{,}24 : 0{,}072) \cdot \frac{3}{10}\right] - 0{,}25 = ?$

A) 1 B) $\frac{3}{2}$ C) $\frac{3}{4}$

D) $\frac{1}{4}$ E) 0

8. $\frac{0{,}35}{0{,}7} - \frac{0{,}56}{0{,}28} - \frac{0{,}001}{0{,}002} = ?$

A) –3 B) –2 C) 1

D) 2 E) 3

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9. $\dfrac{20+\dfrac{0,01}{0,001}}{0,04+\dfrac{0,04}{4}} = ?$

A) 600 B) 60 C) 6

D) 0,6 E) 0,06

10. $\dfrac{0,1\overline{2}}{0,0\overline{1}} + \dfrac{0,0\overline{6}}{0,0\overline{3}} = ?$

A) 11 B) 12 C) 13

D) 14 E) 15

11. $\dfrac{0,1\overline{7}}{0,0\overline{4}} - \dfrac{0,1\overline{3}}{0,0\overline{1}} = ?$

A) –2 B) –4 C) –6

D) –8 E) –10

12. $x = 0,05\overline{2}$

$y = 0,0\overline{52}$

$z = 0,\overline{052} \Rightarrow ? < ? < ?$

A) $x < y < z$ B) $x < z < y$ C) $z < y < x$

D) $z < x < y$ E) $y < x < z$

13. $\dfrac{(2,\overline{3})^2 - (4,\overline{3})^2}{6+\dfrac{2}{3}} = ?$

A) –4 B) –2 C) 0

D) 2 E) 4

14. $\dfrac{1+\dfrac{1-0,4}{3}}{0,2} = ?$

A) 0,6 B) 1,2 C) 1,8

D) 6 E) 8

15. $\dfrac{0,\overline{5}}{0,05} + \dfrac{0,\overline{7}}{0,07} - \dfrac{1,\overline{9}}{0,09} = ?$

A) 0,6 B) 0,4 C) 0,2

D) 0,1 E) 0

16. $0,3 \cdot \dfrac{7,\overline{2}-0,\overline{2}}{6,\overline{4}+0,\overline{5}} : 2,\overline{6} = ?$

A) $\frac{-4}{5}$ B) $\frac{-8}{5}$ C) $\frac{4}{5}$

D) $\frac{3}{40}$ E) $\frac{9}{80}$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	C	D	A	E	D	C	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	C	D	D	B	D	E	E

1. $0,4 + 0,7 - 0,1 = ?$

A) 0,1 B) 0,9 C) 1

D) 1,1 E) 10

2. $0,8 - 0,2 : 0,1 = ?$

A) −1,8 B) −1,2 C) −1

D) 1 E) 1,2

3. $\frac{1}{0,1} + \frac{1}{0,2} - \frac{1}{0,5} = ?$

A) 1,3 B) 1,5 C) 2

D) 10 E) 13

4. $\frac{0,01}{0,001} + \frac{0,2}{0,02} + \frac{0,003}{0,0003} = ?$

A) 10 B) 20 C) 30

D) 40 E) 50

5. $\frac{1,2}{2,4} + 0,5 - 0,1 = ?$

A) 0,8 B) 0,9 C) 1

D) 1,1 E) 1,2

6. $\left(\frac{0,2}{2} + \frac{0,3}{3}\right) : 0,1 = ?$

A) 0,1 B) 0,2 C) 1

D) 1,5 E) 2

7. $\frac{(0,009 - 0,03)\cdot 10^3}{0,7} = ?$

A) −30 B) −15 C) 1

D) 1,5 E) 15

8. $(3\cdot 0,3 - 1,2 : 2) : 6^{-1} = ?$

A) −1,8 B) −1,2 C) 1

D) 1,2 E) 1,8

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9. $\frac{7}{10}+\frac{7}{100}+\frac{7}{1000}+1=?$

A) 0,007 B) 0,077 C) 0,777

D) 1,007 E) 1,777

10. $0,4-\frac{0,1+0,2}{\frac{6}{10}}=?$

A) –0,2 B) –0,1 C) 0,1

D) 0,2 E) 1

11. $\frac{0,a}{a}+\frac{0,0a}{a}+\frac{0,00a}{a}=?$

A) 0,1 B) 0,01 C) 0,11

D) 0,011 E) 0,111

12. $\left(\frac{a}{0,a}+\frac{b}{0,b}+\frac{c}{0,c}\right)\cdot 0,3=?$

A) 0,3 B) 0,9 C) 3

D) 9 E) 30

13. $\frac{ab,cd}{a,bcd}+\frac{a,bcd}{0,abcd}-\frac{abc,d}{ab,cd}=?$

A) 0,1 B) 0,2 C) 1

D) 10 E) 100

14. $\frac{30}{0,6}-\frac{24}{0,8}+\frac{15}{0,5}=?$

A) 10 B) 20 C) 30

D) 40 E) 50

15. $\frac{4,4}{1,1}+\frac{32}{1,6}-\frac{64}{1,6}=?$

A) –24 B) –16 C) –10

D) 10 E) 20

16. A = 4, 821

B = 5, 179

$\Rightarrow A^2+2AB+B^2=?$

A) 0,01 B) 0,1 C) 1

D) 10 E) 100

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	B	E	C	B	E	A	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	B	E	D	D	E	B	E

1. $\frac{0,0025 \cdot 0,05}{125 \cdot 10^{-3}} = ?$

A) 10^{-3} B) 10^{-2} C) 10^{-1}

D) 1 E) 10

2. $\frac{0,048}{0,004} + \frac{0,01}{0,001} - \frac{0,16}{0,04} = ?$

A) 12 B) 14 C) 16

D) 18 E) 21

3. $\frac{0,2 + 0,02 + 0,002\ldots}{0,9 + 0,\overline{1} - 0,5} = ?$

A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$

D) $\frac{4}{5}$ E) 1

4. $\left(\frac{6}{5} + 1,4 + 5^{-1} \cdot 2\right)^{-1} = ?$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1

D) 2 E) 3

5. $\left.\begin{matrix} A = 0,a\overline{2} \\ B = 0,2\overline{a} \end{matrix}\right\}$, $A + B = \frac{8}{9} \Rightarrow a = ?$

A) 2 B) 4 C) 6

D) 8 E) 10

6. $\frac{0,\overline{a}}{0,b} = 6 \Rightarrow a - b = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

7. $\frac{(3,04)^2 - (2,04)^2}{0,508} = ?$

A) 0,1 B) 1 C) 10

D) 1,1 E) 10^2

8. $\frac{0,12}{1,2} + \frac{1,21}{1,1} - 0,1 = ?$

A) 4 B) 1,1 C) 1,2

D) 1,3 E) 1,4

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9. $\frac{1,\overline{1}+2,\overline{2}+3,\overline{3}}{5,5\overline{5}} = ?$

A) $\frac{2}{5}$ B) $\frac{3}{5}$ C) $\frac{4}{5}$

D) 1 E) $\frac{6}{5}$

10. A, $x \in Z$,

$\frac{0,009 \cdot 0,00027}{(0,9)^2} = A \cdot 10^x \Rightarrow \min(A+x) = ?$

A) –4 B) –3 C) –1

D) 2 E) 3

11. $\frac{aaa+bbb+ccc}{aa,a+bb,b+cc,c} = ?$

A) 10^{-1} B) 1 C) 10

D) 10^2 E) 10^3

12. A = 3,801

B = 5,801

$\Rightarrow A^3 - 3A^2B + 3AB^2 - B^3 = ?$

A) –8 B) –4 C) 4

D) 9 E) 16

13. $\frac{0,a\overline{b}+0,b\overline{a}}{3} = \frac{5}{9} \Rightarrow \max(a \cdot b) = ?$

A) 50 B) 52 C) 54

D) 56 E) 58

14. $a = 0,45\overline{6}$

$b = 0,4\overline{56}$

$c = 0,\overline{456}$

$\Rightarrow ? > ? > ?$

A) a > b > c B) b > a > c C) c > a > b

D) b > c > a E) a > c > b

15. $0,x+0,\overline{y} = \frac{29}{45}$

$\Rightarrow y - x = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

16. $\frac{(-0,04)^{-3} \cdot (-5)^2 \cdot (0,001)^2}{(-5)^3 \cdot 4^{-2}} = ?$

A) 0,2 B) 0,1 C) 0,01

D) 0,05 E) 0,001

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	D	B	A	C	E	C	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	B	C	A	D	A	B	D

BÖLÜM CHAPTER 3

TARİHSEL NOT / HISTORICAL NOTE

Omar Khayyam [1048 – 1122]

Ömer Hayyam, anlaşılması güç matematiksel konuları, şiirsel bir dille açıklayan, derin felsefi bilgisi olan bir düşünür ve şairdir. O astronomik tablolar oluşturarak, yeni bir takvimin ortaya çıkmasına katkıda bulunmuştur. Ayrıca bir çemberin, bir parabol ile kesişiminden oluşan ve üçüncü dereceden denklemleri içeren problemleri çözerek yeni geometrik yöntemler de geliştirmiştir.

Omar Khayyam was a scholar who was a poet as well as a mathematician. He compiled astronomical tables and contributed to calendar reform and discovered a geometrical method of solving cubic equations by intersecting a parabola with a circle.

ÜSLÜ SAYILAR
EXPONENTIALS

Bu bölüm, 40 çözümlü örnek, 12 çözümlü test sorusu, 16 çözümlü YÖS sorusu, 128 test sorusu içermektedir.

This chapter includes 40 examples with solutions, 12 tests questions with solutions, 16 YÖS questions with solutions and 128 test questions.

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BÖLÜM
03
CHAPTER

ÜSLÜ SAYILAR
EXPONENTIALS

ÖNEMLİ / IMPORTANT

- Eğer kuvvet 0 ise, ifadenin değeri her zaman 1 dir.
- Eğer kuvvet 1 ise, ifadenin değeri aynı tabandaki sayıya eşittir.
- If the exponent is 0, the expression always has a value of 1.
- If the exponent is 1, the value of the expression is equal to the base number.

ÖNEMLİ / IMPORTANT

- $-a^{2n}$, a^{2n} in negatifi anlamındadır, –a'nın 2n. kuvveti anlamında değildir. –a'nın 2n. kuvveti $(-a)^{2n}$ ile gösterilir.
- $-a^{2n}$ means the negative of a^{2n}, not the $(2n)^{th}$ power of –a which would be represented by $(-a)^{2n}$

$-a^{2n} \neq a^{2n}$

$-4^4 = -4 \cdot 4 \cdot 4 \cdot 4 = -256$

$4^4 = 4 \cdot 4 \cdot 4 \cdot 4 = 256 \Rightarrow -4^4 \neq 4^4$

PEKİŞTİRME / REINFORCE

✓ $2^0 = ?$

..... [1]

✓ $(-7)^0 = ?$

..... [1]

✓ $5^0 = ?$

..... [1]

✓ $0^0 = ?$

..... [Belirsiz]

[Indefinite]

✓ $(2195)^0 = ?$

..... [1]

ÜSLÜ SAYILAR / EXPONENTIALS

Tanım: a sıfırdan farklı bir gerçel sayı ve n bir doğal sayı olmak üzere,

$\underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{\text{n çarpan}} = a^n$ ifadesine, *üslü ifade* denir.

n : Üs (kuvvet) , **a :** Taban

a^n ifadesi, a nın n. kuvveti olarak okunur.

Definition: Provided that $n \in N$ and $a \in R - \{0\}$, the expression

$\underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{\text{n factors}} = a^n$ is called ***exponential expression.***

n : power , **a :** base.

The expression a^n is read as " a to the n^{th} power".

Etkinlikler / Activities

a. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

b. $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

c. $(-4)^2 = (-4) \,.\, (-4) = 16$

d. $1^7 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$

e. $(-4)^3 = (-4) \cdot (-4) \cdot (-4) = -64$

f. $\left(\frac{1}{3}\right)^4 = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{81}$

Özellik / Property

★ Sıfırdan farklı bir sayının sıfırıncı kuvveti 1 dir.
Zeroth power of a number different than zero is 1.
$a \neq 0 \Rightarrow a^0 = 1$

Etkinlikler / Activities

a. $3^0 = 1$

b. $412^0 = 1$

c. $(-382)^0 = 1$

d. $\left(\frac{4}{5}\right)^0 = 1$

Özellik / Property

★ $a \in R^+$ ve $n \in Z$

I. $(-a)^{2n} = a^{2n}$

II. $(-a)^{2n-1} = -a^{2n-1}$

Etkinlikler / Activities

a. $(-3)^4 = (-3)\cdot(-3)\cdot(-3)\cdot(-3) = 81 = 3^4$

b. $(-4)^2 = (-4)\cdot(-4) = 16 = 4^2$

c. $(-5)^3 = (-5)\cdot(-5)\cdot(-5) = -125$

d. $(-2)^5 = (-2)\cdot(-2)\cdot(-2)\cdot(-2)\cdot(-2) = -32$

Örnek / Example 1

$$\frac{(-2)^6\cdot 4^{-2}\cdot(-3)^3}{3^5\cdot(-2)^4} = ?$$

A) $\frac{-1}{36}$ B) $\frac{-1}{24}$ C) $\frac{-1}{6}$ D) $\frac{1}{24}$ E) $\frac{1}{36}$

Çözüm / Solution

Yanıt / Answer **A**

$$\frac{(-2)^6\cdot 2^{-4}\cdot(-3)^3}{3^5\cdot(-2)^4} = \frac{2^6\cdot 2^{-4}\cdot(-3)^3}{3^5\cdot 2^4} = \frac{-1}{36}$$

Örnek / Example 2

$x > 0$

$$\frac{(-x)^3\cdot x^2\cdot(-x^2)\cdot(-x)^4}{x^3\cdot(-x)^2\cdot(-x)^4} = 4 \Rightarrow x = ?$$

A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution

Yanıt / Answer **B**

$$\frac{-x^3\cdot x^2\cdot(-x^2)\cdot x^4}{x^3\cdot x^2\cdot x^4} = 4$$

$$x^2 = 4$$

$$x = 2$$

PEKİŞTİRME / REINFORCE

✓ $(-2)^6 = ?$ ..... [64]

✓ $(-2^6) = ?$ ..... [−64]

✓ $(-3)^3 = ?$ ..... [−27]

✓ $(-3^3) = ?$ ..... [−27]

PEKİŞTİRME / REINFORCE

✓ $(-3)^2 + 3^2 + (-3^2) + (-3)^3 = ?$

..... [−18]

✓ $3^2 - (-2)^2 + 3^4 = ?$

..... [86]

✓ $(-2)^2 + 2^3 - 2^2 = ?$

..... [8]

Örnek / Example 3

$(-2^2) + (-5)^2 + (-5^2) - (-2)^3 = ?$

A) −12 B) 4 C) 46 D) 62 E) 38

Çözüm / Solution Yanıt / Answer **B**

$(-2^2) + (-5)^2 + (-5^2) - (-2)^3 = -4 + 25 - 25 - (-8) = 4$

Örnek / Example 4

$$\frac{(-2)^3 \cdot (-3)^2 \cdot (-5)^3}{2^2 \cdot (-3)^3 \cdot (-5)^2} = ?$$

A) $-\frac{10}{3}$ B) $\frac{10}{3}$ C) $-\frac{5}{3}$ D) $\frac{5}{3}$ E) $-\frac{2}{3}$

Çözüm / Solution Yanıt / Answer **A**

$$\frac{(-8)\cdot 9\cdot (-125)}{4\cdot (-27)\cdot 25} = -2 \cdot \frac{-1}{3} \cdot (-5)$$

$$= -\frac{10}{3}$$

Örnek / Example 5

$$\frac{\left(-\frac{1}{2}\right)^3 \cdot \left(\frac{2}{3}\right)^3 \cdot (-2)^2}{(-2^2) \cdot \left(-\frac{1}{3}\right)^2} = ?$$

A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{1}{5}$ D) $\frac{1}{6}$ E) $\frac{1}{7}$

Çözüm / Solution Yanıt / Answer **A**

$$\frac{\left(-\frac{1}{2}\right)^3 \cdot \left(\frac{2}{3}\right)^3 \cdot (-2)^2}{(-2^2) \cdot \left(-\frac{1}{3}\right)^2} = \frac{-\frac{1}{8} \cdot \frac{8}{27} \cdot 4}{-4 \cdot \frac{1}{9}} = \frac{1}{2} \cdot \frac{8}{27} \cdot \frac{9}{1} \cdot \frac{1}{4} = \frac{1}{3}$$

Özellik / Property

★ I. $x^{2n} \geq 0,\ n \in Z$

II. $(f(x))^{2n} + (g(x))^{2n} = 0$ gibi denklemlerin tek çözümü, $f(x) = 0$ ve $g(x) = 0$ dır.

★ I. $x^{2n} \geq 0,\ n \in Z$

II. The only solution of the equations like $(f(x))^{2n} + (g(x))^{2n} = 0$ is $f(x) = 0$ and $g(x) = 0$

PEKİŞTİRME / REINFORCE

✓ $x, y \in R \Rightarrow \min(x^2 - 2x + y^2 + 4y) = ?$

..... [–5]

Örnek / Example 6

$(x - 2)^2 + (x - y - 5)^4 = 0 \Rightarrow x \cdot y = ?$

A) 6 B) 3 C) 0 D) –1 E) –6

Çözüm / Solution

Yanıt / Answer **E**

$(x - 2)^2 + (x - y - 5)^4 = 0$

$x - 2 = 0$, $x - y - 5 = 0$

$x = 2$ $2 - y - 5 = 0$

$y = -3$

$\Rightarrow x \cdot y = 2 \cdot (-3) = -6$

Örnek / Example 7

$A = x^2 - 4x + 12 \Rightarrow \min(A) = ?$

A) 4 B) 6 C) 8 D) 10 E) 12

Çözüm / Solution

Yanıt / Answer **C**

$A = x^2 - 4x + 4 + 8$

$= (x - 2)^2 + 8$

$\Rightarrow \min(A) = 0 + 8 = 8$

PEKİŞTİRME / REINFORCE

✓ $3x^2 - 4x^2 = ?$

..... [$-x^2$]

✓ $5x - 4x = ?$

..... [x]

✓ $3x^n - 2x^n = ?$

..... [x^n]

✓ $3m^2 - 2m^2 = ?$

..... [m^2]

✓ $5x^2 - 4x^2 + 3x^2 = ?$

..... [$4x^2$]

ÜSLÜ İFADELERDE İŞLEMLER / OPERATIONS in EXPONENTIALS

TOPLAMA ve ÇIKARMA / ADDITION and SUBSTRACTION

Ortak çarpan parantezine alma özelliği kullanılarak üslü sayıların toplam veya farkı tek bir eşitlik ile ifade edilebilir.

Using the common factor brackets property, you can express the sum and difference of exponents as a single form.

$a \cdot x^n + b \cdot x^n - c \cdot x^n = (a + b - c) \cdot x^n$

Etkinlikler / Activities

a. $3x^2 + 2x^2 = (3 + 2) \cdot x^2 = 5x^2$

b. $2 \cdot a^3 - 3 \cdot a^3 + 5 \cdot a^3 = (2 - 3 + 5) \cdot a^3 = 4 \cdot a^3$

c. $3 \cdot 5^4 - 4 \cdot 5^4 = (3 - 4) \cdot 5^4 = -5^4$

d. $5 \cdot 2^5 - 3 \cdot 2^5 + 7 \cdot 2^5 = (5 - 3 + 7) \cdot 2^5 \quad = 9 \cdot 2^5$

ÇARPMA / MULTIPLICATION

▼ Tabanları aynı olan üslü sayıların çarpımında üsler toplanır.

When multiplying of exponentials with same base, add exponents.

$a^n \cdot a^m = a^{n+m}$

▼ Üsleri aynı olan üslü sayıların çarpımında, tabanlar çarpılarak ortak üs, üs olarak yazılır.

In multiplication of exponentials with same power, multiply the base and write the power.

$a^n \cdot b^n = (a \cdot b)^n$

PEKİŞTİRME / REINFORCE

✓ $2^5 \cdot 2^0 \cdot 2^3 = ?$

..... [2^8]

✓ $5^2 \cdot 5^6 = ?$

..... [5^8]

✓ $3^4 \cdot 3^0 = ?$

..... [3^4]

Etkinlikler / Activities

a. $3^4 \cdot 3^3 = 3^{4+3} = 3^7$

b. $2^5 \cdot 2^3 \cdot 2^{-2} = 2^{5+3+(-2)} = 2^6$

c. $4^7 \cdot 4^6 \cdot 4^{-3} = 4^{7+6+(-3)} = 4^{10}$

d. $(-2)^5 \cdot (-2)^4 \cdot (-2)^{-3} = (-2)^{5+4+(-3)} = (-2)^6$

e. $2^{a-b} \cdot 2^{2a-b} \cdot 2^{a+b} = 2^{a-b+2a-b+a+b} = 2^{4a-b}$

f. $2^2 \cdot 3^2 \cdot 7^2 = (2 \cdot 3 \cdot 7)^2 = (42)^2$

Örnek / Example 8

$4 \cdot 2^{a+1} - 3 \cdot 2^{a+3} + 8 \cdot 2^{a+2} = ?$

A) 2^{a+2} B) 2^{a+4} C) 2^{a-2} D) 2^{a-4} E) 2^{a-6}

Çözüm / Solution

Yanıt / Answer **B**

$$4 \cdot 2^{a+1} - 3 \cdot 2^{a+3} + 8 \cdot 2^{a+2} = 4 \cdot 2^a \cdot 2 - 3 \cdot 2^a \cdot 2^3 + 8 \cdot 2^a \cdot 2^2$$
$$= 2^a \cdot (8 - 24 + 32)$$
$$= 2^a \cdot 16$$
$$= 2^a \cdot 2^4 = 2^{a+4}$$

Örnek / Example 9

$2^x + 2^{x+1} = m \cdot 2^{x+2} \Rightarrow m = ?$

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{2}{3}$ D) $\frac{3}{4}$ E) $\frac{7}{8}$

Çözüm / Solution

Yanıt / Answer **D**

$$m = \frac{2^x + 2^{x+1}}{2^{x+2}} = \frac{2^x(1+2)}{2^x \cdot 2^2} = \frac{3}{4}$$

Örnek / Example 10

$a > 1, b > 1$

$$\left.\begin{matrix} a^{x-y} = b^9 \cdot a^{11} \\ b^{x-y} = a^8 \cdot b^{10} \end{matrix}\right\} \Rightarrow x - y = ?$$

A) 35 B) 27 C) 21 D) 19 E) 17

Çözüm / Solution

Yanıt / Answer **D**

$$\begin{matrix} a^{x-y} = b^9 \cdot a^{11} \\ \times\ b^{x-y} = \times\ a^8 \cdot b^{10} \end{matrix}$$

$$a^{x-y} \cdot b^{x-y} = b^9 \cdot a^{11} \cdot a^8 \cdot b^{10}$$
$$(a \cdot b)^{x-y} = (a \cdot b)^{19}$$
$$x - y = 19$$

PEKİŞTİRME / REINFORCE

✓ $\frac{2^{n+1}+2^{n-2}}{6\cdot 2^n-3\cdot 2^{n-1}}=k \Rightarrow k=?$

..... $\left[\frac{1}{2}\right]$

Örnek / Example 11

$\frac{2\cdot 3^{x+1}+3^{x-1}-3^{x+2}}{4\cdot 3^{x-1}}=?$

A) -2 B) -1 C) 0 D) 1 E) 2

Çözüm / Solution Yanıt / Answer **A**

$$\frac{2\cdot 3^{x+1}+3^{x-1}-3^{x+2}}{4\cdot 3^{x-1}}=\frac{2\cdot 3^x\cdot 3+3^x\cdot 3^{-1}-3^x\cdot 3^2}{4\cdot 3^x\cdot 3^{-1}}$$

$$=\frac{3^x\left(6+\frac{1}{3}-9\right)}{3^x\cdot\frac{4}{3}}=\frac{-\frac{8}{3}}{\frac{4}{3}}=-2$$

ÖNEMLİ / IMPORTANT

- Çarpma ve bölme için verilen kurallar, toplama ve çıkarma işlemi için uygulanamaz.
- The rules of multiplying and dividing exponents do not apply to addition or subtraction.

BÖLME / DIVISION

▼ Tabanları aynı olan iki üslü sayının bölümünde, ortak taban üzerinde payın üssünden paydanın üssü çıkarılır.
In the division of exponentials with same base subtract the power of denominator from that of numerator.

$$\frac{a^n}{a^m}=a^{n-m}$$

▼ Üsleri aynı olan iki üslü sayının bölümünde, tabanların bölümüne ortak üs, üs olarak yazılır.
In the division of exponentials with same power, divide bases and write common power as the power.

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n,\ b\neq 0$$

ETKİNLİKLER / ACTIVITIES

a. $\frac{2^7}{2^4}=2^{7-4}=2^3$

b. $\frac{x^6}{x^4}=x^{6-4}=x^2$

c. $5^5:5^3=5^{5-3}=5^2$

d. $\frac{4^5\cdot 4^2}{4^3}=\frac{4^{5+2}}{4^3}=4^{7-3}=4^4$

e. $\frac{6^8}{2^8}=\left(\frac{6}{2}\right)^8=3^8$

f. $\frac{15^3}{3^3}=\left(\frac{15}{3}\right)^3=5^3$

g. $\frac{(x^2)^4}{(x^2)^2}=(x^2)^{4-2}=x^4$

Örnek / Example 12

$\frac{(-2)^{13}-2^{14}}{2^{13}} = ?$

A) 4 B) 3 C) 0 D) –3 E) –4

Çözüm / Solution

Yanıt / Answer **D**

$\frac{(-2)^{13}-2^{14}}{2^{13}} = \frac{-2^{13}-2^{13}\cdot 2}{2^{13}} = \frac{2^{13}(-1-2)}{2^{13}} = -3$

Örnek / Example 13

$\frac{5^x+10^x+15^x}{2^x+4^x+6^x} = \frac{8}{125} \Rightarrow x = ?$

A) 3 B) 2 C) 0 D) –2 E) –3

Çözüm / Solution

Yanıt / Answer **E**

$\frac{5^x+10^x+15^x}{2^x+4^x+6^x} = \frac{8}{125}$

$\frac{5^x\cdot(1+2^x+3^x)}{2^x\cdot(1+2^x+3^x)} = \frac{8}{125}$

$\left(\frac{5}{2}\right)^x = \left(\frac{5}{2}\right)^{-3}$

$x = -3$

Örnek / Example 14

$\frac{3^{44}-3^{22}}{3^{11}-1}\cdot\frac{3^{-22}}{3^{11}+1} = ?$

A) –3 B) –2 C) 1 D) 2 E) 3

Çözüm / Solution

Yanıt / Answer **C**

$\frac{3^{44}-3^{22}}{3^{11}-1}\cdot\frac{3^{-22}}{3^{11}+1} = \frac{3^{22}(3^{22}-1)}{3^{11}-1}\cdot\frac{3^{-22}}{3^{11}+1}$

$= \frac{3^{22}\cdot 3^{-22}\cdot(3^{22}-1)}{3^{22}-1}$

$= 3^0$

$= 1$

PEKİŞTİRME / REINFORCE

✓ $\frac{2^3}{4^3} = ?$ ..... [2^{-3}]

✓ $\frac{4^2}{4^3} = ?$ ..... [4^{-1}]

✓ $\frac{4^6}{4^7} = ?$ ..... [4^{-1}]

✓ $\frac{5^3}{5^9} = ?$ ..... [5^{-6}]

✓ $\frac{2^{1993}}{2^{1992}} = ?$ ..... [2]

PEKİŞTİRME / REINFORCE

✓ $\frac{2\cdot 3^x+4\cdot 3^x}{4\cdot 3^x-3^x} = ?$ ..... [2]

PEKİŞTİRME / REINFORCE

✓ $25^6\cdot 125^2\cdot 500^{10} = 2^x\cdot 5^y \Rightarrow x-y = ?$ ..... [–28]

PEKİŞTİRME / REINFORCE

✓ $\frac{3^4+3^5+3^6}{3^3+3^4+3^5} = ?$

..... [3]

ÖNEMLİ / IMPORTANT

- $(a^n)^m \neq (a)^{n^m}$

BİR ÜSLÜ İFADENİN KUVVETİ
POWER of an EXPONENTIAL

Bir üslü ifadenin kuvveti alınırken, üsler çarpılıp aynı taban üzerine üs olarak yazılır.
To raise an exponential expression to another power, multiply the powers and write the same base.

$(a^m)^n = (a^n)^m = a^{m \cdot n}$

Etkinlikler / Activities

a. $(3^2)^4 = 3^{2 \cdot 4} = 3^8$

b. $(2^3)^4 = 2^{3 \cdot 4} = 2^{12}$

c. $((-3)^2)^4 = (-3)^{2 \cdot 4} = (-3)^8 = 3^8$

d. $\left(\frac{3^2}{5^3}\right)^4 = \frac{3^{2 \cdot 4}}{5^{3 \cdot 4}} = \frac{3^8}{5^{12}}$

e. $[(-5)^3]^{-3} = (-5)^{3 \cdot (-3)} = (-5)^{-9}$

Örnek / Example 15

$9^4 \cdot 81^5 \cdot 243^7 = ?$

A) 3^{63} B) 3^{62} C) 3^{60} D) 3^{58} E) 3^{56}

Çözüm / Solution

Yanıt / Answer **A**

$$9^4 \cdot 81^5 \cdot 243^7 = 3^{2 \cdot 4} \cdot 3^{4 \cdot 5} \cdot 3^{5 \cdot 7}$$
$$= 3^8 \cdot 3^{20} \cdot 3^{35}$$
$$= 3^{8+20+35} = 3^{63}$$

Örnek / Example 16

$$\left.\begin{matrix} 4^{\frac{x}{2}+y} = 4 \\ 2^{x-2y} = 9 \end{matrix}\right\} \Rightarrow 2^x = ?$$

A) 6 B) 8 C) 10 D) 12 E) 16

Çözüm / Solution

Yanıt / Answer **A**

$$4^{\frac{x}{2}+y} = 4 \Rightarrow 2^{x+2y} = 4$$
$$(2^{x+2y}) \cdot (2^{x-2y}) = 4 \cdot 9$$
$$2^{x+2y+x-2y} = 36$$
$$2^{2x} = 6^2$$
$$2^x = 6$$

Örnek / Example 17

$9^a = x$ ve $3^{a+2} = y$

$\Rightarrow$ $y^2 = ?$

A) 9x B) 27x C) 36x D) 72x E) 81x

Çözüm / Solution

Yanıt / Answer **E**

$3^{a+2} = y \Rightarrow 3^a \cdot 3^2 = y$

$3^a = \frac{y}{9}$

$9^a = x \Rightarrow (3^a)^2 = x$

$\left(\frac{y}{9}\right)^2 = x \Rightarrow y^2 = 81x$

PEKİŞTİRME / REINFORCE

✓ $(2^2)^2 = ?$ ..... [2^4]

✓ $(2^3)^2 = ?$ ..... [2^6]

✓ $(2^{13})^1 = ?$ ..... [2^{13}]

✓ $(3^{1987})^1 = ?$ ..... [3^{1987}]

Örnek / Example 18

$(3^2)^5 + (-3^2)^5 - (-3^2)^5 + (-3^{-5})^{-2} = ?$

A) 3^{10} B) $2 \cdot 3^{10}$ C) $4 \cdot 3^{10}$ D) $5 \cdot 3^{10}$ E) $7 \cdot 3^{10}$

Çözüm / Solution

Yanıt / Answer **B**

$(3^2)^5 + (-3^2)^5 - (-3^2)^5 + (-3^{-5})^{-2} = 3^{10} + (-3^{10}) - (-3^{10}) + (3^{10})$

$= 3^{10} - 3^{10} + 3^{10} + 3^{10}$

$= 2 \cdot 3^{10}$

PEKİŞTİRME / REINFORCE

✓ $(-2^4)^3 + \left(-\frac{1}{2}\right)^{-12} - (-2^{-2})^{-3} = ?$ ..... [64]

✓ $12^n = 6^{n+1} \Rightarrow 4^n = ?$ ..... [36]

Örnek / Example 19

$(81)^x = 9^2 \Rightarrow x = ?$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

Çözüm / Solution

Yanıt / Answer **B**

$(81)^x = (9^2)^x = 9^2$

$9^{2x} = 9^2$

$2x = 2$

$x = 1$

ÜSLÜ SAYILARDA NEGATİF KUVVET
NEGATIVE POWERS in EXPONENTIALS

$a^{-1} = \frac{1}{a}$, $a^{-n} = \frac{1}{a^n}$

Etkinlikler / Activities

a. $2^{-1} = \frac{1}{2}$ **b.** $3^{-3} = \frac{1}{3^3}$ **c.** $4^{-2} = \frac{1}{4^2}$

Örnek / Example 20

$$\frac{(-2)^3 + (-2^2)}{\left(\frac{1}{2}\right)^{-2} + \frac{1}{2^{-3}}} = ?$$

A) –3 B) –2 C) –1 D) 1 E) 4

Çözüm / Solution

Yanıt / Answer **C**

$$\frac{(-2)^3 + (-2^2)}{\left(\frac{1}{2}\right)^{-2} + \frac{1}{2^{-3}}} = \frac{-8 + (-4)}{(2)^2 + 2^3} = \frac{-8-4}{4+8} = \frac{-12}{12} = -1$$

Örnek / Example 21

$$\frac{9^{n-2}}{3^{2n} \cdot 3^{-1}} + \frac{3^{m-1}}{3^m} = ?$$

A) $\frac{7}{27}$ B) $\frac{9}{27}$ C) $\frac{10}{27}$ D) $\frac{13}{27}$ E) $\frac{17}{27}$

Çözüm / Solution

Yanıt / Answer **C**

$$\frac{9^{n-2}}{3^{2n} \cdot 3^{-1}} + \frac{3^{m-1}}{3^m} = \frac{9^n \cdot 3}{9^n \cdot 81} + \frac{1}{3}$$

$$= \underset{(1)}{\frac{1}{27}} + \underset{(9)}{\frac{1}{3}}$$

$$= \frac{1+9}{27} = \frac{10}{27}$$

ONDALIK SAYILARIN ÜSLÜ GÖSTERİMİ
EXPONENTIAL NOTATION of DECIMAL NUMBERS

$0.\underbrace{000\ ...\ 000}_{\text{n rakam / n digits}}a = a \cdot 10^{-n}$

[virgülden sonra n rakam] [n digits after comma]

Etkinlikler / Activities

a. $0{,}00024 = 24 \cdot 10^{-5}$ $\quad 0{,}000004 = 4 \cdot 10^{-6}$

$0{,}2 = 2 \cdot 10^{-1}$

b. $0{,}00013 \cdot 0{,}003 = 13 \cdot 10^{-5} \cdot 3 \cdot 10^{-3}$

$= 13 \cdot 3 \cdot 10^{-5} \cdot 10^{-3}$

$= 39 \cdot 10^{-8}$

Örnek / Example 24

$0{,}000213 = 2{,}13 \cdot 10^{-x} \Rightarrow x = ?$

A) 6 B) 5 C) 4 D) – 5 E) – 6

Çözüm / Solution

Yanıt / Answer **C**

$0{,}000213 = 213 \cdot 10^{-6} = 2{,}13 \cdot 10^{-4}$

$\Rightarrow 2{,}13 \cdot 10^{-4} = 2{,}13 \cdot 10^{-x}$

$10^{-4} = 10^{-x}$

$x = 4$

Örnek / Example 25

$3{,}2 \cdot 10^{n} = 0{,}0000032 \Rightarrow n = ?$

A) 4 B) 5 C) 6 D) –6 E) –7

Çözüm / Solution

Yanıt / Answer **D**

$3{,}2 \cdot 10^{n} = 32 \cdot 10^{-7}$

$3{,}2 \cdot 10^{n} = 3{,}2 \cdot 10^{-6}$

$n = -6$

PEKİŞTİRME / REINFORCE

✓ $2^{-1} = ?$

..... $\left[\frac{1}{2}\right]$

✓ $3^{-2} = ?$

..... $\left[\frac{1}{9}\right]$

✓ $(3^{-2})^{3} = ?$

..... $[3^{-6}]$

PEKİŞTİRME / REINFORCE

✓ $\left.\begin{matrix} x = 4^{(-2)^{-1}} \\ y = (0{,}4)^{-2} \end{matrix}\right\} \Rightarrow 2x + 4y = ?$

..... [26]

Örnek / Example 24

$\frac{1,44}{10^{n+1}} = 0,000144 \Rightarrow n = ?$

A) 3 B) 4 C) – 4 D) 0 E) – 5

Çözüm / Solution Yanıt / Answer **A**

$\frac{1,44}{10^{n+1}} = 0,000144$

$\frac{144 \cdot 10^{-2}}{10^{n+1}} = 144 \cdot 10^{-6}$

$10^{-2-n-1} = 10^{-6}$

$-3-n=-6$

$n=3$

PEKİŞTİRME / REINFORCE

✓ $(0,25)^{-6} = (0,008)^{-2n} \Rightarrow 5^n = ?$

..... [4]

Örnek / Example 25

$a, b, c \in Z$

$\frac{(0,009)^{-3} \cdot (0,0081)^3}{3^3} = 2^a \cdot 3^b \cdot 5^c \Rightarrow a+b-c = ?$

A) –6 B) –3 C) 3 D) 6 E) 9

Çözüm / Solution Yanıt / Answer **C**

$\frac{(0,009)^{-3} \cdot (0,0081)^3}{3^3} = 2^a \cdot 3^b \cdot 5^c$

$\frac{(3^2 \cdot 10^{-3})^{-3} \cdot (3^4 \cdot 10^{-4})^3}{3^3} = \frac{3^{-6} \cdot 10^9 \cdot 3^{12} \cdot 10^{-12}}{3^3}$

$= \frac{3^6 \cdot 10^{-3}}{3^3}$

$= 2^{-3} \cdot 3^3 \cdot 5^{-3}$

$\left.\begin{matrix} a=-3 \\ b=3 \\ c=-3 \end{matrix}\right\} \Rightarrow a+b-c=-3+3+3=3$

ÜSLÜ DENKLEMLER / EXPONENTIAL EQUATIONS

Özellik / Property

★ $a \neq \{-1, 0, 1\}$

$a^n = a^m \Rightarrow n = m$

Etkinlikler / Activities

a. $2^x = 2^3 \Rightarrow x = 3$

b. $5^{2x+1} = 5^5 \Rightarrow 2x + 1 = 5$

$x = 2$

c. $27^{3a} = 3^9 \Rightarrow (3^3)^{3a} = 3^9$

$3^{9a} = 3^9 \Rightarrow a = 1$

d. $3^{2m} = \frac{1}{81} \Rightarrow 3^{2m} = 3^{-4}$

$2m = -4$

$m = -2$

Örnek / Example 26

$4^{m-2} = 8^{m+2} \Rightarrow m = ?$

A) 10 B) 5 C) 0 D) –5 E) –10

Çözüm / Solution

Yanıt / Answer **E**

$(2^2)^{m-2} = (2^3)^{m+2} \Rightarrow 2^{2m-4} = 2^{3m+6}$

$2m - 4 = 3m + 6$

$2m - 3m = 6 + 4$

$-m = 10 \Rightarrow m = -10$

Örnek / Example 27

$16^{\frac{x}{2}} = 256 \Rightarrow x = ?$

A) 2 B) 3 C) 4 D) 6 E) 8

Çözüm / Solution

Yanıt / Answer **C**

$16^{\frac{x}{2}} = 256$

$(2^4)^{\frac{x}{2}} = 2^8 \Rightarrow 2^{2x} = 2^8$

$2x = 8$

$x = 4$

PEKİŞTİRME / REINFORCE

✓ $3^x = 3^4 \Rightarrow x = ?$

..... [4]

✓ $5^0 = 5^{2x-4} \Rightarrow x = ?$

..... [2]

✓ $3^6 = 27^x \Rightarrow x = ?$

..... [2]

Örnek / Example 28

$2^x - 2^{x+1} + 2^{x+2} = 384 \Rightarrow x = ?$

A) 3 B) 5 C) 6 D) 7 E) 8

Çözüm / Solution Yanıt / Answer **D**

$2^x - 2^{x+1} + 2^{x+2} = 384$

$2^x - 2^x \cdot 2 + 2^x \cdot 2^2 = 384$

$2^x(1 - 2 + 4) = 384$

$2^x = \frac{384}{3} = 128$

$2^x = 2^7 \Rightarrow x = 7$

Özellik / Property

★ $\left.\begin{matrix} a^m = b^x \\ a^n = b^y \end{matrix}\right\} \Rightarrow \frac{m}{n} = \frac{x}{y}$

PEKİŞTİRME / REINFORCE

✓ $16^n = 81 \Rightarrow 2^{7n} = ?$

..... [3^7]

Örnek / Example 29

$3^x = 25$ ve $3^y = 125 \Rightarrow \frac{2y + x}{7x - 4y} = ?$

A) 1 B) 2 C) 3 D) 4 E) 6

Çözüm / Solution Yanıt / Answer **D**

$\left.\begin{matrix} 3^x = 5^2 \\ 3^y = 5^3 \end{matrix}\right\} \Rightarrow \frac{x}{y} = \frac{2}{3} \Rightarrow x = 2k \text{ , } y = 3k$

$\frac{2y + x}{7x - 4y} = \frac{2 \cdot (3k) + 2k}{7 \cdot (2k) - 4 \cdot (3k)} = \frac{8k}{2k} = 4$

Örnek / Example 30

$a, b \in Z^+$,

$$\left.\begin{matrix} a^{2a-b} = 8 \\ a^b = 2 \end{matrix}\right\} \Rightarrow a = ?$$

A) 0 B) 1 C) 2 D) 3 E) 4

Çözüm / Solution

Yanıt / Answer **C**

$$a^{2a-b} = 2^3$$
$$a^b = 2^1$$
$$\frac{2a-b}{b} = \frac{3}{1}$$
$$2a - b = 3b$$
$$2a = 4b$$
$$a = 2b \Rightarrow b = \frac{a}{2}$$
$$a^b = 2 \Rightarrow \left(a^{\frac{a}{2}}\right)^2 = (2)^2$$
$$a^a = 2^2$$
$$a = 2$$

Örnek / Example 31

$$\left.\begin{matrix} 4^{a+2b} = 8 \\ 8^{a+b} = 16 \end{matrix}\right\} \Rightarrow b = ?$$

A) $\frac{1}{2}$ B) $\frac{1}{9}$ C) $\frac{1}{6}$ D) $\frac{1}{8}$ E) $\frac{1}{16}$

Çözüm / Solution

Yanıt / Answer **C**

$$(2^2)^{a+2b} = 8 \quad , \quad (2^3)^{a+b} = 16$$
$$2^{2a+4b} = 2^3 \quad , \quad 2^{3a+3b} = 2^4$$
$$2a + 4b = 3 \quad , \quad 3a + 3b = 4$$
$$\left.\begin{matrix} -3 \,/\, 2a + 4b = 3 \\ 2 \,/\, 3a + 3b = 4 \end{matrix}\right\} \Rightarrow -6a - 12b = -9$$
$$+ \quad 6a + 6b = 8$$
$$-6b = -1$$
$$b = \frac{1}{6}$$

PEKİŞTİRME / REINFORCE

✓ $\left.\begin{matrix} a^{x-y} = 16 \\ a^{x+y} = 8 \end{matrix}\right\} \Rightarrow \frac{x}{y} = ?$

..... [–7]

PEKİŞTİRME / REINFORCE

✓ $\left.\begin{matrix} 2^a = x^3 \\ 4^b = x^{\frac{1}{2}} \end{matrix}\right\} \Rightarrow \frac{a}{b} = ?$

..... [12]

Özellik / Property

★ $x^n = y^n \Rightarrow \begin{cases} x = y, & \text{n tek tamsayı ise / if n is an odd integer.} \\ x = y \vee x = -y, & \text{n çift tamsayı ise / if n is an even integer.} \end{cases}$

Etkinlikler / Activities

a. $x^3 = 5^3 \Rightarrow x = 5$

b. $x^6 = 3^6 \Rightarrow x = 3$ ve $x = -3$

c. $(2x-1)^7 = 7^7 \Rightarrow 2x - 1 = 7$

$x = 4$

d. $(2x-1)^8 = 5^8 \Rightarrow 2x - 1 = 5 \quad \vee \quad 2x - 1 = -5$

$x = 3 \qquad\qquad x = -2$

Örnek / Example 32

$(2x-4)^2 = 16 \Rightarrow \sum x = ?$

A) –2 B) 0 C) 2 D) 4 E) 8

Çözüm / Solution

Yanıt / Answer **D**

$(2x-4)^2 = 4^2 \Rightarrow 2x - 4 = 4 \quad \vee \quad 2x - 4 = -4$

$2x = 8 \qquad\qquad 2x = 0$

$x_1 = 4 \qquad\qquad x_2 = 0$

$\Rightarrow x_1 + x_2 = 4 + 0 = 4$

Örnek / Example 33

$(x-2)^4 = 1 \Rightarrow \prod x = ?$

A) 1 B) 3 C) 6 D) 9 E) 12

Çözüm / Solution

Yanıt / Answer **B**

$x - 2 = 1 \quad \vee \quad x - 2 = -1$

$x_1 = 3 \qquad\qquad x_2 = 1$

$\Rightarrow x_1 \cdot x_2 = 3 \cdot 1 = 3$

Örnek / Example 34

$(x-3)^{x-2} = 1 \Rightarrow \sum x = ?$

A) 2 B) 4 C) 6 D) 8 E) 10

Çözüm / Solution

Yanıt / Answer **C**

$x - 3 = 1 \Rightarrow x = 4$

$x - 2 = 0 \Rightarrow x = 2$

$x - 3 = -1 \Rightarrow x = 2$

$\sum x = 4 + 2 = 6$

Özellik / Property

★ x ve y aralarında asal sayılardır.
x and y are relatively prime numbers.

$x^a = y^b \Rightarrow a = b = 0$

Örnek / Example 35

$5^{x-2} = 7^{2y-8} \Rightarrow x \cdot y = ?$

A) 3 B) 5 C) 8 D) 16 E) 32

Çözüm / Solution

Yanıt / Answer **C**

$5^{x-2} = 7^{2y-8} \Rightarrow x - 2 = 0 \ , \ 2y - 8 = 0$

$x = 2 \ , \quad y = 4$

$\Rightarrow x \cdot y = 2 \cdot 4 = 8$

Örnek / Example 36

$3^{x-3} - 5^{xy-12} = 0 \Rightarrow x + y = ?$

A) 1 B) 3 C) 5 D) 7 E) 9

Çözüm / Solution

Yanıt / Answer **D**

$3^{x-3} = 5^{xy-12}$

$x - 3 = 0 \qquad xy - 12 = 0$

$x = 3 \qquad 3y - 12 = 0$

$y = 4$

$x + y = 7$

PEKİŞTİRME / REINFORCE

✓ $x^2 = 4 \Rightarrow x = ?$

..... [∓2]

✓ $x^3 = 8 \Rightarrow x = ?$

..... [2]

✓ $2x^2 = 8 \Rightarrow x = ?$

..... [∓2]

✓ $x^3 = 1 \Rightarrow x = ?$

..... [1]

✓ $(2x-3)^2 = x^2 \Rightarrow x = ?$

..... [1, 3]

ÖNEMLİ / IMPORTANT

• $x^y = 1 \Rightarrow \begin{cases} x = 1 \\ y = 0, & x \neq 0 \\ x = -1, & y \text{ çift / y is even} \end{cases}$

ÜSLÜ SAYILARDA EŞİTSİZLİKLER
INEQUALITIES in EXPONENTIALS

- $0 < x < y$ ve $m \in Z^+$ olmak üzere, $x^m < y^m$ dir.
- $1 < x$ ve $m < n$ olmak üzere, $x^m < x^n$ dir.
- $0 < x < 1$ ve $m < n$ olmak üzere, $x^m > x^n$ dir.
- If $0 < x < y$ and $m \in Z^+$ then, $x^m < y^m$
- If $1 < x$ and $m < n$ then, $x^m < x^n$
- If $0 < x < 1$ and $m < n$ then, $x^m > x^n$

Etkinlikler / Activities

a. $2^3 < 3^3 < 4^3$

b. $2^5 < 2^6 < 2^7$

c. $\left(\frac{1}{3}\right)^2 > \left(\frac{1}{3}\right)^3 > \left(\frac{1}{3}\right)^4$

PEKİŞTİRME / REINFORCE

✓ $2^{x-y+1} = 3^{y-2} \Rightarrow x \cdot y = ?$

..... [2]

Örnek / Example 37

$(2,\overline{7})^3 < \left(\frac{5}{3}\right)^{x-2} \Rightarrow \min(x) = ?$

A) 7 B) 8 C) 9 D) 10 E) 11

Çözüm / Solution

Yanıt / Answer **C**

$(2,\overline{7})^3 < \left(\frac{5}{3}\right)^{x-2}$

$\left(\frac{25}{9}\right)^3 < \left(\frac{5}{3}\right)^{x-2}$

$\left(\frac{5}{3}\right)^6 < \left(\frac{5}{3}\right)^{x-2} \Rightarrow 6 < x-2$

$8 < x \Rightarrow \min(x) = 9$

Örnek / Example 38

$a = 5^{24}$, $b = 2^{36}$ ve $c = 3^{48} \Rightarrow ? < ? < ?$

A) $a < b < c$ B) $b < c < a$ C) $c < b < a$

D) $b < a < c$ E) $c < a < b$

Çözüm / Solution

Yanıt / Answer **D**

$a = 5^{24} = (5^2)^{12} = 25^{12}$

$b = 2^{36} = (2^3)^{12} = 8^{12}$

$c = 3^{48} = (3^4)^{12} = 81^{12}$

$8 < 25 < 81 \Rightarrow b < a < c$

Örnek / Example 39

$x^{36} < 3^{54}$

$x \in Z^+ \Rightarrow \sum x = ?$

A) 14 B) 15 C) 16 D) 17 E) 18

Çözüm / Solution

Yanıt / Answer **B**

$x^{36} < 3^{54}$

$(x^2)^{18} < (3^3)^{18}$

$\left.\begin{matrix} x^2 < 27 \\ x \in Z^+ \end{matrix}\right\} \Rightarrow x = 5, 4, 3, 2, 1$

$\sum x = 1 + 2 + 3 + 4 + 5 = 15$

Örnek / Example 40

$x \in Z$

$3^{4-x} \leq 1 \leq 11^{10-x} \Rightarrow \sum x = ?$

A) 47 B) 48 C) 49 D) 50 E) 51

Çözüm / Solution

Yanıt / Answer **C**

$3^{4-x} \leq 1 \Rightarrow 3^{4-x} \leq 3^0$

$4 - x \leq 0 \Rightarrow 4 \leq x$

$1 \leq 11^{10-x} \Rightarrow 11^{10} \leq 11^{40-x}$

$0 \leq 10 - x \Rightarrow x \leq 10$

$4 \leq x \leq 10 \Rightarrow \sum x = 4 + \ldots + 10 = 49$

1. $\dfrac{\left(-\frac{1}{2}\right)^{-2}+\left(-\frac{1}{2}\right)^{-3}}{\left(-\frac{1}{3}\right)^{2}-\left(-\frac{1}{3}\right)^{3}} = ?$

A) $\frac{2}{27}$ B) $\frac{27}{16}$ C) -27

D) −54 E) -108

2. $\dfrac{(-2^{-1})^{4}\cdot(-2)^{-5}\cdot(-2^{2})}{(-2)^{-1}\cdot(-2)^{-6}} = ?$

A) −1 B) 1 C) 2

D) 2^2 E) 2^7

3. $\dfrac{(-2)^{-2}-3^{-1}+2^{-2}}{6^{-2}} = ?$

A) 2 B) 4 C) 6

D) 8 E) 10

4. $2^a = 50 \Rightarrow 2^{2a-2} = ?$

A) 125 B) 250 C) 500

D) 625 E) 750

5. $(4^{a+1} - 2^{2a}) : (3 \cdot 2^{3a}) = ?$

A) 2a B) $\frac{1}{2}$ C) $2\frac{1}{3}$

D) 2^{-a} E) 2^a

6. $8^x = a \Rightarrow (32)^{3x} = ?$

A) a^3 B) $4a^3$ C) $16a^3$

D) a^5 E) a^{15}

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7. a ve b birer tamsayıdır. / a and b are integers.

$\frac{8^3 \cdot 6^4}{18^2} = 2^a \cdot 3^b \Rightarrow a+b = ?$

A) 8
B) 10
C) 11
D) 12
E) 15

8. $a^{-\frac{3}{2}} = 64 \Rightarrow a = ?$

A) $\frac{1}{20}$
B) $\frac{1}{16}$
C) $\frac{1}{12}$
D) $\frac{1}{8}$
E) $\frac{1}{4}$

9. $\frac{3}{2^{1-x}} = 27 \Rightarrow ? < x < ?$

A) $1 < x < 2$
B) $3 < x < 4$
C) $4 < x < 5$
D) $5 < x < 6$
E) $6 < x < 7$

10. $\frac{1}{2^{x-1}} + \frac{6}{2^x} + \frac{2}{2^{x-2}} = 64 \Rightarrow x = ?$

A) –2
B) –1
C) 0
D) 1
E) 2

11. $\frac{5^x}{2^{x+1}} = \frac{1}{4} \Rightarrow \left(\frac{4}{25}\right)^{2x} = ?$

A) $\frac{2}{5}$
B) $\frac{1}{5}$
C) 0
D) 16
E) $\frac{625}{16}$

12. $2^{-x} = 3 \Rightarrow 27 \cdot 2^{2x+1} = ?$

A) $\frac{16}{3}$
B) 6
C) 3
D) 7
E) 9

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
C	A	C	D	D	D	C	B	C	A	D	B

1. $\dfrac{\left(-\frac{1}{2}\right)^{-2}+\left(-\frac{1}{2}\right)^{-3}}{\left(-\frac{1}{3}\right)^{2}-\left(-\frac{1}{3}\right)^{3}} = \dfrac{(-2)^2+(-2)^3}{\frac{1}{9}+\frac{1}{27}}$

$= \dfrac{-4}{\frac{4}{27}} = -27$

Yanıt / The Answer is C

2. $\dfrac{(-2^{-1})^4\cdot(-2)^{-5}\cdot(-2^2)}{(-2)^{-1}\cdot(-2)^{-6}} = \dfrac{2^{-4}\cdot(-2^{-5})\cdot(-2^2)}{-2^{-1}\cdot 2^{-6}}$

$= -\dfrac{2^{-4}\cdot 2^{-5}\cdot 2^2}{2^{-1}\cdot 2^{-6}}$

$= -\dfrac{2^{-7}}{2^{-7}}$

$= -1$

Yanıt / The Answer is A

3. $\dfrac{(-2)^{-2}-3^{-1}+2^{-2}}{6^{-2}} = \dfrac{\left(-\frac{1}{2}\right)^2-\frac{1}{3}+\frac{1}{2^2}}{\frac{1}{6^2}}$

$= \dfrac{\frac{1}{4}-\frac{1}{3}+\frac{1}{4}}{\frac{1}{36}}$

$= \dfrac{\frac{3-4+3}{12}}{\frac{1}{36}}$

$= \dfrac{2}{12}\cdot\dfrac{36}{1} = 2\cdot 3 = 6$

Yanıt / The Answer is C

4. $2^a = 50$

$2^{2a-2} = 2^{2a}\cdot 2^{-2} = (2^a)^2\cdot\dfrac{1}{2^2}$

$= (50)^2\cdot\dfrac{1}{4}$

$= \dfrac{2500}{4} = 625$

Yanıt / The Answer is D

5. $\dfrac{4^{a+1}-2^{2a}}{3\cdot 2^{3a}} = \dfrac{(2^2)^{a+1}-2^{2a}}{3\cdot 2^{3a}}$

$= \dfrac{2^{2a+2}-2^{2a}}{3\cdot 2^{3a}} = \dfrac{2^{2a}(2^2-1)}{3\cdot 2^{2a}\cdot 2^a}$

$= \dfrac{3}{3\cdot 2^a}$

$= \dfrac{1}{2^a} = 2^{-a}$

Yanıt / The Answer is D

6. $8^x = a$

$2^{3x} = a$

$(32)^{3x} = (2^5)^{3x}$

$= (2^{3x})^5$

$= a^5$

Yanıt / The Answer is D

METROPOL YAYINLARI • METROPOL PUBLICATIONS

7. $\frac{8^3 \cdot 6^4}{18^2} = 2^a \cdot 3^b \Rightarrow \frac{(2^3)^3 \cdot 2^4 \cdot 3^4}{3^4 \cdot 2^2} = 2^a \cdot 3^b$

$2^{13-2} \cdot 3^{4-4} = 2^a \cdot 3^b$

$2^{11} \cdot 3^0 = 2^a \cdot 3^b$

$a = 11 \text{ , } b = 0$

$a + b = 11$

Yanıt / The Answer is C

8. $a^{\frac{-3}{2}} = 4^3 \Rightarrow \left(a^{\frac{-3}{2}}\right)^{\frac{-2}{3}} = (4^3)^{\frac{-2}{3}}$

$a = 4^{-2} \Rightarrow a = \frac{1}{4^2} = \frac{1}{16}$

Yanıt / The Answer is B

9. $\frac{3}{2^{1-x}} = 27 \Rightarrow 3 \cdot 2^{x-1} = 27$

$2^{x-1} = 9$

$2^3 < 9 < 2^4$

$2^3 < 2^{x-1} < 2^4$

$4 < x < 5$

Yanıt / The Answer is C

10. $\frac{1}{2^x \cdot 2^{-1}} + \frac{6}{2^x} + \frac{2}{2^x \cdot 2^{-2}} = 64$

$\frac{2}{2^x} + \frac{6}{2^x} + \frac{8}{2^x} = 64$

$\frac{16}{2^x} = 64$

$\frac{1}{2^x} = 4$

$2^{x+2} = 1 \Rightarrow x + 2 = 0 \Rightarrow x = -2$

Yanıt / The Answer is A

11. $\frac{5^x}{2^{x+1}} = \frac{1}{4} \Rightarrow \frac{5^x}{2^x} = \frac{1}{2}$

$\left(\frac{4}{25}\right)^{2x} = \left(\frac{2}{5}\right)^{4x} = \left(\frac{2^x}{5^x}\right)^4 = \left(\frac{5^x}{2^x}\right)^{-4} = \left(\frac{1}{2}\right)^{-4} = 16$

Yanıt / The Answer is D

12. $27 \cdot 2^{2x+1} = 27 \cdot 2^{2x} \cdot 2$

$= 27 \cdot (2^x)^2 \cdot 2$

$= 27 \cdot (2^{-x})^{-2} \cdot 2$

$= 54 \cdot 3^{-2}$

$= \frac{54}{9} = 6$

Yanıt / The Answer is B

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $\frac{3^{21}\cdot 3^{21}\cdot 3^{21}\cdot 3^{21}}{9^3\cdot 9^3} = (ab)^{ba}$

üstte verilen şekilde (ab) ve (ba) iki basamaklı sayılardır.

Bu eşitliğe göre a·b çarpımı kaça eşittir?

In the picture given above, ab and ba are 2-digits numbers.

According to this equality, what is a·b equal to?

A) 9 B) 11 C) 8

D) 14 E) 18

[KARABÜK ÜNİVERSİTESİ – YÖS 2019]

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2. $\left.\begin{matrix} 2^x = 27 \\ 3^y = \frac{1}{64} \end{matrix}\right\} \Rightarrow x\cdot y = ?$

A) 1 B) –18 C) 18

D) –9 E) 9

[ANKARA ÜNİVERSİTESİ – YÖS 2019]

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3. $\left(\frac{1}{12}\right)^{x-1} = \left(\frac{9}{4}\right)^{x} \Rightarrow 27^x = ?$

A) 6 B) 9 C) 12

D) 24 E) 36

[MUĞLA SITKI KOÇMAN ÜNİVERSİTESİ – YÖS 2019]

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4. $\left.\begin{matrix} 2^x = 9 \\ 3^y = 16 \end{matrix}\right\} \Rightarrow x\cdot y = ?$

A) 6 B) 7 C) 8

D) 9 E) 10

[İSTANBUL ÜNİVERSİTESİ – YÖS 2018]

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5. $\left(\frac{1}{27}\right)^{2-x} = 9^{x-1} \Rightarrow x = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

[DUMLUPINAR ÜNİVERSİTESİ – YÖS 2018]

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6. $a, b \in R$

$\frac{5^{a-b}}{27^{a+b}} = 135 \Rightarrow a^2 - b^2 = ?$

A) –3 B) –1 C) 0

D) 1 E) 2

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2018]

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7. $2^a = 5 \Rightarrow \frac{12^{a+2}}{6^{a+1}} = ?$

A) 20 B) 48 C) 60

D) 84 E) 120

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2017]

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8. $5^x = 3^y \Rightarrow 3^{\frac{3y}{x}} + 5^{\frac{2x}{y}} = ?$

A) 8 B) 14 C) 28

D) 134 E) 152

[SELÇUK ÜNİVERSİTESİ – YÖS 2017]

METROPOL YAYINLARI • METROPOL PUBLICATIONS

8. $\frac{2^4+2^5+2^6}{2^2+2^3+2^4} = ?$

A) 2 B) 4 C) 8

D) 16 E) 32

[BALIKESİR ÜNİVERSİTESİ – YÖS 2017]

9. $\left.\begin{matrix} 2^{x+2}+2^{x+1}=48 \\ 3^x+3^{x-y}=36 \end{matrix}\right\} \Rightarrow y = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

10. $\frac{5^{19}-5^7}{(5^6+1)(5^6-1)} = ?$

A) 1 B) 5 C) 5^5

D) 5^7 E) 5^{12}

[KARADENİZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

11. $\frac{9^{3n+1}+27^{2n}}{3^{6n-1}} + \frac{4^{3n}-8^{2n-1}}{2^{6n-5}} = ?$

A) 18 B) 30 C) 36

D) 43 E) 58

[İSTANBUL ÜNİVERSİTESİ – YÖS 2015]

12. $\left.\begin{matrix} 3^{-x}\cdot 3^{y}\cdot 3^{2x}=243 \\ \frac{1}{2^{-7x}\cdot 2^{x}\cdot 2^{2y}}=64 \end{matrix}\right\} \Rightarrow y-x = ?$

A) –2 B) 1 C) 3

D) 4 E) 5

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2014]

13. $\frac{2^{2002}+2^{2001}+2^{2000}}{2^{-2004}+2^{-2003}+2^{-2002}} = ?$

A) 2^{2000} B) 2^{2001} C) 2^{2002}

D) 2^{2003} E) 2^{4004}

[İSTANBUL ÜNİVERSİTESİ – YÖS 2014]

14. $(3^{-1}+2^{-2})^{-1} : (-2^2) = ?$

A) $\frac{2}{3}$ B) $-\frac{1}{4}$ C) $-\frac{3}{5}$

D) $-\frac{3}{2}$ E) $-\frac{3}{7}$

[AKDENİZ ÜNİVERSİTESİ – YÖS 2014]

15. $5^x = 2$

$\Rightarrow \frac{5^{x-1}+5^{-x}}{25^x} = ?$

A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{9}{40}$

D) $\frac{25}{81}$ E) 3

[KARADENİZ TEKNİK ÜNİVERSİTESİ – YÖS 2013]

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	B	C	C	D	B	E	D
9	10	11	12	13	14	15	16
B	A	D	E	B	E	E	C

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $\frac{3^{21}\cdot 3^{21}\cdot 3^{21}\cdot 3^{21}}{9^3\cdot 9^3} = \frac{(3^{21})^4}{(9^3)^2} = \frac{3^{84}}{3^{12}}$

$\frac{3^{84}}{3^{12}} = 3^{72} = (3^4)^{18} = (81)^{18}$

$a = 8, b = 1 \Rightarrow a\cdot b = 8$

Yanıt / The Answer is C

---

2. $\left.\begin{matrix} 2^x = 3^3 \\ 3^y = 2^{-6} \end{matrix}\right\} \Rightarrow \begin{matrix} 2^x = 3^3 \\ 2^{-6} = 3^y \end{matrix}$

$\Rightarrow \frac{x}{-6} = \frac{3}{y}$

$\Rightarrow x\cdot y = -18$

Yanıt / The Answer is B

---

3. $\frac{1}{12^{x-1}} = \frac{3^{2x}}{4^x}$

$\frac{12}{3^x\cdot 4^x} = \frac{3^{2x}}{4^x}$

$12 = 3^{3x}$

$12 = 27^x$

Yanıt / The Answer is C

---

4. $2^x = 9$

$3^y = 16$

$\left.\begin{matrix} 2^x = 3^2 \\ 2^4 = 3^y \end{matrix}\right\} \Rightarrow \frac{x}{4} = \frac{2}{y}$

$x\cdot y = 8$

Yanıt / The Answer is C

---

5. $\left(\frac{1}{27}\right)^{2-x} = 9^{x-1}$

$3^{-6+3x} = 3^{2x-2}$

$-6 + 3x = 2x - 2$

$x = 4$

Yanıt / The Answer is D

---

6. $\frac{5^{a-b}}{2^{a+b}} = 135$

$5^{a-b}\cdot 27^{-(a+b)} = 5^1\cdot 27^1$

$\left.\begin{matrix} a - b = 1 \\ a + b = -1 \end{matrix}\right\} \Rightarrow \begin{matrix} a = 0 \\ b = -1 \end{matrix}$

$a^2 - b^2 = -1$

Yanıt / The Answer is B

---

7. $2^a = 5$

$\frac{12^a\cdot 12^2}{6^a\cdot 6} = \frac{6^a\cdot 2^a\cdot 12\cdot 12}{6^a\cdot 6} = 5\cdot 2\cdot 12$

$= 120$

Yanıt / The Answer is E

---

8. $5^x = 3^y, \quad 3^{\frac{y}{x}} = 5$

$5^{\frac{x}{y}} = 3, \quad 3^{\frac{3y}{x}} = 125$

$5^{\frac{2x}{y}} = 9 \Rightarrow 125 + 9 = 134$

Yanıt / The Answer is D

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9. $\frac{2^4+2^5+2^6}{2^2+2^3+2^4}=\frac{2^4(1+2+4)}{2^2(1+2+4)}=\frac{16}{4}=4$

Yanıt / The Answer is B

---

10. $2^{x+2}+2^{x+1}=48 \Rightarrow 2^x\cdot 4+2\cdot 2^x=48$

$3^3+3^{3-y}=36 \qquad 6\cdot 2^x=48$

$3^{3-y}=9=3^2 \qquad 2^x=8$

$x=3$

$3-y=2 \Rightarrow y=1$

Yanıt / The Answer is A

---

11. $\frac{5^{19}-5^7}{(5^6+1)(5^6-1)}=\frac{5^7(5^{12}-1)}{(5^{12}-1)}=5^7$

Yanıt / The Answer is D

---

12. $\frac{9^{3n+1}+27^{2n}}{3^{6n-1}}+\frac{4^{3n}-8^{2n-1}}{2^{6n-5}}=\frac{3^{6n}\cdot 9+3^{6n}}{\frac{3^{6n}}{3}}+\frac{2^{6n}-2^{6n}\cdot 2^{-3}}{2^{6n}\cdot 2^{-5}}$

$=\frac{3\cdot 3^{6n}(9+1)}{3^{6n}}+\frac{2^{6n}\left(1-\frac{1}{8}\right)}{2^{6n}\cdot\frac{1}{32}}$

$=30+\frac{7}{8}\cdot\frac{32}{1}=58$

Yanıt / The Answer is E

---

13. $3^{-x+y+2x}=243 \qquad 2^{7x-x-2y}=2^6$

$3^{x+y}=3^5 \qquad 6x-2y=6$

$x+y=5 \qquad 3x-y=3$

$x+y=5 \rightarrow 2+y=5 \Rightarrow y=3$

$+\ 3x-y=3$

$4x=8$

$x=2$

$\Rightarrow y-x=3-2=1$

Yanıt / The Answer is B

---

14. $\frac{2^{2002}+2^{2001}+2^{2000}}{2^{-2004}+2^{-2003}+2^{-2002}}=\frac{2^{2000}(2^2+2^1+1)}{2^{-2004}(1+2^1+2^2)}$

$=2^{4004}$

Yanıt / The Answer is E

---

15. $(3^{-1}+2^{-2})^{-1}:(-2^2)=\left(\frac{1}{3}+\frac{1}{4}\right)^{-1}:(-4)$

$=\left(\frac{7}{12}\right)^{-1}\cdot\left(-\frac{1}{4}\right)$

$=\frac{12}{7}\cdot\left(-\frac{1}{4}\right)$

$=-\frac{3}{7}$

Yanıt / The Answer is E

---

16. $\frac{5^{x-1}+5^{-x}}{25^x}=\frac{2\cdot\frac{1}{5}+\frac{1}{2}}{2^2}=\frac{\frac{9}{10}}{4}=\frac{9}{40}$

Yanıt / The Answer is C

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1. $\left[\left(-\frac{1}{2}\right)^{-1}\right]^3 = ?$

A) $(-2)^{-1}$ B) -2^3 C) 16

D) 2^3 E) $\frac{1}{2}$

2. $\frac{3^3 - 3^2}{9} \cdot (2^{-3})^{-2} = ?$

A) 32 B) 64 C) 128

D) 256 E) 512

3. $\frac{(-2)^2 \cdot 2^3 \cdot 2^{-9}}{8^{-3}} = ?$

A) 2 B) 4 C) 16

D) 32 E) 64

4. $\frac{2^{-2} \cdot 2^4 (-2)^3 \cdot (-2^6)}{-2^5 \cdot (-2)^2} = ?$

A) -2^2 B) $\frac{-1}{2^4}$ C) -2^4

D) 2^8 E) 32

5. $(-a)^5 \cdot (-a)^4 \cdot -a^3 = ?$

A) $-a^{12}$ B) a^{-3} C) a^4

D) a^{12} E) a^{60}

6. $(0,5)^{2x} = 3 \Rightarrow (0,25)^{2x} = ?$

A) 27 B) 9 C) 3

D) 3^{-1} E) $3^{\frac{1}{2}}$

7. $\frac{(-a^5) \cdot (-a)^3 + 2 \cdot (-a)^5 \cdot (-a^3) \cdot (-a^0)}{(-a^3)^2} = ?$

A) $-a^3$ B) $-a^2$ C) $-a$

D) a^2 E) a^3

8. $\frac{12^3 + 12^3 + 12^3}{6^4 + 6^4 + 6^4 + 6^4} = ?$

A)1 B) 2 C) 3

D) 4 E) 5

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9. $x = 2$ ve / and $y = -2 \Rightarrow x - y^{x-y} = ?$

A) 64 B) 16 C) 14

D) −14 E) −16

10. $2^{3a+9} = 8^{-b-3} \Rightarrow a + b = ?$

A) −6 B) −2 C) 0

D) 4 E) 12

11. $10^x = 16$

$\Rightarrow 2^{x-1} \cdot 5^{x+3} = ?$

A) 125 B) $\frac{125}{2}$ C) 250

D) 625 E) 1000

12. $6^{x-1} = 3^{x-2} \Rightarrow 2^x = ?$

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{4}{9}$

D) $\frac{3}{4}$ E) 3

13. $4^{x+1} + 4^{x+1} + 4^{x+1} + 4^{x+1} = 2^{x+1} + 2^{x+1}$

$\Rightarrow x = ?$

A) −3 B) −2 C) −1

D) 2 E) 3

14. $6^m = 10 \Rightarrow (0{,}5)^{2-m} \cdot 3^{m+1} = ?$

A) $\frac{15}{2}$ B) $\frac{7}{4}$ C) $\frac{10}{3}$

D) $\frac{5}{8}$ E) 18

15. $\left.\begin{matrix} 2^{x+2} = a \\ 3^{x+3} = b \end{matrix}\right\} \Rightarrow 6^{x-2} = ?$

A) $\frac{3ab}{16}$ B) 16ab C) ab

D) 48ab E) $\frac{ab}{3^5 \cdot 2^4}$

16. $\left.\begin{matrix} 5^{3x+2} = a \\ 125^{2y+x-1} = 5a \end{matrix}\right\} \Rightarrow y = ?$

A) −2 B) −1 C) 0

D) 1 E) 2

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	C	D	C	D	B	B	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	A	E	B	B	A	E	D

1. $6^x = 3^{x+1} \Rightarrow 8^x = ?$

A) 2 B) 3 C) 9 D) 27 E) 36

2. $(x-3)^x = 1 \Rightarrow \sum x = ?$

A) 3 B) 4 C) 5 D) 6 E) 7

3. $2^x + 3 \cdot 2^{x+1} - 2^{x+2} = 12 \Rightarrow x = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

4. $\left.\begin{matrix} 2^x \cdot 3^y = 5 \\ 4^x \cdot 6^y = 80 \end{matrix}\right\} \Rightarrow x + y = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

5. $\dfrac{\left(-\frac{1}{2}\right)^3 \cdot (-2)^5}{(-2)^4} = ?$

A) $\frac{1}{2}$ B) 2^{-2} C) $\frac{1}{6}$ D) 2^4 E) 2^{-3}

6. $(-2)^2 \cdot (-2)^3 \cdot (-2^{-1})\,(-2)^{-3} = ?$

A) -2 B) 2^{-5} C) 2^{-6} D) 2^{-2} E) 2^{-3}

7. $2^{a-1} = 4 \Rightarrow 4^{a-1} = ?$

A) 2^{-2} B) 2^{-3} C) 2^{-4} D) 2 E) 16

8. $2^{a+1} = 16$

$2^b = 8 \Rightarrow b - a = ?$

A) -2 B) -1 C) 0 D) 1 E) 2

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $8^{x-1} = 2^{x+1} \Rightarrow 2^x = ?$

A) –8 B) –4 C) 0

D) 2 E) 4

10. $\frac{2^x + 2^x + 2^x}{2^x \cdot 2^x} = 24 \Rightarrow x = ?$

A) –1 B) –2 C) –3

D) $\frac{1}{6}$ E) $\frac{1}{16}$

11. $\left(\frac{1}{2}\right)^{x-2} \cdot 8^{x+1} = 2^{4-x} \Rightarrow x = ?$

A) $-\frac{1}{2}$ B) $-\frac{1}{3}$ C) –4

D) 8 E) 16

12. $4^x = 5 \Rightarrow 16^{x-1} = ?$

A) –2 B) $\frac{1}{8}$ C) $\frac{1}{5}$

D) $\frac{25}{16}$ E) 4

13. $3^x = 32,\ 9^y = 4 \Rightarrow \frac{x+2y}{x-y} = ?$

A) $\frac{4}{7}$ B) $\frac{3}{5}$ C) $\frac{5}{3}$

D) $\frac{6}{5}$ E) $\frac{7}{4}$

14. $5^x + 5^{-x} = a,$
$5^x - 5^{-x} = b \Rightarrow a^2 - b^2 = ?$

A) 1 B) 2 C) 3

D) 4 E) 8

15. $a^{2x} + b^{2x} = 169$ ve / and $a^x - b^x = 5 \Rightarrow a^x \cdot b^x = ?$

A) 36 B) 68 C) 72

D) 108 E) 144

16. $12^x = 9,\ 3^y = 2 \Rightarrow y = ?$

A) $\frac{2x}{x+1}$ B) $\frac{2-x}{2x}$ C) $\frac{x-1}{x+1}$

D) $\frac{x+1}{2}$ E) $\frac{x-1}{3}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	D	B	C	B	A	E	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	C	B	D	E	D	C	B

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1. $3^{x+1} - 3^{x-2} + 3^x = 105 \Rightarrow x = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

2. $2^x = 3 \Rightarrow 2^{2x+1} = ?$

A) 9 B) 14 C) 16 D) 18 E) 20

3. $\frac{3^{x+1} + 3^{x-1}}{3^x - 3^{x+2}} = ?$

A) $-\frac{7}{5}$ B) $-\frac{6}{7}$ C) $-\frac{7}{12}$ D) $-\frac{5}{12}$ E) $-\frac{8}{9}$

4. $10^{x+1} = 2^{x+2} \Rightarrow 5^{-2x} = ?$

A) $\frac{5}{4}$ B) $\frac{25}{4}$ C) $\frac{125}{4}$ D) $\frac{625}{4}$ E) 1

5. $2^x = \frac{1}{y} \Rightarrow 16^{1-\frac{x}{4}} = ?$

A) $\frac{16}{y}$ B) 16y C) 4y D) $\frac{4}{y}$ E) 8y

6. $\frac{a^{x+y+3} - a^{x+3}}{a^{x+y} - a^x} = ?$

A) a B) a^2 C) a^3 D) a^x E) a^{x+y}

7. $3^{x-1} = m \Rightarrow 3^{x+3} - 3^{x+2} - 3^{x+1} = ?$

A) 7m B) 15m C) 34m D) 45m E) 63m

8. $\frac{2 \cdot 5^{22} - 9 \cdot 5^{21}}{25^{10}} = ?$

A) 5^{-1} B) 1 C) 5 D) 5^2 E) 5^3

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9. $\frac{4\cdot 9^{n-1}-2\cdot 3^{n-2}}{2\cdot 3^{2n}-3^{n}} = ?$

A) $\frac{2}{3}$ B) $\frac{3}{4}$ C) $\frac{2}{7}$ D) $\frac{5}{7}$ E) $\frac{2}{9}$

10. $\frac{a^{-1}-b^{-1}}{a^{-1}\cdot b^{-1}} = ?$

A) a B) b C) a – b D) b – a E) ab

11. $\frac{4^{2n+2}\cdot 3^{2n-1}}{6^{n-1}\cdot 2^{3n+1}} : 3^{n} = ?$

A) 16 B) 12 C) 8 D) 5 E) 3

12. $20^2 \cdot 40^4 \cdot 80^5 = ?$

A) $2^{11}\cdot 5^{11}$ B) $2^{36}\cdot 5^{11}$ C) $2^{15}\cdot 10^{11}$ D) $4^{8}\cdot 5^{8}$ E) $2^{18}\cdot 5^{10}$

13. $x \in Z$

$3^{x+1} < 3^{2x-4} \Rightarrow$ **min(x) = ?**

A) 4 B) 5 C) 6 D) 7 E) 8

14. $x \in Z$

$\left(\frac{1}{4}\right)^{x} < \left(\frac{1}{8}\right)^{x-1} \Rightarrow$ **max(x) = ?**

A) –2 B) –1 C) 0 D) 1 E) 2

15. $5\cdot 2^{-x} + \frac{1}{2^{x}} = 192 \Rightarrow$ **x = ?**

A) –5 B) –4 C) –3 D) 3 E) 5

16. $\left.\begin{matrix} 2^{x} = 125 \\ 5^{-y} = 32 \end{matrix}\right\} \Rightarrow$ **x·y = ?**

A) –20 B) –15 C) –10 D) 10 E) 15

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	D	D	B	B	C	D	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	D	A	B	C	E	A	B

1. $\dfrac{(4{,}01)^2-(3{,}99)^2}{(0{,}5)^2-(0{,}3)^2}=?$

A) 2 B) 1,2 C) 1
D) 0,1 E) 1,3

2. $8^x=27 \Rightarrow 2^{x+1}=?$

A) 2 B) 4 C) 6
D) 8 E) 12

3. $9^x=a \Rightarrow 3^{2x+1}=?$

A) a B) 2a C) 3a
D) 6a E) 9a

4. $3^x=2 \Rightarrow 9^x+4^{\frac{1}{x}}=?$

A) 7 B) 9 C) 41
D) 13 E) 25

5. $4^{a+3}\cdot 16^{a+1}\cdot 32^{1-a}=1 \Rightarrow a=?$

A) –20 B) –15 C) –2
D) 15 E) 20

6. $2^x=m \Rightarrow 8^{x+3}=?$

A) $(8m)^3$ B) $2m^2$ C) $4m^2$
D) $(5m)^3$ E) $(4m)^3$

7. $\left.\begin{matrix}3^x\cdot 2^y=81\\ 2^y\cdot 9^x=27\end{matrix}\right\} \Rightarrow x=?$

A) –2 B) –1 C) 1
D) 2 E) 3

8. $\dfrac{5^{x+2}-125}{5^{2x}-25}=\dfrac{5}{6} \Rightarrow x=?$

A) 5 B) 4 C) 3
D) 2 E) 0

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9. $(0,6)^{2x-1} = \left(\frac{5}{3}\right)^{x-8} \Rightarrow x = ?$

A) 5 B) 4 C) 3

D) 2 E) 0

10. $\frac{3\cdot 2^{x-1} - 3\cdot 2^{x} + 2^{x+1}}{2^{x}} = 2^{x-1} \Rightarrow x = ?$

A) –1 B) 0 C) 1

D) 2 E) 3

11. $3^{x+1} + 3^{x+2} = 36 \cdot 3^{y} \Rightarrow x = ?$

A) y + 1 B) y C) y – 1

D) 2y E) $\frac{y+1}{3}$

12. $\frac{2^{x+1}+6}{4^{x}-9} = \frac{2}{5} \Rightarrow x = ?$

A) 2 B) 3 C) 4

D) 5 E) 6

13. $3^{x+1} = 6 \Rightarrow 4^{\frac{1}{x}} = ?$

A) $\frac{1}{3}$ B) $\frac{1}{9}$ C) 3

D) 6 E) 9

14. $3^{x+y-7} = 7^{x-y-3}$

$x,y \in Z \Rightarrow x\cdot y = ?$

A) 5 B) 10 C) 12

D) 15 E) 20

15. $\left(\frac{27}{8}\right)^{x-1} = \left(\frac{4}{9}\right)^{x+4} \Rightarrow x = ?$

A) –5 B) –3 C) –1

D) 3 E) 4

16. $6^{\frac{1}{x+1}} = 2 \Rightarrow 8^{x} = ?$

A) 3 B) 6 C) 9

D) 27 E) 36

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	C	C	D	B	A	B	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	B	A	B	E	B	C	D

1. $3^{x-1} = 5 \Rightarrow 15^{\frac{1}{x}} = ?$

A) 4 B) 5 C) 3

D) 4 E) 5

2. $\left.\begin{matrix} 7^x = 49 \\ 5^y = 125 \end{matrix}\right\} \Rightarrow \frac{x+y}{x-y} = ?$

A) –5 B) –3 C) 1

D) 3 E) 5

3. $\left.\begin{matrix} 5^a = 16 \\ 25^b = 8 \end{matrix}\right\} \Rightarrow \frac{3a+b}{b} = ?$

A) 4 B) 5 C) 7

D) 8 E) 9

4. $2^{-x} = 3 \Rightarrow 2^x + 3^{-\frac{1}{x}} = ?$

A) 1 B) $\frac{4}{3}$ C) $\frac{5}{3}$

D) 2 E) $\frac{7}{3}$

5. $\left.\begin{matrix} A = 5^{28} \\ B = 4^{56} \\ C = 3^{70} \end{matrix}\right\} \Rightarrow ? > ? > ?$

A) A > B > C B) A > C > B C) B > A > C

D) B > C > A E) C > A > B

6. $\left.\begin{matrix} 3^x = 10 \\ 2^y = 3 \\ 5^z = \frac{1}{2} \\ 7^t = \frac{1}{8} \end{matrix}\right\} \Rightarrow ? > ? > ? > ?$

A) x > y > z > t B) x > y > t > z

C) x > z > t > y D) y > x > z > t

E) y > x > t > z

7. $x \in Z$,

$\left(\frac{4}{9}\right)^{-x} < \left(\frac{3}{2}\right)^{x+1} \Rightarrow \max(x) = ?$

A) –1 B) 0 C) 1

D) 2 E) 3

8. $\left.\begin{matrix} 3^x \cdot 5^y = 45 \\ 3^y \cdot 5^x = 75 \end{matrix}\right\} \Rightarrow x + y = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

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9. $\left.\begin{matrix} a=-1 \\ b=-\frac{1}{2} \end{matrix}\right\} \Rightarrow (b)^a+(a)^{b^{-1}} = ?$

A) –4 B) –3 C) –1
D) 1 E) 2

10. $(3^x-27)^4+(2^{4y}-16)^6=0 \Rightarrow x-y=?$

A) 1 B) 2 C) 3
D) 4 E) 5

11. $\frac{32^x-8^x}{(2^x-1)(2^x+1)}=\frac{1}{8} \Rightarrow x=?$

A) –2 B) –1 C) 1
D) 2 E) 3

12. $8^{2x-1}=9^{3y-6}=11^{z-1} \Rightarrow x\cdot y\cdot z=?$

A) –2 B) –1 C) $\frac{1}{2}$
D) 1 E) 4

13. $\frac{3,6\cdot10^{-6}+0,64\cdot10^{-5}}{5\cdot10^{-2}+3\cdot10^{-2}+2\cdot10^{-2}}=A\cdot10^n \Rightarrow A+n=?$

A) –4 B) –3 C) –2
D) –1 E) 1

14. $\frac{3^x-3^{-x}}{3^{x+1}+3}=\frac{8}{27} \Rightarrow x=?$

A) 1 B) 2 C) 3
D) 4 E) 5

15. $\frac{5^{2x}+5^x-2}{5^{2x}-1}=\frac{7}{6} \Rightarrow x=?$

A) 1 B) 2 C) 3
D) 4 E) 5

16. $3^{\frac{1}{x+1}}=2 \Rightarrow 2^x=?$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$
D) 2 E) $\frac{5}{2}$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	A	E	E	D	A	B	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	B	B	D	B	B	A	C

1. $8^x \cdot 16 = 2^{5x} \Rightarrow x^x = ?$

A) 1 B) 4 C) 9

D) 16 E) 25

2. $\frac{5^x + 5^x + 5^x + 5^x}{10^x + 10^x} = 8 \Rightarrow x = ?$

A) -3 B) -2 C) -1

D) 1 E) 2

3. $3^{x-1} = 2$,

$9^{2x-1} = 32$

$\Rightarrow 2^x = ?$

A) $\frac{1}{2}$ B) 1 C) $\sqrt{2}$

D) 2 E) 8

4. $125^{x+0,\overline{3}} = 20 \Rightarrow 5^{\frac{3x}{2}} = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

5. A, $n \in N$,

$(8)^3 \cdot (25)^4 = A \cdot 10^n \Rightarrow \min(A + n) = ?$

A) 8 B) 10 C) 12

D) 14 E) 16

6. $\left.\begin{array}{l} a = \frac{b}{2} \\ a^b = 2^{16} \end{array}\right\} \Rightarrow a + b = ?$

A) 6 B) 8 C) 10

D) 12 E) 14

7. $25^{25} : 5 = ?$

A) 5^{49} B) 5^{39} C) 5^{25}

D) 5^{24} E) 5^{19}

8. $\left.\begin{array}{l} 3^x = a \\ 2^x = b \end{array}\right\} \Rightarrow (108)^x = ?$

A) ab B) a^3b C) ab^3

D) a^2b^2 E) $a^3 \cdot b^2$

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9. $\left.\begin{matrix} x = 4 - 2^{a-1} \\ y = 3 + 2^{a+1} \end{matrix}\right\} \Rightarrow \frac{x}{y} = ?$

A) $\frac{16-y}{y}$ B) $\frac{17-y}{2y}$ C) $\frac{19-y}{y}$

D) $\frac{19-y}{4y}$ E) $\frac{21-y}{2y}$

10. $(((5^{-1})^4)^0)^5 = ?$

A) 0 B) $\frac{1}{2}$ C) 1

D) 5 E) 5^2

11. $\frac{2\cdot2\cdot2\cdot2\cdot2\cdot2}{\underbrace{8+8+\ldots+8}_{x \text{ tane/x times}}} = 16 \Rightarrow x = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 2

D) 4 E) 5

12. $\left.\begin{matrix} 4^x = 25 \\ 9^y = 49 \end{matrix}\right\} \Rightarrow 2^{x+1} + 3^{y+1} = ?$

A) 28 B) 29 C) 30

D) 31 E) 32

13. $2^1 \cdot 2^2 \cdot \ldots \cdot 2^x = 16^9 \Rightarrow x = ?$

A) 5 B) 6 C) 7

D) 8 E) 9

14. $3^{81} = x^x \Rightarrow \frac{x}{3} = ?$

A) 1 B) 3 C) 9

D) 27 E) 81

15. $\left[\frac{a^{x-y}}{a^x} : \frac{a^y}{a^{y-x}}\right]^{\frac{1}{x+y}} = ?$

A) $-a$ B) $\frac{1}{a}$ C) a

D) a^{x-y} E) a^{x+y}

16. $\left.\begin{matrix} a = 1 + x^{-1} \\ b = x^{-1} - 1 \end{matrix}\right\} \Rightarrow \frac{a}{b} = ?$

A) $\frac{x-1}{x+1}$ B) $\frac{1-x}{x+1}$ C) $\frac{x+1}{x-1}$

D) $\frac{x+1}{1-x}$ E) 1

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	B	E	B	B	D	A	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	C	B	D	D	C	B	D

1. $(x+6)^{x^2-x-6}=1 \Rightarrow \sum x = ?$

A) −11 B) −10 C) −9
D) −8 E) −7

2. $\dfrac{2\cdot3^{x+1}-5\cdot3^x+3^{x-1}}{3\cdot3^{x-1}+4\cdot3^{x+1}-3^x}=?$

A) $\frac{1}{12}$ B) $\frac{1}{9}$ C) $\frac{1}{6}$
D) $\frac{1}{3}$ E) $\frac{1}{2}$

3. $\dfrac{25^x+15^x+10^x}{20^x+12^x+8^x}=\dfrac{4}{5} \Rightarrow x = ?$

A) −2 B) −1 C) $\frac{1}{2}$
D) 1 E) 2

4. $\dfrac{4}{7^{1-x}+1}+\dfrac{4}{7^{x-1}+1}=?$

A) 1 B) 2 C) 3
D) 4 E) 5

5. $3^{x+1}+3^x=36,$
$2^x+16^y=8 \Rightarrow y = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1
D) 2 E) 3

6. $10^x=980 \Rightarrow ? < ? < ?$

A) $-1 < x < 0$ B) $0 < x < 1$ C) $1 < x < 2$
D) $2 < x < 3$ E) $3 < x < 4$

7. $x \neq 0$

$\left(\dfrac{8x^a}{x^{-b}}\right)^{-2}\cdot\left(\dfrac{x^{a+1}}{x^{-b+1}}\right)^2=?$

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{16}$
D) $\frac{1}{64}$ E) $\frac{1}{128}$

8. $\left.\begin{array}{l}\dfrac{3^x}{9^y}=81^2\\ x+y=17\end{array}\right\} \Rightarrow y = ?$

A) 1 B) 2 C) 3
D) 4 E) 5

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9. $(16)^x = a^2 \Rightarrow 2^{8x} = ?$

A) a B) a^2 C) a^3

D) a^4 E) a^5

10. $\left.\begin{array}{l} x = 5^{-36} \\ y = 6^{-24} \\ z = 7^{-12} \end{array}\right\} \Rightarrow ? > ? > ?$

A) x > y > z B) x > z > y C) y > x > z

D) z > y > x E) z > x > y

11. $\dfrac{5^{37} - 5^9}{(5^{14} - 1)(5^{14} + 1)} = 5^{3a} \Rightarrow a = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

12. $6^{\frac{1}{64}} = x \Rightarrow \left(\dfrac{6^{\frac{1}{16}} + 1}{6^{\frac{1}{8}} - 1}\right) : \dfrac{1}{6^{\frac{1}{32}}} = ?$

A) $\dfrac{x}{1+x^2}$ B) $\dfrac{x^2}{1+x}$ C) $\dfrac{x^3}{x^2+1}$

D) $\dfrac{x^2}{1-x^4}$ E) $\dfrac{x^2}{x^4-1}$

13. $5^x = 4 \Rightarrow 2^{\frac{x+1}{x}} = ?$

A) 2 B) 4 C) $2\sqrt{5}$

D) 8 E) 16

14. $3^{x+y} = 15^x \Rightarrow 9^{\frac{y}{x}} = ?$

A) $\dfrac{1}{5}$ B) 1 C) 5

D) 25 E) 125

15. $\left.\begin{array}{l} x = (-2^3)^6 \\ y = (-2)^{(3^2)} \\ z = (-8^2)^5 \end{array}\right\} \Rightarrow ? > ? > ?$

A) x > y > z B) x > z > y C) y > x > z

D) y > z > x E) z > y > x

16. $\left.\begin{array}{l} a^{x+y} = 48 \\ a^{x-y} = 12 \end{array}\right\} \Rightarrow a^x = ?$

A) 2 B) 8 C) 16

D) 24 E) 32

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	B	B	D	B	D	D	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	D	C	E	C	D	A	D

1. $2^{x-1} = a \Rightarrow 2^{x+1} + 2^{x+2} + 2^x = ?$

A) 10a B) 12a C) 14a

D) 16a E) 18a

2. $A, n \in Z$

$(0{,}0005)^4 \cdot (0{,}4)^4 = A \cdot 10^n \Rightarrow \min(A + n) = ?$

A) −12 B) −8 C) −4

D) 0 E) 16

3. $x^y + \frac{1}{x^y} = 6 \Rightarrow (x^{3y} + x^{-3y}) \cdot 3^{-2} = ?$

A) 11 B) 14 C) 16

D) 19 E) 22

4. $\left.\begin{array}{l} 2^{x^2-y^2} = 27 \\ 4^{x-y} = 9 \end{array}\right\} \Rightarrow x + y = ?$

A) 2 B) 3 C) 4

D) 5 E) 6

5. $5^{x-1} + \frac{24}{5^{1-x}} = 125 \Rightarrow x = ?$

A) 2 B) 3 C) 4

D) 5 E) 6

6. $\left.\begin{array}{l} a = 2^{x-1} \\ b = 3^{x-1} \\ c = 5^{x+1} \end{array}\right\} \Rightarrow 180^x = ?$

A) a^2b^2c B) ab^3c C) $36a^2bc$

D) $\frac{5a^2b^2c}{36}$ E) $\frac{36a^2b^2c}{5}$

7. $x \in Z^+$

$(4x - 11)^{x^2-x-6} = 1 \Rightarrow \sum x = ?$

A) −1 B) 0 C) 1

D) 2 E) 3

8. $\left.\begin{array}{l} 5^{-x} = \frac{2a}{b} \\ 5^x = \frac{a+4}{2b} \end{array}\right\} \Rightarrow a = ?$

A) $\sqrt{b^2+4} - 2$ B) $b^2 + 2$ C) $\sqrt{b^2+8}$

D) $\sqrt{b^2-4} + 2$ E) $\sqrt{b^2+4} - 4$

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9. $3^{x-1} = a \Rightarrow 3^{x-2} + 3^{x+1} = ?$

A) 8a B) $\frac{25a}{3}$ C) $\frac{26a}{3}$

D) 9a E) $\frac{28a}{3}$

10. $\frac{2^{x+1} - 2^{x-1}}{2^{x+2} + 2^{x+1}} = ?$

A) $\frac{1}{6}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$

D) 1 E) 2

11. $3^x \cdot 2^y = 54$,
$2^x \cdot 3^y = 24$
$\Rightarrow x + y = ?$

A) 8 B) 7 C) 6

D) 5 E) 4

12. $\left.\begin{matrix} 9^x = 125 \\ 3^y = 25 \end{matrix}\right\} \Rightarrow \frac{x}{y} = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{3}{4}$

D) 1 E) $\frac{5}{4}$

13. $\left.\begin{matrix} 2^x = 5 \\ 5^y = 4 \end{matrix}\right\} \Rightarrow x \cdot y = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

14. $\left.\begin{matrix} 2^{x+1} - 3^{y+2} = -11 \\ 3^{y+1} - 2^x = 1 \end{matrix}\right\} \Rightarrow x + y = ?$

A) 4 B) 5 C) 6

D) 7 E) 8

15. $x^{0,4} = 4 \Rightarrow x = ?$

A) 16 B) 24 C) 32

D) 48 E) 64

16. $(0,2)^{x+1} \cdot 25^{-x+2} = 125^{x-1} \Rightarrow x = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	D	E	B	A	E	E	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	B	E	C	B	A	C	A

BÖLÜM CHAPTER 4

TARİHSEL NOT / HISTORICAL NOTE

Leonhard Euler [1707 – 1783]

Leonhard Euler, matematik, fizik, analitik geometri, trigonometri ve sayı teorisi konusunda katkıları bulunan bir İsviçre matematikçisidir.

Leonhard Euler was a Swiss mathematician who made enormous contibutions to a wide range of mathematics and physics topics including analytic geometry, trigonometry, geometry, calculus and number theory.

KÖKLÜ SAYILAR
RADICALS

Bu bölüm, 31 çözümlü örnek, 12 çözümlü test sorusu, 16 çözümlü YÖS sorusu, 128 test sorusu içermektedir.
This chapter includes 31 examples with solutions, 12 tests questions with solutions, 16 YÖS questions with solutions and 128 test questions.

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BÖLÜM
04
CHAPTER

KÖKLÜ SAYILAR
RADICALS

KÖKLÜ SAYILAR / RADICALS

Tanım: $n \in Z^+$ $n \geq 2$ olmak üzere,

$x^n = a$ eşitliğini sağlayan x sayısına, *a'nın n'inci dereceden kökü* denir ve $x = \sqrt[n]{a}$ ile gösterilir.

$n = 2$ için $\sqrt{a}$ ifadesi karekök a,

$n = 3$ için $\sqrt[3]{a}$ ifadesi küpkök a diye okunur.

Definition: $n \in Z^+$ $n \geq 2$

For any positive integer n, if $x^n = a$, then x is called the *n^{th} root of the number a.* The symbol $\sqrt[n]{a}$ is used to denote the n^{th} root of a.

For $n = 2$, $\sqrt{a}$ is the square root of a,

For $n = 3$, $\sqrt[3]{a}$ is the cube root of a.

ÖNEMLİ / IMPORTANT

• Çift dereceli köklerin sonucu negatif olamayacağından $\sqrt{x} = -4$ şeklindeki denklemler reel sayılarda tanımsızdır.

• The results of even roots can not be negative, so equations like $\sqrt{x} = -4$ are undefined in real numbers.

Örneğin; $x^3 = 64$ denkleminde $64 = 4^3$ olduğundan, $x = \sqrt[3]{64} = 4$ tür.

$x^4 = 16$ denkleminde ise $16 = 2^4$ veya $16 = (-2)^4$ olduğundan x hem 2, hem de (–2) değerlerini alabilir. Fakat, 16'nın 4. dereceden kökü 2 dir. ($\sqrt[4]{16} = 2$)

For example; In equation $x^3 = 64$ since $64 = 4^3$

$x = \sqrt[3]{64} = 4$.

In equation $x^4 = 16$, $16 = 2^4$ or $16 = (-2)^4$, x can be both 2 and –2. However, the fourth index of 16 is 2. ($\sqrt[4]{16} = 2$)

ÖNEMLİ / IMPORTANT

• Reel sayılarda bir negatif sayının çift dereceli kökü yoktur.

• A negative number does not have a square real number root.

O halde, kök alma işlemi, kök derecesinin tek veya çift olması durumlarında, aşağıdaki gibi tanımlanır.

Therefore, the definition of root according to n's being odd or even can be defined as follow.

Özellik / Property

★ n tek doğal sayı ise, $\sqrt[n]{a^n} = a$ dır.

★ n çift doğal sayı ise, $\sqrt[n]{a^n} = |a|$ dır.

★ If n is an odd natural number, then $\sqrt[n]{a^n} = a$.

★ If n is an even natural number, then $\sqrt[n]{a^n} = |a|$.

Etkinlikler / Activities

a. $\sqrt[3]{216} = \sqrt[3]{6^3} = 6$

b. $\sqrt[3]{-125} = \sqrt[3]{(-5)^3} = -5$

c. $\sqrt{9} = \sqrt{(\mp 3)^2} = |\mp 3| = 3$

d. $\sqrt[4]{(-2)^4} = |-2| = 2$

e. $\sqrt{(\sqrt{3}-2)^2} = |\sqrt{3}-2| = 2-\sqrt{3}\ \ (2 > \sqrt{3})$

Örnek / Example 1

$\sqrt{(-3)^2} + \sqrt[3]{(-2)^3} + \sqrt[4]{16} = ?$

A) 1 B) 3 C) 5 D) 7 E) 9

Çözüm / Solution

Yanıt / Answer **B**

$\sqrt{(-3)^2} = |-3| = 3$

$\sqrt[3]{(-2)^3} = -2$

$\sqrt[4]{16} = \sqrt[4]{(2)^4} = 2$

$\sqrt{(-3)^2} + \sqrt[3]{(-2)^3} + \sqrt[4]{16} = |-3| + (-2) + |2|$

$= 3 - 2 + 2 = 3$

Örnek / Example 2

$\sqrt{0,25} + \sqrt{1,21} + \sqrt{1,44} = ?$

A) 1,8 B) 2 C) 2,2 D) 2,6 E) 2,8

Çözüm / Solution

Yanıt / Answer **E**

$\sqrt{0,25} + \sqrt{1,21} + \sqrt{1,44} = \sqrt{(0,5)^2} + \sqrt{(1,1)^2} + \sqrt{(1,2)^2}$

$= 0,5 + 1,1 + 1,2$

$= 2,8$

PEKİŞTİRME / REINFORCE

✓ $\sqrt{4} = ?$

..... [2]

✓ $\sqrt[4]{81} = ?$

..... [3]

✓ $\sqrt[3]{27} = ?$

..... [3]

PEKİŞTİRME / REINFORCE

✓ $\sqrt{0,25} = ?$

..... [0,5]

✓ $\sqrt{1,21} = ?$

..... [1,1]

✓ $\sqrt{1,44} = ?$

..... [1,2]

ÖNEMLİ / IMPORTANT

- $(-a)^{2n} = a^{2n}$
- $(-a)^{2n+1} = -a^{2n+1}$

Örnek / Example 3

$\sqrt[4]{(-2)^3+(-2)^2+(-2)^2} = ?$

A) 0 B) 1 C) 2 D) 3 E) 4

Çözüm / Solution

Yanıt / Answer **A**

$(-2)^3 = (-2)\cdot(-2)\cdot(-2) = -8$

$(-2)^2 = (-2)\cdot(-2) = 4$

$\sqrt[4]{(-2)^3+(-2)^2+(-2)^2} = \sqrt[4]{-8+4+4}$

$= \sqrt[4]{0}$

$= 0$

PEKİŞTİRME / REINFORCE

- ✓ $a,\overline{b} = \frac{ab-a}{9}$
- ✓ $\sqrt{\frac{25}{9}} = \sqrt{\left(\frac{5}{3}\right)^2} = \frac{5}{3}$
- ✓ $\sqrt{\frac{49}{9}} = \sqrt{\left(\frac{7}{3}\right)^2} = \frac{7}{3}$
- ✓ $\sqrt{\frac{1}{9}} = \sqrt{\left(\frac{1}{3}\right)^2} = \frac{1}{3}$
- ✓ $\sqrt{\frac{4}{9}} = \sqrt{\left(\frac{2}{3}\right)^2} = \frac{2}{3}$

Örnek / Example 4

$\frac{\sqrt{2,\overline{7}}+\sqrt{5,\overline{4}}}{\sqrt{0,\overline{1}}+\sqrt{0,\overline{4}}} = ?$

A) 2 B) 4 C) 6 D) 8 E) 10

Çözüm / Solution

Yanıt / Answer **B**

$\frac{\sqrt{\frac{27-2}{9}}+\sqrt{\frac{54-5}{9}}}{\sqrt{\frac{1}{9}}+\sqrt{\frac{4}{9}}} = \frac{\sqrt{\frac{25}{9}}+\sqrt{\frac{49}{9}}}{\frac{1}{3}+\frac{2}{3}}$

$= \frac{\frac{5}{3}+\frac{7}{3}}{\frac{3}{3}}$

$= \frac{12}{3} = 4$

Özellik / Property

★ $\sqrt{a} + \sqrt{b} = 0 \Rightarrow a = 0 , b = 0$

Örnek / Example 5

$\sqrt{a-3} + \sqrt{b-4} = 0 \Rightarrow a \cdot b = ?$

A) 9 B) 10 C) 12 D) 14 E) 16

Çözüm / Solution

Yanıt / Answer **C**

$\sqrt{a-3} + \sqrt{b-4} = 0$

$a - 3 = 0 , \quad b - 4 = 0$

$a = 3 \qquad b = 4$

$\Rightarrow a \cdot b = 3 \cdot 4 = 12$

Örnek / Example 6

$\sqrt[4]{x-2y} + x^2 = 12x - 36 \Rightarrow x + y = ?$

A) 3 B) 4 C) 5 D) 8 E) 9

Çözüm / Solution

Yanıt / Answer **E**

$\sqrt[4]{x-2y} + x^2 = 12x - 36$

$\sqrt[4]{x-2y} + x^2 - 12x + 36 = 0$

$\sqrt[4]{x-2y} + (x-6)^2 = 0$

$x - 2y = 0 , \quad x - 6 = 0$

$x = 2y , \quad x = 6$

$6 = 2y$

$y = 3$

$\Rightarrow x + y = 6 + 3 = 9$

PEKİŞTİRME / REINFORCE

✓ $\sqrt[2]{2^3} = ?$

..... [$2^{3/2}$]

✓ $\sqrt[3]{3^2} = ?$

..... [$3^{2/3}$]

✓ $\sqrt[4]{4^2} = ?$

..... [2]

✓ $\sqrt[3]{9^2} = ?$

..... [$3^{4/3}$]

✓ $\sqrt[3]{27^2} = ?$

..... [9]

Özellik / Property

★ **I.** p tek sayı ise, $\sqrt[p]{a^n} = a^{\frac{n}{p}}$ dır.

If p is an odd number, $\sqrt[p]{a^n} = a^{\frac{n}{p}}$

II. p ve n çift sayı ise, $\sqrt[p]{a^n} = |a|^{\frac{n}{p}}$ dır.

If p and n are even numbers, $\sqrt[p]{a^n} = |a|^{\frac{n}{p}}$

Özellik / Property

★ **I.** x tek doğal sayı ve a > 0 ise, $\sqrt[n]{a^n} = \sqrt[nx]{a^{nx}}$ dır.

II. a < 0 ve x çift doğal sayı ise, $\sqrt[n]{a^n} \neq \sqrt[nx]{a^{nx}}$ dır.

I. If x is an odd natural number and a > 0, then $\sqrt[n]{a^n} = \sqrt[nx]{a^{nx}}$

II. If a < 0 and x is an even natural number, then $\sqrt[n]{a^n} \neq \sqrt[nx]{a^{nx}}$

Etkinlikler / Activities

a. $\sqrt[4]{5^2} = \sqrt[4 \cdot 3]{5^{2 \cdot 3}}$

b. $\sqrt[5]{(-5)^3} \neq \sqrt[5 \cdot 2]{(-5)^{3 \cdot 2}}$

Örnek / Example 7

$$\frac{\sqrt{3} \cdot \sqrt[3]{3} \cdot \sqrt[4]{3}}{\sqrt[6]{243}} = ?$$

A) $4\sqrt{3}$ B) $2\sqrt{3}$ C) 3 D) $\sqrt{3}$ E) $\sqrt[4]{3}$

Çözüm / Solution

Yanıt / Answer **E**

$$\frac{\sqrt{3} \cdot \sqrt[3]{3} \cdot \sqrt[4]{3}}{\sqrt[6]{243}} = \frac{3^{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}}{3^{\frac{5}{6}}}$$

$$= \frac{3^{\frac{13}{12}}}{3^{\frac{5}{6}}}$$

$$= 3^{\frac{13}{12}-\frac{5}{6}}$$

$$= 3^{\frac{1}{4}} = \sqrt[4]{3}$$

Özellik / Property

★ $\sqrt[n]{a^n \cdot b} = \begin{cases} a \cdot \sqrt[n]{b}, & n \text{ tek ise / if n is odd} \\ |a| \cdot \sqrt[n]{b}, & n \text{ çift ise / if n is even} \end{cases}$

Etkinlikler / Activities

a. $\sqrt{240} = ?$

240	2
120	2
60	2
30	2
15	3
5	5
1	

$$\sqrt{240} = \sqrt{2^4 \cdot 3 \cdot 5} = 2^2\sqrt{15} = 4\sqrt{15}$$

b. $5\sqrt{144} = ?$

144	2
72	2
36	2
18	2
9	3
3	3
1	

$$5\sqrt{2^4 \cdot 3^2} = 5\sqrt{2^4} \cdot \sqrt{3^2} = 5 \cdot 4 \cdot 3 = 60$$

c. $\sqrt[4]{32} = \sqrt[4]{16 \cdot 2} = \sqrt[4]{16} \cdot \sqrt[4]{2} = 2\sqrt[4]{2}$

d. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$

e. $\sqrt[4]{48} = \sqrt[4]{16 \cdot 3} = \sqrt[4]{16} \cdot \sqrt[4]{3} = 2\sqrt[4]{3}$

f. $\sqrt[3]{135} = \sqrt[3]{27 \cdot 5} = \sqrt[3]{27} \cdot \sqrt[3]{5} = 3\sqrt[3]{5}$

Özellik / Property

★ $a\sqrt[n]{b} = \sqrt[n]{a^n b}$

Etkinlikler / Activities

a. $4\sqrt{3} = \sqrt{4^2 \cdot 3} = \sqrt{16 \cdot 3} = \sqrt{48}$

b. $2\sqrt{5} = \sqrt{2^2 \cdot 5} = \sqrt{4 \cdot 5} = \sqrt{20}$

c. $3\sqrt[3]{2} = \sqrt[3]{3^3 \cdot 2} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{54}$

d. $3\sqrt[4]{3} = \sqrt[4]{3^4 \cdot 3} = \sqrt[4]{81 \cdot 3} = \sqrt[4]{243}$

PEKİŞTİRME / REINFORCE

✓ $\sqrt{169} = ?$

..... [13]

✓ $\sqrt{243} = ?$

..... [$9\sqrt{3}$]

✓ $\sqrt[3]{64} = ?$

..... [4]

✓ $\sqrt[4]{243} = ?$

..... [$3\sqrt[4]{3}$]

ÖNEMLİ / IMPORTANT

- $\sqrt{a+b} \neq \sqrt{a}+\sqrt{b}$
- $\sqrt{a-b} \neq \sqrt{a}-\sqrt{b}$

PEKİŞTİRME / REINFORCE

✓ $2\sqrt{3}+3\sqrt{3}=?$

..... [$5\sqrt{3}$]

✓ $2\sqrt{5}-3\sqrt{5}=?$

..... [$-\sqrt{5}$]

✓ $2\sqrt{5}+\sqrt{5}-3\sqrt{5}=?$

..... [0]

✓ $\sqrt{12}+\sqrt{27}-\sqrt{48}=?$

..... [$\sqrt{3}$]

✓ $3\sqrt{7}-2\sqrt{7}+\sqrt{7}=?$

..... [$2\sqrt{7}$]

KÖKLÜ SAYILARDA DÖRT İŞLEM
FOUR OPERATIONS in RADICALS

TOPLAMA ve ÇIKARMA
ADDITION and SUBTRACTION

Kök içerisinde yer alan ifade ve kök derecesi aynı olan köklü sayılara benzer köklü sayılar denir. Sadece benzer köklü ifadelerde, toplama veya çıkarma yapılabildiğinden benzer kökler toplanacağı veya çıkarılacağı zaman, kök katsayılarının toplamı veya farkı alınır ve sonra benzer olan köklü ifade ile çarpılır.

Radicals are said to be like (or similar) radicals if they are in the same index, and have the same radicand. Only like radicals can be added (or subtracted) , so when we add or subtract like radicals, we find the algebraic sum or difference of the coefficients of the radicals and multiply that sum by the like radical.

Örnek / Example 8

$\sqrt{48}-\sqrt{12}+2\sqrt{27}=?$

A) 0 B) $2\sqrt{3}$ C) $4\sqrt{3}$ D) $8\sqrt{3}$ E) $10\sqrt{3}$

Çözüm / Solution Yanıt / Answer **D**

$\sqrt{48}-\sqrt{12}+2\sqrt{27}=\sqrt{16\cdot 3}-\sqrt{4\cdot 3}+2\sqrt{9\cdot 3}$

$=4\sqrt{3}-2\sqrt{3}+6\sqrt{3}$

$=(4-2+6)\sqrt{3}$

$=8\sqrt{3}$

Örnek / Example 9

$3\sqrt{147}+2\sqrt{75}-5\sqrt{108}=?$

A) 0 B) $\sqrt{3}$ C) $-2\sqrt{7}$ D) $2\sqrt{7}$ E) $6\sqrt{3}$

Çözüm / Solution Yanıt / Answer **B**

$3\sqrt{147}+2\sqrt{75}-5\sqrt{108}=3\cdot\sqrt{49\cdot 3}+2\cdot\sqrt{25\cdot 3}-5\cdot\sqrt{36\cdot 3}$

$=3\cdot 7\sqrt{3}+2\cdot 5\sqrt{3}-5\cdot 6\sqrt{3}$

$=21\sqrt{3}+10\sqrt{3}-30\sqrt{3}$

$=(21+10-30)\sqrt{3}$

$=\sqrt{3}$

KÖKLÜ SAYILARDA ÇARPMA İŞLEMİ
MULTIPLICATION in RADICALS

Özellik / Property

★ İki veya daha fazla köklü ifade aynı kök derecesine sahipse kök içerisinde çarpma işlemi yapılabilir.
If two or more radicals have the same root degree, they can be multiplied inside radical.
$a, b \in R^+$

I. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

II. $\sqrt{a} \cdot \sqrt{a} = \sqrt{a^2}$

Etkinlikler / Activities

a. $\sqrt{2} \cdot \sqrt{5} = \sqrt{2 \cdot 5} = \sqrt{10}$

b. $\sqrt[3]{3} \cdot \sqrt[3]{13} = \sqrt[3]{39}$

c. $2\sqrt{15} \cdot \sqrt{5} = 2\sqrt{15 \cdot 5}$
$= 2\sqrt{5 \cdot 5 \cdot 3} = 10\sqrt{3}$

d. $\sqrt[3]{18} \cdot \sqrt[3]{2} = \sqrt[3]{36}$

e. $\sqrt[4]{40} \cdot \sqrt[4]{4} = \sqrt[4]{160}$
$= \sqrt[4]{16 \cdot 10} = 2\sqrt[4]{10}$

f. $\sqrt[7]{2^4 \cdot 3^2} \cdot \sqrt[7]{2^4 \cdot 3^5} = \sqrt[7]{2^7 \cdot 2 \cdot 3^7}$
$= \sqrt[7]{2 \cdot 6^7} = 6\sqrt[7]{2}$

Özellik / Property

★ Eğer köklü ifadeler aynı kök derecesine sahip değilse, ilk olarak kök dereceleri eşitlenir.
If the radicals do not have the same index, they should first be reduced to radicals with the same index.

$\sqrt[n]{a} \cdot \sqrt[m]{a} = \sqrt[m \cdot n]{a^m} \cdot \sqrt[m \cdot n]{a^n} = \sqrt[m \cdot n]{a^{m+n}}$

Örnek / Example 10

$\sqrt{a} \cdot \sqrt[3]{a^2} = ?$

A) $\sqrt[6]{a}$ B) $\sqrt[4]{a}$ C) $\sqrt[3]{a}$ D) $a\sqrt[6]{a}$ E) 2a

Çözüm / Solution

Yanıt / Answer **D**

$\sqrt{a} \cdot \sqrt[3]{a^2} = \sqrt[3 \cdot 2]{a^3} \cdot \sqrt[2 \cdot 3]{a^{2 \cdot 2}}$

$= \sqrt[6]{a^3} \cdot \sqrt[6]{a^4}$

$= \sqrt[6]{a^3 \cdot a^4}$

$= \sqrt[6]{a^7} = a\sqrt[6]{a}$

PEKİŞTİRME / REINFORCE

✓ $\sqrt{3} \cdot \sqrt{7} = ?$

..... [$\sqrt{21}$]

✓ $\sqrt{15} \cdot \sqrt{9} = ?$

..... [$3\sqrt{15}$]

✓ $\sqrt{3} \cdot \sqrt{81} = ?$

..... [$9\sqrt{3}$]

✓ $\sqrt[4]{3} \cdot \sqrt[4]{27} = ?$

..... [3]

✓ $\sqrt[3]{8} \cdot \sqrt[3]{64} = ?$

..... [8]

Özellik / Property

★ $a \in R^+$, $n \in N^+$ ise, $(\sqrt{a})^n = \sqrt{a^n}$ dir.

If $a \in R^+$ and $n \in N^+$, then $(\sqrt{a})^n = \sqrt{a^n}$

PEKİŞTİRME / REINFORCE

✓ $(\sqrt{5})^2 = ?$

..... [5]

✓ $(\sqrt{5})^6 = ?$

..... [125]

✓ $(\sqrt{7})^4 = ?$

..... [49]

✓ $(\sqrt{1})^2 = ?$

..... [1]

✓ $(\sqrt{6})^4 = ?$

..... [36]

Etkinlikler / Activities

a. $(\sqrt{7})^2 = \sqrt{7^2} = 7$

b. $(\sqrt{2})^4 = \sqrt{2^4} = \sqrt{2^2 \cdot 2^2} = 2 \cdot 2 = 4$

c. $(\sqrt{3})^6 = \sqrt{3^6} = \sqrt{3^2 \cdot 3^2 \cdot 3^2} = 3 \cdot 3 \cdot 3 = 27$

Örnek / Example 11

$p, q \in Z^+$

$\sqrt[12]{a} \cdot \sqrt[32]{b} = \sqrt[x]{a^p \cdot b^q} \Rightarrow \dfrac{x+q}{p+q} = ?$

A) 1 B) 3 C) 6 D) 9 E) 11

Çözüm / Solution

Yanıt / Answer **D**

$\sqrt[12]{a} \cdot \sqrt[32]{b} = \sqrt[12 \cdot 8]{a^8} \cdot \sqrt[32 \cdot 3]{b^3}$

$= \sqrt[96]{a^8 \cdot b^3}$

$\left.\begin{matrix} x = 96 \\ p = 8 \\ q = 3 \end{matrix}\right\} \Rightarrow \dfrac{96+3}{8+3} = \dfrac{99}{11} = 9$

Örnek / Example 12

$\sqrt{2} \cdot \sqrt{3} \cdot \ldots \cdot \sqrt{n} = 2\sqrt{30} \Rightarrow n = ?$

A) 2 B) 3 C) 5 D) 8 E) 16

Çözüm / Solution

Yanıt / Answer **C**

$\sqrt{2 \cdot 3 \cdot 4 \cdot \ldots \cdot n} = \sqrt{4 \cdot 30}$

$\sqrt{n!} = \sqrt{120}$

$n! = 120$

$n = 5$

KÖKLÜ SAYILARDA BÖLME İŞLEMİ
DIVISION in RADICALS

İki köklü ifade aynı kök derecesine sahip ise ortak kök içerisinde bölme işlemi yapılabilir.

$a, b \in R^+ , b \neq 0$

If two radicals have same index, the operation of division can be done inside the common radical.

$a, b \in R^+ , b \neq 0$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Etkinlikler / Activities

a. $\frac{\sqrt{15}}{\sqrt{3}} = \sqrt{\frac{15}{3}} = \sqrt{5}$

b. $\frac{\sqrt{225}}{\sqrt{81}} = \sqrt{\frac{225}{81}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$

c. $\sqrt{\frac{3}{5}} : \sqrt{\frac{27}{125}} = \sqrt{\frac{3}{5} : \frac{27}{125}} = \sqrt{\frac{3}{5} \cdot \frac{125}{27}}$

$= \sqrt{\frac{25}{9}} = \frac{5}{3}$

d. $\sqrt{a^3} : \sqrt{\frac{1}{a}} = \sqrt{a^3 : \frac{1}{a}} = \sqrt{a^3 \cdot a} = \sqrt{a^4} = a^2$

Örnek / Example 13

$$\frac{\sqrt{7+\frac{1}{9}}}{0,1\overline{3}} = ?$$

A) 10 B) 15 C) 18 D) 20 E) 30

Çözüm / Solution

Yanıt / Answer **D**

$$\frac{\sqrt{7+\frac{1}{9}}}{0,1\overline{3}} = \frac{\sqrt{\frac{64}{9}}}{\frac{13-1}{90}}$$

$$= \frac{8}{3} \cdot \frac{90}{12} = 20$$

PEKİŞTİRME / REINFORCE

✓ $\frac{\sqrt{27}}{\sqrt{3}} = ?$

..... [3]

✓ $\frac{\sqrt{21}}{\sqrt{3}} = ?$

..... [$\sqrt{7}$]

✓ $\frac{\sqrt{6}}{\sqrt{3}} = ?$

..... [$\sqrt{2}$]

✓ $\frac{\sqrt{18}}{\sqrt{2}} = ?$

..... [3]

ÖNEMLİ / IMPORTANT

- $0,a\overline{b} = \frac{ab-a}{90}$

EŞLENİK İFADELER / CONJUGATE TERMS

Eşlenik: Rasyonel olmayan bir ifadeyi rasyonel yapan çarpana eşlenik denir.

Conjugate: a multiplier that vationalizes an irrational expresson is called conjugate.

ifade / Expression	Eşlenik / Conjugate
$\sqrt{a}$	$\sqrt{a}$
$\sqrt[n]{a^t}$	$\sqrt[n]{a^{n-t}}$
$\sqrt{a}+\sqrt{b}$	$\sqrt{a}-\sqrt{b}$
$\sqrt[3]{a}-\sqrt[3]{b}$	$\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}$
$\sqrt{a}+\sqrt{b}+\sqrt{c}$	$\sqrt{a}+\sqrt{b}-\sqrt{c}$, $\sqrt{a}-\sqrt{b}+\sqrt{c}$, $-\sqrt{a}+\sqrt{b}+\sqrt{c}$

PEKİŞTİRME / REINFORCE

✓ $\frac{6}{\sqrt{3}} = ?$ ..... [$2\sqrt{3}$]

✓ $\frac{3}{\sqrt{2}} = ?$ ..... [$\frac{3\sqrt{2}}{2}$]

✓ $\frac{6}{\sqrt{6}} = ?$ ..... [$\sqrt{6}$]

✓ $\frac{5}{\sqrt{6}} = ?$ ..... [$\frac{5\sqrt{6}}{6}$]

Etkinlikler / Activities

a. $\frac{1}{\sqrt{3}} = \frac{1\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3^2}} = \frac{\sqrt{3}}{3}$

b. $\frac{4}{\sqrt{2}} = \frac{4\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

c. $\frac{3}{\sqrt{5}-\sqrt{2}} = \frac{3\cdot(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})} = \frac{3(\sqrt{5}+\sqrt{2})}{3} = \sqrt{5}+\sqrt{2}$

Örnek / Example 14

$\frac{1}{\sqrt{3}-\sqrt{2}} - \frac{2}{\sqrt{2}} = ?$

A) $\frac{\sqrt{3}}{2}$ B) $\sqrt{3}$ C) $2\sqrt{3}$ D) $3\sqrt{3}$ E) $4\sqrt{3}$

Çözüm / Solution

Yanıt / Answer **B**

$\underset{(\sqrt{3}+\sqrt{2})}{\frac{1}{\sqrt{3}-\sqrt{2}}} - \underset{(\sqrt{2})}{\frac{2}{\sqrt{2}}} = \frac{\sqrt{3}+\sqrt{2}}{3-2} - \frac{2\sqrt{2}}{2}$

$= \sqrt{3}+\sqrt{2}-\sqrt{2} = \sqrt{3}$

Özellik / Property

★ $\sqrt{a \mp 2\sqrt{b}}$ nin $\sqrt{m} \mp \sqrt{n}$ biçiminde yazılışı

$\sqrt{a \mp 2\sqrt{b}}$ ifadesi için a, b $\in R^+$ olmak üzere,

b = m·n , a = m + n koşulunu sağlayan m ve n gibi iki doğal sayı bulunabiliyorsa,

m > n olmak üzere, $\sqrt{a \mp 2\sqrt{b}} = \sqrt{m} \mp \sqrt{n}$ şeklinde yazılabilir.

★ Writing the expression $\sqrt{a \mp 2\sqrt{b}}$ in the form of $\sqrt{m} \mp \sqrt{n}$

For the expression $\sqrt{a \mp 2\sqrt{b}}$,

If you can find two natural numbers m and n in a way that b = m·n and a = m + n when

m > n, m,n $\in N^+$.

We can write as follows: $\sqrt{a \mp 2\sqrt{b}} = \sqrt{m} \mp \sqrt{n}$.

ÖNEMLİ / IMPORTANT

- $a, b \in R^+$

$$\left.\begin{array}{l} a = \dfrac{m+n}{2} \\ b = m \cdot n \end{array}\right\} \Rightarrow \sqrt{a \pm \sqrt{b}} = \sqrt{\frac{m}{2}} \pm \sqrt{\frac{n}{2}}$$

Örnek / Example 15

$\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}} = ?$

A) 1 B) 2 C) 4 D) 6 E) 8

Çözüm / Solution

Yanıt / Answer **B**

$$\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}} = \sqrt{3}+1-(\sqrt{3}-1)$$

$$= \sqrt{3}+1-\sqrt{3}+1$$

$$= 2$$

PEKİŞTİRME / REINFORCE

✓ $\sqrt{7+2\sqrt{12}} = ?$

..... [$2+\sqrt{3}$]

✓ $\sqrt{10+2\sqrt{21}} = ?$

..... [$\sqrt{7}+\sqrt{3}$]

✓ $\sqrt{14-2\sqrt{45}} = ?$

..... [$3-\sqrt{5}$]

PEKİŞTİRME / REINFORCE

✓ $\sqrt{4-\sqrt{12}} = ?$

..... [$\sqrt{3}-1$]

✓ $\sqrt{3-\sqrt{8}} = ?$

..... [$\sqrt{2}-1$]

✓ $\sqrt{10+4\sqrt{6}} = ?$

..... [$\sqrt{6}+2$]

✓ $\sqrt{12+8\sqrt{2}} = ?$

..... [$2\sqrt{2}+2$]

Örnek / Example 16

$\sqrt{2+\sqrt{3}} - \sqrt{2-\sqrt{3}} = ?$

A) $\sqrt{2}$ B) $2\sqrt{2}$ C) 4 D) $4\sqrt{2}$ E) 8

Çözüm / Solution

Yanıt / Answer **A**

$$\sqrt{2+\sqrt{3}} - \sqrt{2-\sqrt{3}} = \sqrt{\frac{4+2\sqrt{3}}{2}} - \sqrt{\frac{4-2\sqrt{3}}{2}}$$

$$= \frac{\sqrt{4+2\sqrt{3}}}{\sqrt{2}} - \frac{\sqrt{4-2\sqrt{3}}}{\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{2}} - \frac{\sqrt{3}-1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}+1-\sqrt{3}+1}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} = \frac{\sqrt{2}\cdot\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

Örnek / Example 17

$\sqrt{14+8\sqrt{3}} = 2\sqrt{2}+b \Rightarrow 2\sqrt{3}\cdot b = ?$

A) $\sqrt{2}$ B) $2\sqrt{2}$ C) $3\sqrt{2}$ D) $4\sqrt{2}$ E) $6\sqrt{2}$

Çözüm / Solution

Yanıt / Answer **E**

$$\sqrt{14+8\sqrt{3}} = 2\sqrt{2}+b$$

$$\sqrt{14+2\cdot\sqrt{48}} = 2\sqrt{2}+b$$

$$\sqrt{8}+\sqrt{6} = \sqrt{8}+b$$

$$b = \sqrt{6}$$

$$2\sqrt{3}\cdot b = 2\sqrt{3}\cdot\sqrt{6}$$

$$= 2\sqrt{18}$$

$$2\cdot 3\sqrt{2} = 6\sqrt{2}$$

Örnek / Example 18

$\sqrt{5} = x \Rightarrow \sqrt{9-4\sqrt{5}} = ?$

A) $x-1$ B) $x-2$ C) $x+1$ D) $x+2$ E) $x+4$

Çözüm / Solution Yanıt / Answer **B**

$$\sqrt{9-4\sqrt{5}} = \sqrt{9-2\sqrt{20}}$$
$$= \sqrt{5} - \sqrt{4}$$
$$= \sqrt{5} - 2$$
$$= x - 2$$

İÇ İÇE KÖKLER / NESTED RADICALS

Özellik / Property

★ $\sqrt[n]{\sqrt[m]{a}} = \sqrt[n \cdot m]{a}$

★ $\sqrt[n]{a^k\sqrt[x]{b^x\sqrt{c}}} = \sqrt[n \cdot k \cdot x]{c \cdot b^x \cdot a^{k \cdot x}}$

Örnek / Example 19

$\sqrt{2^3\sqrt{2^5\sqrt{\frac{1}{4}}}} = ?$

A) $\sqrt[5]{2^3}$ B) $\sqrt[4]{2^3}$ C) $\sqrt{2^3}$ D) $\sqrt[5]{2^2}$ E) $\sqrt[4]{2^5}$

Çözüm / Solution Yanıt / Answer **A**

$$\sqrt{2^3\sqrt{2^5\sqrt{\frac{1}{4}}}} = \sqrt{\sqrt[3]{2^3 \cdot 2^5\sqrt{\frac{1}{4}}}}$$
$$= \sqrt{\sqrt[3]{\sqrt[5]{2^{20} \cdot \frac{1}{4}}}}$$
$$= \sqrt[30]{2^{18}}$$
$$= 2^{\frac{18}{30}} = 2^{\frac{3}{5}} = \sqrt[5]{2^3}$$

Örnek / Example 20

$\sqrt[x+3]{\sqrt[x-3]{81}} = \sqrt[4]{3} \Rightarrow x = ?$

A) 1 B) 3 C) 5 D) 7 E) 9

Çözüm / Solution Yanıt / Answer **C**

$\sqrt[x^2-9]{81} = \sqrt[4]{3}$

$3^{\frac{4}{x^2-9}} = 3^{\frac{1}{4}}$

$\frac{4}{x^2-9} = \frac{1}{4}$

$x^2 - 9 = 16 \Rightarrow x^2 = 25$

$x = 5$

Örnek / Example 21

$\sqrt{x\sqrt[3]{x}} = 16 \Rightarrow \sqrt[3]{x\sqrt{x}} = ?$

A) 24 B) 20 C) 16 D) 12 E) 8

Çözüm / Solution Yanıt / Answer **E**

$\sqrt{x\sqrt[3]{x}} = 16 \Rightarrow \sqrt[6]{x \cdot x^3} = 16$

$x = 2^6$

$\sqrt[3]{x\sqrt{x}} = \sqrt[6]{x \cdot x^2} = x^{\frac{1}{2}}$

$\Rightarrow (2^6)^{\frac{1}{2}} = 2^3 = 8$

Örnek / Example 22

$A = \sqrt{114+\sqrt{44+\sqrt{22+\sqrt{3+\sqrt{36}}}}} \Rightarrow A = ?$

A) 5 B) 7 C) 9 D) 11 E) 13

Çözüm / Solution

Yanıt / Answer **D**

$\sqrt{3+\sqrt{36}} = \sqrt{3+6} = 3$

$\sqrt{22+3} = \sqrt{25} = 5$

$\sqrt{44+5} = \sqrt{49} = 7$

$A = \sqrt{114+7} = \sqrt{121} \Rightarrow A = 11$

Örnek / Example 23

$\sqrt{\frac{28}{81}+\frac{1}{3}\sqrt{7+\sqrt{\frac{1}{81}}}} = ?$

A) $\frac{5}{9}$ B) $\frac{7}{9}$ C) $\frac{8}{9}$ D) 1 E) $\frac{10}{9}$

Çözüm / Solution

Yanıt / Answer **E**

$$\sqrt{\frac{28}{81}+\frac{1}{3}\sqrt{7+\sqrt{\frac{1}{81}}}} = \sqrt{\frac{28}{81}+\frac{1}{3}\sqrt{7+\frac{1}{9}}}$$

$$= \sqrt{\frac{28}{81}+\frac{1}{3}\cdot\frac{8}{3}}$$

$$= \sqrt{\frac{100}{81}}$$

$$= \frac{10}{9}$$

PEKİŞTİRME / REINFORCE

✓ $\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}} = ?$

.....[2]

✓ $\sqrt{12+\sqrt{12+\sqrt{12+\ldots}}} = ?$

.....[4]

✓ $\sqrt{20+\sqrt{20+\ldots}} = ?$

.....[5]

✓ $\sqrt{42+\sqrt{42+\ldots}} = ?$

.....[7]

Özellik / Property

★ $\sqrt{a+\sqrt{a+\sqrt{a+\ldots}}} = \dfrac{1+\sqrt{1+4\cdot a}}{2}$

$\sqrt{a-\sqrt{a-\sqrt{a-\ldots}}} = \dfrac{-1+\sqrt{1+4\cdot a}}{2}$

★ $a \in R^+,\ n\cdot(n+1) = a$

$\sqrt{a+\sqrt{a+\sqrt{a+\ldots}}} = n+1$

$\sqrt{a-\sqrt{a-\sqrt{a-\ldots}}} = n$

Örnek / Example 24

$A = \sqrt{30+\sqrt{30+\sqrt{30+\ldots}}} \Rightarrow A = ?$

A) 6 B) 16 C) 32 D) 64 E) 128

Çözüm / Solution

Yanıt / Answer **A**

$30 = 5\cdot6$

$\sqrt{5\cdot6+\sqrt{5\cdot6+\sqrt{5\cdot6+\ldots}}} = 6$

PEKİŞTİRME / REINFORCE

✓ $\sqrt{2-\sqrt{2-\ldots}} = ?$

.....[1]

✓ $\sqrt{12-\sqrt{12-\ldots}} = ?$

.....[3]

✓ $\sqrt{20-\sqrt{20-\ldots}} = ?$

.....[4]

✓ $\sqrt{30-\sqrt{30-\ldots}} = ?$

.....[5]

✓ $\sqrt{72-\sqrt{72-\sqrt{72-\ldots}}} = ?$

.....[8]

Örnek / Example 25

$\sqrt{56-\sqrt{56-\sqrt{56-\ldots}}} = ?$

A) 5 B) 6 C) 7 D) 8 E) 9

Çözüm / Solution

Yanıt / Answer **C**

$56 = 7\cdot8$

$\sqrt{7\cdot8-\sqrt{7\cdot8-\sqrt{7\cdot8-\ldots}}} = 7$

Örnek / Example 26

$\sqrt{a+\sqrt{a+\sqrt{a+\ldots}}} = 5 \Rightarrow a = ?$

A) 6 B) 7 C) 13 D) 17 E) 20

Çözüm / Solution

Yanıt / Answer **E**

$a = 5 \cdot (5 - 1) = 20$

Örnek / Example 27

$x = \sqrt{a-\sqrt{a-\sqrt{a\ldots}}} = 8 \Rightarrow a = ?$

A) 56 B) 64 C) 72 D) 80 E) 88

Çözüm / Solution

Yanıt / Answer **C**

$a = 8 \cdot (8 + 1) = 72$

Özellik / Property

★ $y = \sqrt[n]{a\sqrt[n]{a\sqrt[n]{a\ldots}}} = \sqrt[n-1]{a}$

PEKİŞTİRME / REINFORCE

✓ $\sqrt{2\sqrt{2\sqrt{2\ldots}}} = ?$

.....[2]

✓ $\sqrt[3]{5\sqrt[3]{5\sqrt[3]{5\ldots}}} = ?$

.....[$\sqrt{5}$]

✓ $\sqrt[4]{27\sqrt[4]{27\sqrt[4]{27\ldots}}} = ?$

.....[3]

✓ $\sqrt[4]{64\sqrt[4]{64\sqrt[4]{64\ldots}}} = ?$

.....[4]

Örnek / Example 28

$\sqrt[6]{32\sqrt[6]{32\sqrt[6]{32\ldots}}} + \sqrt{5-\sqrt{13+\sqrt{9}}} = ?$

A) 7 B) 6 C) 5 D) 4 E) 3

Çözüm / Solution Yanıt / Answer **E**

$\sqrt[6]{32\sqrt[6]{32\sqrt[6]{32\ldots}}} = \sqrt[5]{32} = 2$

$\sqrt{5-\sqrt{13+\sqrt{9}}} = \sqrt{5-\sqrt{16}} = \sqrt{5-4} = 1$

$\sqrt[6]{32\sqrt[6]{32\sqrt[6]{32\ldots}}} + \sqrt{5-\sqrt{13+\sqrt{9}}} = 2+1 = 3$

Örnek / Example 29

$\sqrt{3\sqrt{2\sqrt{3\sqrt{2\ldots}}}} = x \Rightarrow x = ?$

A) $\sqrt[4]{6}$ B) $\sqrt[4]{18}$ C) $\sqrt[3]{18}$ D) $3\sqrt{2}$ E) 3

Çözüm / Solution Yanıt / Answer **C**

$$\underbrace{\sqrt{3\sqrt{2\sqrt{3\sqrt{2\ldots}}}}}_{x} = x$$

$$\sqrt{3\sqrt{2\cdot x}} = x$$
$$3\cdot\sqrt{2\cdot x} = x^2$$
$$9\cdot 2\cdot x = x^4$$
$$18 = x^3$$
$$x = \sqrt[3]{18}$$

Özellik / Property

★ $y = \sqrt[n]{a : \sqrt[n]{a : \sqrt[n]{a : \ldots}}} = \sqrt[n+1]{a}$

Örnek / Example 30

$\sqrt[3]{16 : \sqrt[3]{16 : \sqrt[3]{16 : \ldots}}} = ?$

A) 1 B) 2 C) 3 D) 9 E) 27

Çözüm / Solution

Yanıt / Answer **B**

$\sqrt[3]{16 : \sqrt[3]{16 : \sqrt[3]{16 : \ldots}}} = \sqrt[4]{16} = 2$

Örnek / Example 31

$\sqrt{7 + \sqrt[4]{8 \cdot \sqrt[4]{8 \ldots}}} + \sqrt[3]{x : \sqrt[3]{x : \sqrt[3]{x : \ldots}}} = 5 \Rightarrow x = ?$

A) 4 B) 16 C) 25 D) 36 E) 49

Çözüm / Solution

Yanıt / Answer **B**

$\sqrt{7 + \sqrt[4]{8 \cdot \sqrt[4]{8 \ldots}}} + \sqrt[3]{x : \sqrt[3]{x : \sqrt[3]{x : \ldots}}} = 5$

$\sqrt{7 + \sqrt[3]{8}} + \sqrt[4]{x} = 5$

$3 + \sqrt[4]{x} = 5$

$\sqrt[4]{x} = 2$

$x = 16$

PEKİŞTİRME / REINFORCE

✓ $\sqrt[3]{81 : \sqrt[3]{81 : \sqrt[3]{81 : \ldots}}} = ?$

.....[3]

✓ $\sqrt{8 : \sqrt{8 : \sqrt{8 : \ldots}}} = ?$

.....[2]

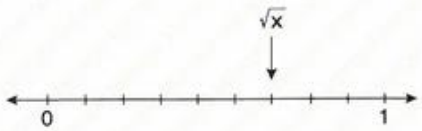
✓ $\sqrt[5]{64 : \sqrt[5]{64 : \ldots}} = ?$

.....[2]

✓ $\sqrt[4]{32 : \sqrt[4]{32 : \ldots}} = ?$

.....[2]

1.



Yukarıdaki sayı doğrusunda 0 ile 1 arasında 9 eşit aralık vardır.

Buna göre, x'in değeri kaçtır?

On the number line above, there are 9 equal intervals between 0 and 1. Accordingly, what is the value of x ?

A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{1}{9}$

D) $\frac{4}{9}$ E) $\frac{4}{25}$

2. $18\sqrt{\frac{8}{27}} - \sqrt{150} = ?$

A) 0

B) $-\sqrt{6}$

C) $\sqrt{6}$

D) $2-\sqrt{3}$

E) $2\sqrt{3}$

3. $\frac{1}{2}\sqrt{32} - \frac{1}{3}\sqrt{18} + \frac{\sqrt{6}}{\sqrt{3}} = ?$

A) 0

B) –1

C) $\sqrt{2}$

D) $2\sqrt{2}$

E) $\sqrt{3}$

4. $\frac{10}{\sqrt{5}} - \frac{8}{\sqrt{20}} - \sqrt{\frac{1}{5}} = ?$

A) $5\sqrt{5}$

B) $\sqrt{5}$

C) $\frac{1}{5}$

D) $\frac{1}{\sqrt{5}}$

E) 5

5. $\frac{2\sqrt{5}}{\sqrt{5}-\sqrt{3}} = ?$

A) 10

B) $5-\sqrt{15}$

C) $3+\sqrt{3}$

D) $3\sqrt{5}$

E) $5+\sqrt{15}$

6. $\sqrt{1,44} - \sqrt{19,6} + \sqrt{\frac{490}{25}} = ?$

A) 0,2

B) 0,9

C) 1

D) 1,2

E) 1,4

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7. $\sqrt{\frac{1}{16}+\frac{1}{9}}\cdot\sqrt{\frac{1}{9}-\frac{1}{25}} = ?$

A) 1

B) $\frac{1}{3}$

C) 16

D) $\frac{1}{9}$

E) $\frac{1}{20}$

8. $\frac{\sqrt{28}+\sqrt{63}+\sqrt{7}}{\sqrt{150}+\sqrt{24}} = ?$

A) $\sqrt{6}$

B) $\sqrt{7}$

C) $\sqrt{\frac{6}{7}}$

D) 6

E) 7

9. $\frac{\sqrt{2}}{\sqrt{2}+\frac{1}{\sqrt{2}+\frac{1}{\sqrt{2}}}} = ?$

A) $\frac{2}{3}$

B) $\frac{3}{4}$

C) $\frac{4}{5}$

D) $\frac{5}{4}$

E) $\frac{4}{3}$

10. $\frac{1}{\sqrt{3}+1}-\frac{3}{\sqrt{3}-1}+\frac{3}{\sqrt{3}} = ?$

A) $-2\sqrt{3}$

B) -2

C) 1

D) 2

E) $2\sqrt{3}$

11. $\sqrt[3]{3^{x+6}}+9\cdot\sqrt[3]{8\cdot 3^x} = 9 \Rightarrow x = ?$

A) -3

B) -1

C) 1

D) 3

E) 4

12. $x+y=0 \Rightarrow \left(x+\sqrt{x^2+1}\right)\cdot\left(y+\sqrt{y^2+1}\right) = ?$

A) 1

B) x^2-1

C) $2x^2-1$

D) -1

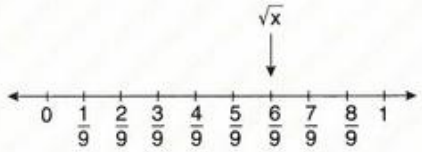
E) $2x^2$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
D	B	D	B	E	D	D	C	B	B	A	A

1.



$$\sqrt{x} = \frac{6}{9} = \frac{2}{3} \Rightarrow x = \frac{4}{9}$$

Yanıt / The Answer is D

2. $18\sqrt{\frac{8}{27}} - \sqrt{150} = 18 \cdot \frac{\sqrt{4 \cdot 2}}{\sqrt{9 \cdot 3}} - \sqrt{25 \cdot 6}$

$$= 18 \cdot \frac{2\sqrt{2}}{3\sqrt{3}} - 5\sqrt{6}$$

$$= 12 \cdot \frac{\sqrt{2}}{\underset{(\sqrt{3})}{\sqrt{3}}} - 5\sqrt{6}$$

$$= 12 \cdot \frac{\sqrt{6}}{3} - 5\sqrt{6}$$

$$= 4\sqrt{6} - 5\sqrt{6} = -\sqrt{6}$$

Yanıt / The Answer is B

3. $\frac{1}{2}\sqrt{32} - \frac{1}{3}\sqrt{18} + \underset{(\sqrt{3})}{\frac{\sqrt{6}}{\sqrt{3}}}$

$$= \frac{1}{2} \cdot \sqrt{16 \cdot 2} - \frac{1}{3} \cdot \sqrt{9 \cdot 2} + \frac{\sqrt{18}}{3}$$

$$= \frac{1}{2} \cdot 4\sqrt{2} - \frac{1}{3} \cdot 3 \cdot \sqrt{2} + \frac{3 \cdot \sqrt{2}}{3}$$

$$= 2\sqrt{2} - \sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

Yanıt / The Answer is D

4. $\underset{(\sqrt{5})}{\frac{10}{\sqrt{5}}} - \underset{(\sqrt{20})}{\frac{8}{\sqrt{20}}} - \underset{(\sqrt{5})}{\sqrt{\frac{1}{5}}}$

$$= \frac{10 \cdot \sqrt{5}}{5} - \frac{8 \cdot \sqrt{20}}{20} - \frac{\sqrt{5}}{5}$$

$$= 2\sqrt{5} - \frac{4\sqrt{5}}{5} - \frac{\sqrt{5}}{5}$$

$$= 2\sqrt{5} - \frac{5\sqrt{5}}{5} = 2\sqrt{5} - \sqrt{5} = \sqrt{5}$$

Yanıt / The Answer is B

5. $\underset{(\sqrt{5}+\sqrt{3})}{\frac{2\sqrt{5}}{\sqrt{5}-\sqrt{3}}} = \frac{2 \cdot \sqrt{5}(\sqrt{5}+\sqrt{3})}{5-3}$

$$= \frac{2 \cdot \sqrt{5}(\sqrt{5}+\sqrt{3})}{2}$$

$$= \sqrt{5}(\sqrt{5}+\sqrt{3})$$

$$= 5 + \sqrt{15}$$

Yanıt / The Answer is E

6. $\sqrt{\frac{144}{100}} - \sqrt{\frac{196}{10}} + \sqrt{\frac{490}{25}} = \frac{12}{10} - \frac{14}{\sqrt{10}} + \frac{7\sqrt{10}}{5}$

$$= \frac{12}{10} - \frac{14 \cdot \sqrt{10}}{10} + 7 \cdot \frac{\sqrt{10}}{5}$$

$$= \frac{12}{10} - \frac{7\sqrt{10}}{5} + \frac{7\sqrt{10}}{5}$$

$$= \frac{12}{10} = 1{,}2$$

Yanıt / The Answer is D

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7. $\sqrt{\frac{1}{16}+\frac{1}{9}}\cdot\sqrt{\frac{1}{9}-\frac{1}{25}}=\sqrt{\frac{25}{144}}\cdot\sqrt{\frac{16}{225}}$

$=\frac{5}{12}\cdot\frac{4}{15}$

$=\frac{20}{180}=\frac{1}{9}$

Yanıt / The Answer is D

8. $\frac{\sqrt{28}+\sqrt{63}+\sqrt{7}}{\sqrt{150}+\sqrt{24}}=\frac{\sqrt{4\cdot7}+\sqrt{9\cdot7}+\sqrt{7}}{\sqrt{25\cdot6}+\sqrt{4\cdot6}}$

$=\frac{2\cdot\sqrt{7}+3\cdot\sqrt{7}+\sqrt{7}}{5\cdot\sqrt{6}+2\cdot\sqrt{6}}$

$=\frac{(2+3+1)\cdot\sqrt{7}}{(5+2)\cdot\sqrt{6}}$

$=\frac{6\cdot\sqrt{7}}{7\cdot\sqrt{6}}$

$=\sqrt{\frac{6}{7}}$

Yanıt / The Answer is C

9. $\frac{\sqrt{2}}{\sqrt{2}+\frac{1}{\sqrt{2}+\frac{1}{\sqrt{2}}}}=\frac{\sqrt{2}}{\sqrt{2}+\frac{1}{\frac{3}{\sqrt{2}}}}=\frac{\sqrt{2}}{\sqrt{2}+\frac{\sqrt{2}}{3}}$

$=\frac{\sqrt{2}}{\frac{4\sqrt{2}}{3}}=\sqrt{2}\cdot\frac{3}{4\sqrt{2}}=\frac{3}{4}$

Yanıt / The Answer is B

10. $\underset{(\sqrt{3}-1)}{\frac{1}{\sqrt{3}+1}}-\underset{(\sqrt{3}+1)}{\frac{3}{\sqrt{3}-1}}+\underset{(\sqrt{3})}{\frac{3}{\sqrt{3}}}$

$=\frac{\sqrt{3}-1}{3-1}-\frac{3\cdot(\sqrt{3}+1)}{3-1}+\frac{3\sqrt{3}}{3}$

$=\frac{\sqrt{3}-1}{2}-\frac{3\cdot(\sqrt{3}+1)}{2}+\sqrt{3}$

$=\frac{\sqrt{3}-1-3\sqrt{3}-3}{2}+\sqrt{3}$

$=\frac{-2\sqrt{3}-4}{2}+\sqrt{3}$

$=\frac{2(-\sqrt{3}-2)}{2}+\sqrt{3}$

$=-\sqrt{3}-2+\sqrt{3}$

$=-2$

Yanıt / The Answer is B

11. $\sqrt[3]{3^x\cdot3^6}+9\cdot\sqrt[3]{8\cdot3^x}=9$

$3^2\cdot\sqrt[3]{3^x}+2\cdot9\cdot\sqrt[3]{3^x}=9$

$27\cdot\sqrt[3]{3^x}=9$

$3^{\frac{x}{3}}=3^{-1}$

$\frac{x}{3}=-1 \Rightarrow x=-3$

Yanıt / The Answer is A

12. $x+y=0 \Rightarrow y=-x$

$(x+\sqrt{x^2+1})\cdot(-x+\sqrt{x^2+1})=(\sqrt{x^2+1})^2-x^2$

$=x^2+1-x^2=1$

Yanıt / The Answer is A

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1. $a, b, c \in R$

$$\left.\begin{matrix} a^b = b^a \\ 6a = b \end{matrix}\right\} \Rightarrow a = ?$$

A) $5\sqrt[5]{6}$ B) $\sqrt[5]{6}$ C) $6\sqrt{6}$ D) $6\sqrt[5]{6}$ E) $\sqrt{6}$

[GAZİANTEP ÜNİVERSİTESİ – YÖS 2020]

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2. $\sqrt{0,006} \cdot \sqrt{0,08} \cdot \sqrt{0,3} = ?$

A) 0,9 B) 0,024 C) 0,08 D) 0,012 E) 0,36

[KARABÜK ÜNİVERSİTESİ – YÖS 2020]

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3. $\sqrt{11 + \sqrt{21}} - \sqrt{11 - \sqrt{21}} = ?$

A) $-\sqrt{21}$ B) $-\sqrt{2}$ C) 0 D) $\sqrt{2}$ E) $\sqrt{21}$

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2020]

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4. $x < 0$

$\sqrt[3]{x^3} - \sqrt[4]{x^4} + \sqrt{(-x)^2} + \sqrt[5]{x^5} = ?$

A) 2x B) –2x C) 0 D) x E) –3x

[KARADENİZ TEKNİK ÜNİVERSİTESİ – YÖS 2019]

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5. $\sqrt[5]{2 \cdot \sqrt[3]{x}} = \sqrt[5]{2} \cdot \sqrt[15]{15}$

$\Rightarrow x = ?$

A) 3 B) 9 C) 15 D) 27 E) 45

[MANİSA CELAL BAYAR ÜNİVERSİTESİ – YÖS 2019]

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6. $\dfrac{\sqrt{4,4} + \sqrt{1,1} + \sqrt{9,9}}{\sqrt{1,1}} = ?$

A) 14 B) 13 C) 9 D) 8 E) 6

[İNÖNÜ ÜNİVERSİTESİ – YÖS 2018]

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7. $\dfrac{\sqrt{2 \cdot \sqrt[3]{4}}}{\sqrt[3]{2} \cdot \sqrt{8}} = ?$

A) 2 B) $\sqrt{2}$ C) 1 D) $\dfrac{1}{\sqrt{2}}$ E) $\dfrac{1}{2}$

[İSTANBUL ÜNİVERSİTESİ – YÖS 2018]

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8. $\dfrac{a}{b}\sqrt{\dfrac{b}{a}\sqrt{\left(\dfrac{a}{b}\right)^3}} = ?$

A) $\sqrt[4]{\left(\dfrac{a}{b}\right)^5}$ B) $\sqrt[3]{\left(\dfrac{a}{b}\right)^4}$ C) $\sqrt{\left(\dfrac{a}{b}\right)^3}$ D) $\sqrt{\dfrac{a}{b}}$ E) $\dfrac{a}{b}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2017]

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $3\sqrt{32}-\sqrt{27}+\sqrt{3}-\sqrt{2}=?$

A) $2\sqrt{2}-11\sqrt{3}$ B) $2\sqrt{2}+\sqrt{3}$ C) $11\sqrt{2}-2\sqrt{3}$

D) $11\sqrt{2}-\sqrt{3}$ E) $11\sqrt{2}+\sqrt{3}$

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2017]

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10. $x\sqrt{0,08}=2 \Rightarrow x=?$

A) $\sqrt{2}$ B) $3\sqrt{2}$ C) $5\sqrt{2}$ D) $7\sqrt{2}$ E) $8\sqrt{2}$

[KARADENİZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

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11. $\dfrac{2^5-1}{\sqrt[3]{2^5}}=?$

A) $\dfrac{31-\sqrt[3]{8}}{8}$ B) $\dfrac{2^5-\sqrt[3]{2}}{4}$ C) $\dfrac{31\sqrt[3]{2}}{4}$

D) $\dfrac{2^{16/3}-\sqrt[5]{4}}{8}$ E) $\dfrac{2^{15}-1}{2^5}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2015]

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12. $x \in R^+$

$\Rightarrow \dfrac{\sqrt[3]{x}\cdot\sqrt[3]{x}\cdot\sqrt[3]{x}\cdot\sqrt[3]{x}}{\sqrt[6]{x}\cdot\sqrt[6]{x}\cdot\sqrt[6]{x}}=?$

A) $\sqrt[12]{x^7}$ B) $\sqrt[4]{x^9}$ C) $\sqrt{x}$

D) $\sqrt[9]{x}$ E) $\sqrt[6]{x^5}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – 2014]

13. $\sqrt{7+\sqrt{24}}+\sqrt{7-\sqrt{24}}=?$

A) $\sqrt{6}$ B) $2\sqrt{6}$ C) $\sqrt{14}$

D) $4\sqrt{6}$ E) $6\sqrt{6}$

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2014]

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14. $a=3+\sqrt{3}$

$b=\dfrac{\sqrt{3a}\cdot\sqrt[4]{a^2}}{\sqrt{3}+\sqrt[3]{27}} \Rightarrow b=?$

A) $3\sqrt{a}$ B) $a\sqrt{3}$ C) $3\sqrt{3}$

D) $2\sqrt{3}$ E) $\sqrt{3}$

[AKDENİZ ÜNİVERSİTESİ – YÖS 2013]

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15. $x \in R^+$

$\Rightarrow \dfrac{\sqrt[4]{x^3}\cdot\sqrt[3]{x^4}}{\sqrt[12]{x^{24}}}=?$

A) $\sqrt[7]{x^{12}}$ B) $\sqrt[4]{x^{12}}$ C) $\sqrt{x^5}$

D) $\sqrt[12]{x}$ E) $\sqrt[12]{x^7}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2012]

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16. $\sqrt{\dfrac{0,04}{0,01}}-\sqrt{\dfrac{6,25}{25}}=?$

A) $\dfrac{1}{2}$ B) 1 C) $\dfrac{3}{2}$

D) 2 E) $\dfrac{5}{2}$

[EGE ÜNİVERSİTESİ – YÖS 2012]

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	D	D	A	C	E	E	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	C	C	E	B	E	D	C

1. $a^{6a} = (6a)^a$

$(a^6)^a = (6a)^a$

$a^6 = 6a$

$a^5 = 6 \Rightarrow a = \sqrt[5]{6}$

Yanıt / The Answer is B

---

2. $\sqrt{0,006}\cdot\sqrt{0,08}\cdot\sqrt{0,3} = \frac{\sqrt{6}}{10\sqrt{10}}\cdot\frac{\sqrt{8}}{10}\cdot\frac{\sqrt{3}}{\sqrt{10}}$

$= \frac{12}{1000} = 0,012$

Yanıt / The Answer is D

---

3. $\sqrt{11+\sqrt{21}} - \sqrt{11-\sqrt{21}} = x$

$11+\sqrt{21} - 2\cdot\sqrt{11+\sqrt{21}}\cdot\sqrt{11-\sqrt{21}} + 11-\sqrt{21} = x^2$

$22 - 20 = x^2 \Rightarrow x = \sqrt{2}$

Yanıt / The Answer is D

---

4. $\sqrt[3]{x^3} - \sqrt[4]{x^4} + \sqrt{(-x)^2} + \sqrt[5]{x^5}$

$= x - |x| + |-x| + x$

$= x - (-x) + (-x) + x \ ,[x < 0]$

$= 2x$

Yanıt / The Answer is A

---

5. $\sqrt[5]{2\cdot\sqrt[3]{x}} = \sqrt[5]{2}\cdot\sqrt[15]{15}$

$\sqrt[15]{2^3\cdot x} = \sqrt[15]{2^3\cdot 15}$

$2^3\cdot x = 2^3\cdot 15$

$x = 15$

Yanıt / The Answer is C

---

6. $\frac{\sqrt{44}+\sqrt{11}+\sqrt{99}}{\sqrt{11}} = \frac{\sqrt{11}\cdot(\sqrt{4}+1+\sqrt{9})}{\sqrt{11}}$

$= 2+1+3$

$= 6$

Yanıt / The Answer is E

---

7. $\frac{\sqrt{2\sqrt[3]{4}}}{\sqrt[3]{2}\cdot\sqrt{8}} = \frac{\sqrt[6]{8\cdot 4}}{\sqrt[6]{4\cdot 8^3}} = \sqrt[6]{\frac{8\cdot 4}{4\cdot 8^3}} = \sqrt[6]{\frac{1}{64}} = \frac{1}{2}$

Yanıt / The Answer is E

---

8. $\frac{a}{b}\sqrt{\frac{b}{a}\sqrt{\left(\frac{a}{b}\right)^3}} = \sqrt[4]{\left(\frac{a}{b}\right)^4\cdot\left(\frac{b}{a}\right)^2\cdot\left(\frac{a}{b}\right)^3}$

$= \sqrt[4]{\frac{a^4\cdot b^2\cdot a^3}{b^4\cdot a^2\cdot b^3}}$

$= \sqrt[4]{\left(\frac{a}{b}\right)^5}$

Yanıt / The Answer is A

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9. $3\sqrt{16\cdot 2}-\sqrt{9\cdot 3}+\sqrt{3}-\sqrt{2}=12\sqrt{2}-3\sqrt{3}+\sqrt{3}-\sqrt{2}$

$=11\sqrt{2}-2\sqrt{3}$

Yanıt / The Answer is C

---

10. $x\cdot\sqrt{\frac{8}{100}}=2$

$x\cdot\frac{2\sqrt{2}}{10}=2$

$x=\frac{10}{\sqrt{2}}=\frac{10\sqrt{2}}{2}=5\sqrt{2}$

Yanıt / The Answer is C

---

11. $\frac{2^5-1}{\sqrt[3]{2^5}}=\frac{(32-1)\sqrt[3]{2}}{\sqrt[3]{(2^2)^3}}$

$=\frac{31\cdot\sqrt[3]{2}}{4}$

Yanıt / The Answer is C

---

12. $\frac{\sqrt[3]{x}\cdot\sqrt[3]{x}\cdot\sqrt[3]{x}\cdot\sqrt[3]{x}}{\sqrt[6]{x}\cdot\sqrt[6]{x}\cdot\sqrt[6]{x}}=\frac{\left(x^{\frac{1}{3}}\right)^4}{\left(x^{\frac{1}{6}}\right)^3}$

$=x^{\frac{4}{3}-\frac{1}{2}}$

$=x^{\frac{5}{6}}$

$=\sqrt[6]{x^5}$

Yanıt / The Answer is E

---

13. $\sqrt{7+\sqrt{24}}+\sqrt{7-\sqrt{24}}=\sqrt{7+\sqrt{4\cdot 6}}+\sqrt{7-\sqrt{4\cdot 6}}$

$=\sqrt{7+2\sqrt{6}}+\sqrt{7-2\sqrt{6}}$

$=\sqrt{6}+1+\sqrt{6}-1$

$=2\sqrt{6}$

Yanıt / The Answer is B

---

14. $a=3+\sqrt{3}$

$\Rightarrow b=\frac{\sqrt{3a}\cdot\sqrt[4]{a^2}}{\sqrt{3}+\sqrt[3]{27}}=\frac{\sqrt{3}\cdot\sqrt{a}\cdot\sqrt{a}}{\sqrt{3}+3}=\frac{\sqrt{3}\cdot a}{a}=\sqrt{3}$

Yanıt / The Answer is E

---

15. $\frac{\sqrt[4]{x^3}\cdot\sqrt[3]{x^4}}{\sqrt[12]{x^{24}}}=a$

$\left(\frac{\sqrt[4]{x^3}\cdot\sqrt[3]{x^4}}{\sqrt[12]{x^{24}}}\right)^{12}=a^{12}$

$\frac{(x^3)^3\cdot(x^4)^4}{x^{24}}=a^{12}$

$\frac{x^{25}}{x^{24}}=a^{12}\Rightarrow a=x^{\frac{1}{12}}=\sqrt[12]{x}$

Yanıt / The Answer is D

---

16. $\sqrt{\frac{0,04}{0,01}}-\sqrt{\frac{6,25}{25}}=\sqrt{\frac{(0,2)^2}{(0,1)^2}}-\sqrt{\frac{(2,5)^2}{5^2}}$

$=\frac{2}{1}-\frac{2,5}{5}$

$=2-\frac{1}{2}$

$=\frac{3}{2}$

Yanıt / The Answer is C

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1. $\sqrt{9}+\sqrt{4}-\sqrt{(-4)^2}-\sqrt{(-2)^2}=?$

A) -1 B) 11 C) -10 D) -11 E) -5

2. $\sqrt{0,16}+\sqrt{0,64}=?$

A) $\frac{3}{2}$ B) $\frac{6}{5}$ C) $\frac{4}{5}$ D) $\frac{9}{10}$ E) $\frac{12}{7}$

3. $\sqrt{\frac{3^{-1}}{0,3}:\frac{0,09}{10}}=?$

A) $\frac{10}{3}$ B) $\frac{10}{9}$ C) $\left(\frac{9}{10}\right)^{-2}$ D) $\left(\frac{3}{10}\right)^{-2}$ E) $\frac{3}{5}$

4. $3\sqrt{8}+5\sqrt{8}-4\sqrt{8}=?$

A) $\sqrt{2}$ B) $3\sqrt{2}$ C) $4\sqrt{2}$ D) $4\sqrt{3}$ E) $8\sqrt{2}$

5. $\sqrt{27}-\sqrt{48}=?$

A) 1 B) $-\sqrt{2}$ C) $\sqrt{2}$ D) $-\sqrt{3}$ E) $\sqrt{3}$

6. $3\sqrt{2}+4\sqrt{8}-5\sqrt{50}+8\sqrt{32}=?$

A) $10\sqrt{2}$ B) $12\sqrt{2}$ C) $4\sqrt{3}$ D) $16\sqrt{2}$ E) $18\sqrt{2}$

7. $\sqrt{1+\frac{9}{16}}+\sqrt{1-\frac{9}{25}}=?$

A) $\frac{32}{13}$ B) $\frac{41}{19}$ C) $\frac{41}{20}$ D) $\frac{33}{13}$ E) $\frac{42}{19}$

8. $\frac{\sqrt{32}-\sqrt{45}+\sqrt{2}-\sqrt{20}}{\sqrt{5}-\sqrt{2}}=?$

A) -5 B) $\sqrt{5}+\sqrt{2}$ C) 4 D) $\sqrt{2}$ E) $3\sqrt{5}$

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9. $\sqrt{48}-\sqrt{12}-\sqrt{\frac{4}{3}}-\sqrt{\frac{1}{3}}=?$

A) 0 B) 1 C) $\sqrt{3}$

D) $\frac{1}{3}\sqrt{3}$ E) $\frac{2}{3}\sqrt{3}$

10. $10\sqrt{1,21}-35\cdot\sqrt[3]{0,008}-10\cdot\sqrt[4]{\frac{0,0016}{(0,25)^2}}=?$

A) 0 B) $\frac{1}{2}$ C) $\frac{2}{3}$

D) 5 E) 11

11. $\frac{\sqrt{1,44}}{0,6}+\frac{\sqrt{1,44}}{0,2}-\frac{\sqrt{0,64}}{0,4}=?$

A) 2 B) 4 C) 6

D) 8 E) 10

12. $\sqrt{8,1}+\sqrt{4,9}-\sqrt{12,1}=?$

A) $\sqrt{10}$ B) $2\sqrt{10}$ C) $\frac{\sqrt{10}}{2}$

D) $\frac{\sqrt{10}}{5}$ E) 5

13. $x, y \in Z^+$

$\sqrt{4-2x}+(y^2-9)^2=0 \Rightarrow x+y=?$

A) 2 B) 3 C) 4

D) 5 E) 6

14. $x=\sqrt[3]{\frac{3}{\sqrt{3}}} \Rightarrow x^{-6}=?$

A) $\frac{1}{9}$ B) $\frac{1}{3}$ C) 1

D) 3 E) 9

15. $\left.\begin{matrix}A=23,81\\B=13,81\end{matrix}\right\} \Rightarrow \sqrt{(A+B)^2-4AB}=?$

A) 9 B) 10 C) 11

D) 12 E) 13

16. $\sqrt{100\cdot101-102\cdot99}=?$

A) 1 B) $\sqrt{2}$ C) 2

D) $\sqrt{3}$ E) 3

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	B	D	E	D	E	C	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	A	C	C	D	B	B	B

1. $\frac{\sqrt{169}+\sqrt{144}}{\sqrt{25}-\sqrt{16}} = ?$

A) 1 B) 3 C) 5 D) 25 E) 27

2. $\frac{\sqrt{2}}{\sqrt{5}}+\frac{\sqrt{5}}{\sqrt{2}} = ?$

A) $\frac{3}{\sqrt{10}}$ B) $\frac{4}{\sqrt{10}}$ C) $\frac{5}{\sqrt{10}}$ D) $\frac{7}{\sqrt{10}}$ E) $\frac{9}{\sqrt{10}}$

3. $\sqrt{0,04}-2\sqrt[3]{0,008}-\sqrt[4]{0,0016}+\sqrt{1,69} = ?$

A) 0,5 B) 0,6 C) 0,7 D) 0,8 E) 0,9

4. $\sqrt{\sqrt{0,0036}+\sqrt{0,09}}\cdot\frac{10}{\sqrt{2}} = ?$

A) 3 B) $5\sqrt{2}$ C) $\sqrt{3}$ D) $3\sqrt{2}$ E) 5

5. $2-[3:(\sqrt{3:2})^2-2:(\sqrt{2:3})^2]-3 = ?$

A) -3 B) -2 C) 0 D) $\frac{2}{3}$ E) $\frac{3}{2}$

6. $\sqrt{1,69}+\sqrt{1,44}-\sqrt{0,36} = ?$

A) 1,2 B) 1,3 C) 1,5 D) 1,9 E) 2,1

7. $3\sqrt{0,64}+8\sqrt{0,49} = ?$

A) 4 B) 5 C) 6 D) 7 E) 8

8. $\frac{7}{\sqrt{7}-\frac{3}{\sqrt{7}-\frac{3}{\sqrt{7}}}} = ?$

A) 1 B) $\frac{1}{4}$ C) $4\sqrt{7}$ D) 4 E) $2\sqrt{7}$

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9. $4\sqrt{45} - 2\sqrt{80} - \sqrt[4]{25} = ?$

A) $3\sqrt{5}$ B) $2\sqrt{5}$ C) $\sqrt{5}$

D) 0 E) $9\sqrt{5}$

10. $\sqrt{45} - 10\sqrt{\frac{1}{5}} + \sqrt{80} - \sqrt[4]{25} = ?$

A) $3\sqrt{5}$ B) $7\sqrt{5}$ C) $4\sqrt{5}$

D) $5\sqrt{5}$ E) $3\sqrt{5}$

11. $3 \cdot \sqrt{\frac{4}{3}} - 2 \cdot \sqrt{\frac{25}{3}} + \sqrt{\frac{49}{3}} = ?$

A) $\sqrt{3}$ B) $2\sqrt{3}$ C) $3\sqrt{3}$

D) $\frac{\sqrt{3}}{3}$ E) $5\sqrt{3}$

12. $\frac{\sqrt{5,76} + \sqrt{2,89} + \sqrt{1,96}}{\sqrt{0,49} + \sqrt{0,16}} = ?$

A) $\frac{1}{2}$ B) 2 C) 3

D) 4 E) 5

13. $\frac{a-b}{\sqrt{a}+\sqrt{b}} = x \Rightarrow x + \sqrt{b} = ?$

A) 0 B) $\sqrt{ab}$ C) $\sqrt{a}$

D) $\sqrt{b}$ E) 36

14. $\left.\begin{array}{l} 9a - 4b = 17 \\ 3\sqrt{a} + 2\sqrt{b} = 17 \end{array}\right\} \Rightarrow b = ?$

A) 4 B) 9 C) 16

D) 25 E) 36

15. $(\sqrt{3}+\sqrt{2})^3 \cdot (\sqrt{15}-\sqrt{10})^3 = ?$

A) 1 B) $\sqrt{5}$ C) $2\sqrt{5}$

D) 5 E) $5\sqrt{5}$

16. $x, y \in R^+$,

$\left.\begin{array}{l} x^2 \cdot \sqrt{y} = a^3 \\ y^2 \cdot \sqrt{x} = a^2 \end{array}\right\} \Rightarrow \sqrt{x \cdot y} = ?$

A) $\sqrt{a}$ B) a C) $2\sqrt{a}$

D) $a\sqrt{a}$ E) a^2

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	D	E	D	C	D	E	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	C	A	E	C	C	E	B

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1. $5^{x+1} = \sqrt{25^{3x}} \Rightarrow x = ?$

A) 0 B) 1 C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) 4

2. $d^2 = \sqrt{2^{n+2}} \Rightarrow \sqrt[3]{d^6} = ?$

A) 2^n B) $2^{\frac{1}{2}+n}$ C) $2^{\frac{n+2}{2}}$ D) 2^{n-2} E) 2^{3n+6}

3. $\sqrt{\frac{3^{x+2}}{9^{x-1}}} = 27 \Rightarrow x = ?$

A) –5 B) –2 C) 2 D) 3 E) 8

4. $\frac{(\sqrt{5}-2)\cdot(\sqrt{9+2\sqrt{20}})}{\sqrt{2}} = ?$

A) $\frac{-\sqrt{2}}{3}$ B) 1 C) $\frac{\sqrt{2}}{2}$ D) $\frac{-3}{2}$ E) $\frac{\sqrt{3}}{5}$

5. $\sqrt{3-2\sqrt{2}} + \sqrt{6+2\sqrt{5}} = ?$

A) $\sqrt{5}-\sqrt{2}$ B) $\sqrt{5}-1$ C) $\sqrt{5}+1$ D) $\sqrt{5}+\sqrt{2}$ E) $\sqrt{2}-\sqrt{5}$

6. $\sqrt{5+2\sqrt{6}} + \sqrt{8-2\sqrt{15}} - \sqrt{9-4\sqrt{5}} = ?$

A) $3+\sqrt{3}$ B) $5\sqrt{2}$ C) $2+\sqrt{2}$ D) $4+\sqrt{5}$ E) $\sqrt{3}$

7. $\sqrt{6-\sqrt{11}} + \sqrt{6+\sqrt{11}} = ?$

A) $\sqrt{6}$ B) $\sqrt{11}$ C) $\sqrt{12}$ D) $\sqrt{22}$ E) $\sqrt{33}$

8. $\frac{5\sqrt{\frac{1}{2}} - \sqrt{0,5} + \sqrt{200}}{\sqrt{8}} = ?$

A) 12 B) $6\sqrt{2}$ C) 6 D) 4 E) $2\sqrt{2}$

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9. $\frac{5}{5-\sqrt{5}} \cdot (5+\sqrt{5})^{-1} = ?$

A) $\frac{1}{2}$ B) $\frac{5}{2}$ C) $\frac{1}{4}$ D) $\frac{7}{4}$ E) $\frac{7}{2}$

10. $x = \sqrt[3]{\frac{2}{\sqrt[3]{2}}} \Rightarrow x^{18} = ?$

A) 8 B) 16 C) 21 D) 32 E) 64

11. $\sqrt{\frac{4}{25} - \frac{3}{5} + \frac{9}{16}} = ?$

A) -1 B) 2 C) $\frac{1}{3}$ D) $\frac{7}{20}$ E) $\frac{1}{4}$

12. $\sqrt{2010 \cdot 1998 + 36} = ?$

A) 1997 B) 1999 C) 2000 D) 2002 E) 2004

13. $\sqrt{12+\sqrt{12+\sqrt{12+\ldots}}} = \sqrt{x : \sqrt{x : \sqrt{x : \ldots}}}$
$\Rightarrow x = ?$

A) 2 B) 4 C) 16 D) 64 E) 128

14. $\sqrt{5-2\sqrt{6}} - \sqrt{7+2\sqrt{12}} = ?$

A) $-\sqrt{2}-3$ B) $-\sqrt{2}-2$ C) $2-\sqrt{2}$ D) $\sqrt{3}$ E) $2-\sqrt{3}$

15. $\sqrt{13+\sqrt{12-\sqrt[3]{27}}} = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

16. $\frac{11-\sqrt{11}}{\sqrt{33}-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{11}} = ?$

A) -1 B) 1 C) 2 D) 3 E) 4

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	C	B	C	D	C	D	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	B	D	E	D	B	C	B

1. $(4\sqrt{5}-2\sqrt{3})^2-(4\sqrt{5}+2\sqrt{3})^2=?$

A) $32\sqrt{15}$ B) $16\sqrt{15}$ C) $-20\sqrt{5}$ D) $-24\sqrt{15}$ E) $-32\sqrt{15}$

2. $\sqrt{56-\sqrt{56-\sqrt{56-\ldots}}}=?$

A) 2 B) 5 C) 7 D) 9 E) 11

3. $\sqrt{6+\sqrt{6+\sqrt{6\ldots}}}=?$

A) -2 B) -3 C) 0 D) 2 E) 3

4. $\sqrt{3\sqrt{3\sqrt{3\ldots}}}=a \Rightarrow a^2=?$

A) 3 B) 6 C) 9 D) 12 E) 36

5. $\sqrt{a:\sqrt{a:\sqrt{a:\ldots}}}=4 \Rightarrow a=?$

A) 2 B) 8 C) 16 D) 64 E) 128

6. $\dfrac{1}{\sqrt{6-2\sqrt{8}}}-\dfrac{1}{\sqrt{6+2\sqrt{8}}}=?$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) $2\sqrt{3}$ D) $2\sqrt{2}$ E) $3\sqrt{2}$

7. $\sqrt{2\sqrt[3]{2\sqrt{2}}}=?$

A) $\sqrt[3]{2^2}$ B) $\sqrt[6]{2^5}$ C) $\sqrt[12]{2^5}$ D) $\sqrt[4]{2^3}$ E) $\sqrt[3]{2^4}$

8. $\sqrt{2\sqrt{2\sqrt{2}}}=?$

A) $\sqrt{2}$ B) $\sqrt[4]{2^3}$ C) $\sqrt[8]{2^7}$ D) $\sqrt[8]{2^5}$ E) $\sqrt[8]{2^3}$

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9. $\frac{\sqrt{252}}{\sqrt{7}} + \frac{\sqrt{27}}{\sqrt{\frac{1}{3}}} = ?$

A) 1 B) 3 C) 6
D) 9 E) 15

10. $(3\sqrt{7} - 2\sqrt{7})^2 + (3\sqrt{7} + 2\sqrt{7})^2 = ?$

A) 163 B) 175 C) 182
D) 190 E) 224

11. $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}} - \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}+\sqrt{5}} = ?$

A) $-\sqrt{15}$ B) $-2\sqrt{15}$ C) $-3\sqrt{15}$
D) $-4\sqrt{15}$ E) $\sqrt{15}$

12. $\frac{2\sqrt{2}+\sqrt{3}}{2} - \frac{3\sqrt{2}-\sqrt{3}}{3} = ?$

A) $\frac{\sqrt{3}}{6}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{3}}{3}$
D) $\frac{5\sqrt{3}}{6}$ E) $\sqrt{3}$

13. $\sqrt{2+\sqrt{16+2\sqrt{48}}} - 1 = ?$

A) 1 B) $\sqrt{2}$ C) $\sqrt{3}$
D) 2 E) 3

14. $\sqrt{x-2} + x = 2 \Rightarrow x = ?$

A) 1 B) 2 C) 3
D) 4 E) 5

15. $\sqrt{\frac{9}{16} + \frac{4}{25} - \frac{3}{5}} = ?$

A) $\frac{3}{30}$ B) $\frac{1}{5}$ C) $\frac{1}{4}$
D) $\frac{3}{10}$ E) $\frac{7}{20}$

16. $\frac{\sqrt{0,16}+\sqrt{0,25}}{\sqrt{0,81}-\sqrt{0,09}} \cdot \left(\frac{1}{2}\right)^{-1} = ?$

A) 1 B) 2 C) 3
D) 4 E) 5

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1	2	3	4	5	6	7	8
E	C	E	C	D	A	D	C
9	10	11	12	13	14	15	16
E	C	B	D	C	B	E	C

1. $(3\sqrt{2}-2)^2-(3\sqrt{2}+2)^2=?$

A) $-24\sqrt{2}$ B) $-12\sqrt{2}$ C) $-6\sqrt{2}$

D) $-3\sqrt{7}$ E) $\sqrt{2}+2$

2. $\sqrt{72-\sqrt{72-\sqrt{72...}}}=?$

A) 6 B) 8 C) 10

D) 12 E) 15

3. $\sqrt{x+\sqrt{x^2}}\cdot\sqrt[3]{x+\sqrt{x^2}}=16^{\frac{5}{12}} \Rightarrow x=?$

A) 0 B) 1 C) 2

D) 3 E) 4

4. $\left.\begin{array}{l}\sqrt[4]{27\sqrt[4]{27\sqrt[4]{27....}}}=x\\ \sqrt{5\sqrt{5\sqrt{5...}}}=y\end{array}\right\} \Rightarrow y^2-x^2=?$

A) 8 B) 12 C) 16

D) 21 E) 27

5. $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+...}}}}=\sqrt{x\cdot\sqrt{x\cdot\sqrt{x\cdot...}}} \Rightarrow x=?$

A) 8 B) 12 C) 16

D) 27 E) 64

6. $\left.\begin{array}{l}a=\sqrt[4]{4}-\sqrt{2}\\ b=\sqrt[4]{4}+\sqrt{2}\end{array}\right\} \Rightarrow \frac{a+b}{a-b}=?$

A) 0 B) –1 C) 1

D) 2 E) $\sqrt{2}$

7. $\sqrt{3}+\frac{2}{\sqrt{3}-2}-\frac{1}{\sqrt{3}+2}=?$

A) –8 B) –6 C) –3

D) 0 E) 1

8. $\sqrt{(1+\sqrt{5})^2}\cdot\sqrt{6-2\sqrt{5}}=?$

A) $-\sqrt{5}$ B) –1 C) 1

D) 4 E) 9

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9. $x \in R^+$

$\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 3x + 3y \Rightarrow x \cdot y = ?$

A) $\frac{1}{9}$ B) $\frac{1}{3}$ C) 1

D) 3 E) 9

10. $\frac{1}{\sqrt{7-4\sqrt{3}}} + \frac{1}{2+\sqrt{3}} = ?$

A) 2 B) $2\sqrt{3}$ C) 1

D) 4 E) $\sqrt{3}$

11. $\left.\begin{array}{l} x = 3\sqrt{3} - \sqrt{12} + \sqrt{48} \\ y = 3\sqrt{3} + \sqrt{108} \end{array}\right\} \Rightarrow x \cdot y = ?$

A) 120 B) 135 C) 150

D) 165 E) 185

12. $x = \frac{1}{\sqrt{3}+\sqrt{2}} \Rightarrow 5 - 2\sqrt{6} = ?$

A) 10x B) $10x^2$ C) x^2

D) $2x^2$ E) $3x^2$

13. $\sqrt{(2-\sqrt{5})^2} - \sqrt{7-2\sqrt{10}} = ?$

A) –2 B) $-\sqrt{2}$ C) $\sqrt{2}-2$

D) $\sqrt{2}$ E) 2

14. $\sqrt{4+\sqrt{15}} + \sqrt{4-\sqrt{15}} = ?$

A) 3 B) 4 C) $\sqrt{5}$

D) $\sqrt{6}$ E) $\sqrt{10}$

15. $\frac{1}{\sqrt{2^x}} = 2 \Rightarrow \sqrt{4^x + 3} = ?$

A) $\sqrt{5}$ B) $\sqrt{7}$ C) $\frac{7}{4}$

D) $\frac{3}{2}$ E) $\frac{10}{3}$

16. $2 \cdot \sqrt{\sqrt{8} - \sqrt{2} + \frac{3}{2}} = ?$

A) $2-\sqrt{2}$ B) $2 + \sqrt{2}$ C) $2\sqrt{2} - 2$

D) $\sqrt{2}$ E) $2\sqrt{2}$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	B	C	C	D	B	B	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	D	B	C	C	E	C	B

1. $\sqrt{2x-1}-x=0 \Rightarrow x=?$

A) 1 B) 2 C) 3 D) 4 E) 5

2. $\sqrt[3]{8x-8}+\sqrt[3]{27x-27}-\sqrt[3]{125-125x}=30 \Rightarrow x=?$

A) 25 B) 26 C) 27 D) 28 E) 29

3. $\sqrt{30-5^x}-5^x=0 \Rightarrow x=?$

A) 1 B) 2 C) 3 D) 4 E) 5

4. $\left.\begin{matrix}A=\sqrt{3}-2\\B=\sqrt{3}+2\end{matrix}\right\} \Rightarrow \sqrt{A^2-2AB+B^2}=?$

A) –4 B) –2 C) 2 D) 4 E) 8

5. $\sqrt{2001\cdot1999+1}=?$

A) 1999 B) 2000 C) 2001 D) 2002 E) 2003

6. $\sqrt[3]{81x+27}-4=\sqrt[3]{24x+8} \Rightarrow x=?$

A) 19 B) 20 C) 21 D) 22 E) 23

7. $\sqrt{\frac{1}{81}+\frac{1}{36}+\frac{1}{27}}=?$

A) $\frac{1}{18}$ B) $\frac{1}{9}$ C) $\frac{1}{6}$ D) $\frac{2}{9}$ E) $\frac{5}{18}$

8. $\left.\begin{matrix}x=\sqrt{3}\\y=\sqrt{5}\\z=\sqrt{7}\end{matrix}\right\} \Rightarrow \sqrt{315}=?$

A) xyz B) x^2yz C) x^2y^2z D) xy^2z E) xyz^2

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9. $\left.\begin{array}{l} a=\sqrt{2} \\ b=\sqrt{3} \\ c=\sqrt{5} \end{array}\right\} \Rightarrow \sqrt{1500} = ?$

A) a^2b^2c B) a^3b^2c C) a^2bc^3

D) $a^2b^2c^2$ E) $a^2b^3c^2$

10. $\left.\begin{array}{l} \sqrt{20+\sqrt{20+\sqrt{20+\ldots}}} = x \\ \sqrt{72-\sqrt{72-\sqrt{72-\ldots}}} = y \end{array}\right\} \Rightarrow x+y = ?$

A) 10 B) 11 C) 12

D) 13 E) 14

11. $\left.\begin{array}{l} \sqrt{x+\sqrt{x+\sqrt{x+\ldots}}} = 4 \\ \sqrt{y-\sqrt{y-\sqrt{y-\ldots}}} = 5 \end{array}\right\} \Rightarrow x+y = ?$

A) 40 B) 41 C) 42

D) 43 E) 44

12. $\dfrac{\sqrt[3]{16 : \sqrt[3]{16 : \sqrt[3]{16 : \ldots}}}}{\sqrt[4]{27 \cdot \sqrt[4]{27 \cdot \sqrt[4]{27 \ldots}}}} = ?$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1

D) $\frac{4}{3}$ E) $\frac{5}{3}$

13. $\sqrt[x-1]{8 : \sqrt[x-1]{8 : \sqrt[x-1]{8 : \ldots}}} = 2 \Rightarrow x = ?$

A) 3 B) 4 C) 5

D) 6 E) 7

14. $\sqrt[3]{\sqrt{32} \cdot \sqrt[3]{\cdot \sqrt{32} \ldots}} = x \Rightarrow x = ?$

A) $\sqrt{2}$ B) 2 C) 4

D) $2\sqrt{2}$ E) 8

15. $\sqrt{3 \cdot \sqrt{2 \cdot \sqrt{3 \cdot \sqrt{2 \cdot \ldots}}}} = \sqrt[6]{x} \Rightarrow x = ?$

A) 9 B) 18 C) 18^2

D) $2 \cdot 9^2$ E) 18^4

16. $x > 2$

$\sqrt{x+2\sqrt{x-1}} - \sqrt{x-2\sqrt{x-1}} = ?$

A) 1 B) 2 C) $\sqrt{x}$

D) $\sqrt{x-1}$ E) $x-1$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	D	A	D	B	C	E	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	D	C	B	A	B	C	B

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1. $x + x\sqrt{2} = 3 \Rightarrow \frac{x}{3} + 1 = ?$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) 2

D) 3 E) 4

2. $\sqrt{\sqrt[4]{18} - 2} \cdot \sqrt{\sqrt[4]{18} + 2} \cdot \sqrt{\sqrt{18} + 4} = ?$

A) 1 B) $\sqrt{2}$ C) 2

D) $\sqrt{3}$ E) 3

3. $4 < x < 5$

$\sqrt{x^2 - 8x + 16} + \sqrt{x^2 - 10x + 25} = ?$

A) -1 B) 0 C) 1

D) $2x + 1$ E) $-2x + 9$

4. $\frac{3}{\sqrt{7 - \sqrt{40}}} - \frac{3}{\sqrt{7 + \sqrt{40}}} = ?$

A) 1 B) $\sqrt{2}$ C) $2\sqrt{2}$

D) $3\sqrt{2}$ E) $4\sqrt{2}$

5. $\sqrt{8 + \sqrt{6}} - \sqrt{8 - \sqrt{6}} = ?$

A) $16 - 4\sqrt{3}$ B) $8 + 4\sqrt{3}$ C) $16 - 2\sqrt{3}$

D) $\sqrt{16 - 2\sqrt{58}}$ E) $24 - 2\sqrt{58}$

6. $A = \sqrt{27} \cdot \sqrt[3]{9^{-1}} \cdot \sqrt[4]{81}$

$B = \sqrt[4]{27^{-1}} \cdot \sqrt{0,3} \cdot \sqrt[4]{243}$

$\frac{A}{B} = 3^x \Rightarrow x = ?$

A) $\frac{3}{2}$ B) $\frac{5}{2}$ C) $\frac{11}{6}$

D) 2 E) $\frac{13}{6}$

7. $\frac{\sqrt{2}}{2 - \sqrt{6}} + \frac{\sqrt{3}}{3 + \sqrt{6}} = ?$

A) $-2\sqrt{2}$ B) $-\sqrt{2}$ C) 0

D) $\sqrt{2}$ E) $2\sqrt{2}$

8. $\sqrt{2 + \sqrt{x}} - \sqrt{2 - \sqrt{x}} = 2 \Rightarrow x = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $x > 0, \sqrt{x-12} = \frac{8}{\sqrt{x}} \Rightarrow x = ?$

A) 10 B) 12 C) 14
D) 16 E) 18

10. $A = \sqrt{3} - 2 \Rightarrow A\cdot(A+4)\cdot(A+2) = ?$

A) $-2\sqrt{3}$ B) $-\sqrt{3}$ C) 0
D) $\sqrt{3} - 1$ E) $\sqrt{3} + 1$

11. $x, y \in R^+ \Rightarrow$

$$\frac{x\sqrt{y} - y\sqrt{x}}{\sqrt{xy}} - \sqrt{x} = ?$$

A) $-\frac{\sqrt{xy}}{xy}$ B) $-\sqrt{y}$ C) $\sqrt{xy}$
D) $\frac{\sqrt{xy}}{xy}$ E) $\sqrt{y}$

12. $$\frac{a^2\sqrt{\frac{b}{a^2}} + \frac{a\cdot\sqrt{b}}{\sqrt{a}}}{a+\sqrt{a}} = ?$$

A) $\sqrt{a}$ B) $\sqrt{b}$ C) b
D) $b\sqrt{b}$ E) $\sqrt{ab}$

13. $\sqrt{3}^{(\sqrt{10}-\sqrt{5})\cdot(\sqrt{10}+\sqrt{5})} = ?$

A) $3\sqrt{3}$ B) $4\sqrt{3}$ C) $6\sqrt{3}$
D) $8\sqrt{3}$ E) $9\sqrt{3}$

14. $x < 0 \Rightarrow$

$$\frac{\sqrt[4]{x^4}}{x} - \frac{\sqrt[3]{x^3}}{x} = ?$$

A) –2 B) –x C) 0
D) 2 E) x

15. $\left.\begin{matrix}\sqrt{2} = x \\ \sqrt{3} = y\end{matrix}\right\} \Rightarrow \sqrt{648} = ?$

A) xy^4 B) x^2y^2 C) x^3y^3
D) x^3y^4 E) x^4y^4

16. $\left.\begin{matrix}x = 2-\sqrt{3} \\ y = 2+\sqrt{3}\end{matrix}\right\} \Rightarrow \frac{\sqrt{x}}{\sqrt{y}} + \frac{\sqrt{y}}{\sqrt{x}} = ?$

A) 1 B) $\sqrt{2}$ C) 2
D) $\sqrt{3}$ E) 4

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	B	C	C	D	C	A	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	B	B	B	E	A	D	E

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $\sqrt{23+\sqrt[4]{19-\sqrt{9}}} = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

2. $\sqrt[3]{\sqrt{4^x}} = 8^{0.3} \Rightarrow x = ?$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 3

3. $\left.\begin{array}{l} A=\sqrt{x^{-2}\sqrt{x^3\sqrt{x}}} \\ B=\sqrt[4]{x\sqrt{x^{-1}\sqrt{x}}} \end{array}\right\} \Rightarrow \frac{B}{A} = ?$

A) $x^{\frac{1}{16}}$ B) $x^{\frac{1}{8}}$ C) $x^{\frac{3}{16}}$ D) $x^{\frac{1}{4}}$ E) $x^{\frac{5}{16}}$

4. $\sqrt[6]{\frac{1}{3}\sqrt[3]{3^2\sqrt{3}}} = \left(\frac{1}{3}\right)^{\frac{x}{12}} \Rightarrow x = ?$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 3

5. $\sqrt[x]{4\sqrt[x]{16}} = 4 \Rightarrow x = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

6. $a = \sqrt[3]{\frac{2}{\sqrt{2}}} \Rightarrow a^{12} = ?$

A) 2 B) 4 C) 6 D) 8 E) 10

7. $x \geq 1$

$\sqrt{4x-4}+\sqrt{9x-9} = 25 \Rightarrow x = ?$

A) 10 B) 13 C) 16 D) 20 E) 26

8. $n > 1,\ n \in Z^+,$

$\sqrt[n+1]{\sqrt[n-1]{16}} = 2^{\frac{4}{15}} \Rightarrow n = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $\sqrt[5]{\frac{5^4+5^4+5^4+5^4+5^4}{2^9+2^9}} = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{3}{4}$

D) 1 E) $\frac{5}{4}$

10. $\left.\begin{matrix} A=\sqrt{10}+3 \\ B=\sqrt{10}-3 \end{matrix}\right\} \Rightarrow \frac{A}{B}+\frac{B}{A} = ?$

A) 40 B) 38 C) 36

D) 34 E) 32

11. $\left.\begin{matrix} x=2-\sqrt{3} \\ y=2+\sqrt{3} \end{matrix}\right\} \Rightarrow \sqrt{(x-y)^2} = ?$

A) 1 B) 2 C) $\sqrt{6}$

D) 4 E) $2\sqrt{3}$

12. $\sqrt{-a+\sqrt{2a-1}}+\sqrt{-b+2\sqrt{b-1}}=0 \Rightarrow a+b = ?$

A) 1 B) 3 C) 4

D) 7 E) 9

13. $\sqrt{7+2\sqrt{10}}-\sqrt{2} = ?$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) 3

D) $\sqrt{5}$ E) 5

14. $\frac{\sqrt{8-2\sqrt{15}}}{\sqrt{5}+\sqrt{3}}+\sqrt{15} = ?$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) 3

D) 4 E) $\sqrt{5}$

15. $A=\sqrt{6+\sqrt{35}} \Rightarrow (\sqrt{7}-\sqrt{5})\cdot A = ?$

A) $\sqrt{2}$ B) 2 C) $\sqrt{5}$

D) 10 E) 12

16. $(\sqrt{9+4\sqrt{5}})\cdot(\sqrt{5}-2) = ?$

A) 1 B) $\sqrt{2}$ C) 2

D) $\sqrt{3}$ E) 3

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	E	E	A	A	B	E	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	B	E	B	D	D	A	A

BÖLÜM CHAPTER 5

TARİHSEL NOT / HISTORICAL NOTE

Atiyah, Sir Michael Francis [1929 –]

Topoloji, Geometri, Analiz, cebirsel değişimlerin transdantal teorisi, diferansiyel, işlemler ve kuantum alan teorisine önemli katkıları olan ingiliz matematikçidir. 1966 da Field Medal, 2004 da Abel ödülü ile ödüllendirilmiştir.

British mathematician who made important contributions in topology, geometry, analysis, the transcenolantal theory of algebralc varieties, dlfferential, operations and quantum field theory. he was awarded the Fields Madel in 1966 and the Abel Prize in 2004.

ÖZDEŞLİKLER ve ÇARPANLARA AYIRMA
IDENTITIES and FACTORING

Bu bölüm, 42 çözümlü örnek, 12 çözümlü test sorusu, 16 çözümlü YÖS sorusu, 144 test sorusu içermektedir.
This chapter includes 42 examples with solutions, 12 tests questions with solutions, 16 YÖS questions with solutions and 144 test questions.

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BÖLÜM
05
CHAPTER

ÖZDEŞLİKLER ve ÇARPANLARA AYIRMA
IDENTITIES and FACTORING

ÖZDEŞLİKLER / IDENTITIES

Tanım: Bilinmeyenlerinin her gerçek değeri için doğru olan eşitliklere ***özdeşlik*** denir.

Definition: The equalities that holds true for all of their unknowns' real values are called *identities*.

Örneğin / For example:

$(x + 3)^2 = x^2 + 6x + 9$ eşitliğinde / in equality,

x = 1 için / for: $(1 + 3)^2 = 1^2 + 6 \cdot 1 + 9$

$4^2 = 1 + 6 + 9$

$16 = 16$

x = –2 için / for: $(-2 + 3)^2 = (-2)^2 + 6 \cdot (-2) + 9$

$(1)^2 = 4 - 12 + 9$

$1 = 1$

Burada x'e verilen her değer için eşitlik sağlanır. O halde, bu eşitlik bir özdeşliktir.

For all values assigned to x, the equality holds true. Therefore, this equality is an identity.

TAM KARE İFADELER / PREFECT SQUARE EXPRESSIONS

i. $(a + b)^2 = a^2 + 2ab + b^2$

ii. $(a - b)^2 = a^2 - 2ab + b^2$

PEKİŞTİRME / REINFORCE

✓ $(x - 5)^2 = ?$

.....$[x^2 - 10x + 25]$

✓ $(x + 6)^2 = ?$

.....$[x^2 + 12x + 36]$

✓ $(2x - 1)^2 = ?$

.....$[4x^2 - 4x + 1]$

✓ $(x^2 + 4)^2 = ?$

.....$[x^4 + 8x^2 + 16]$

Etkinlikler / Activities

a. $(x + 1)^2 = x^2 + 2(x) \cdot (1) + 1^2 = x^2 + 2x + 1$

b. $(2x - y)^2 = (2x)^2 - 2\,(2x)y + (y)^2 = 4x^2 - 4xy + y^2$

c. $(x + 2y)^2 = x^2 + 2(x)\,(2y) + (2y)^2 = x^2 + 4xy + 4y^2$

d. $(2x - 3y)^2 = (2x)^2 - 2\,(2x)\,(3y) + (3y)^2 = 4x^2 - 12xy + 9y^2$

Örnek / Example 1

$(x-3)^2 = x^2 - 7x + 13 \Rightarrow$ **x = ?**

A) 3 B) 4 C) 6 D) 8 E) 11

Çözüm / Solution

Yanıt / Answer **B**

$(x-3)^2 = x^2 - 7x + 13$

$x^2 - 6x + 9 = x^2 - 7x + 13$

$7x - 6x = 13 - 9$

$x = 4$

Örnek / Example 2

$a = \sqrt{7} + 1$ ve / and $b = \sqrt{7} - 1 \Rightarrow$ **$a^2 - 2ab + b^2$ = ?**

A) 4 B) 8 C) 12 D) 16 E) 20

Çözüm / Solution

Yanıt / Answer **A**

$a^2 - 2ab + b^2 = (a-b)^2$

$= (\sqrt{7} + 1 - \sqrt{7} + 1)^2$

$= (2)^2 = 4$

Örnek / Example 3

$x, y \in Z^+$,

$x^2 + y^2 = 16$, $x \cdot y = 10 \Rightarrow$ **x + y = ?**

A) 2 B) 3 C) 4 D) 6 E) 7

Çözüm / Solution

Yanıt / Answer **D**

$(x+y)^2 = x^2 + y^2 + 2xy$

$(x+y)^2 = 16 + 2 \cdot 10$

$(x+y)^2 = 36 = 6^2$

$x + y = 6$

PEKİŞTİRME / REINFORCE

✓ $(a+b)(a+b) = ?$

.....$[(a+b)^2]$

✓ $34^2 + 2 \cdot (34)(6) + (6)^2 = ?$

.....$[40^2]$

✓ $(a-b)(a-b) = ?$

.....$[(a-b)^2]$

✓ $24^2 - 2 \cdot (24)(4) + (4)^2 = ?$

.....$[20^2]$

✓ $23^2 - 2 \cdot (23)(3) + (3)^2 = ?$

.....$[20^2]$

✓ $41^2 - 2 \cdot (41) + 1 = ?$

.....$[40^2]$

Örnek / Example 4

$a > b$,

$a^2 + b^2 = 14$,

$a \cdot b = 5 \Rightarrow$ **a – b = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution

Yanıt / Answer **B**

$(a - b)^2 = a^2 - 2ab + b^2$

$= 14 - 2 \cdot 5$

$= 4 \Rightarrow a - b = 2$

Özellik / Property

★ $(a + b)^2 = a^2 + 2ab + b^2 \Rightarrow \mathbf{a^2 + b^2 = (a + b)^2 - 2ab}$

★ $(a - b)^2 = a^2 - 2ab + b^2 \Rightarrow \mathbf{a^2 + b^2 = (a - b)^2 + 2ab}$

PEKİŞTİRME / REINFORCE

✓ $a + b = 10$, $a \cdot b = 21$

$\Rightarrow \mathbf{a^2 + b^2 = ?}$

.....[58]

✓ $a - b = 8$, $a \cdot b = 48$

$\Rightarrow \mathbf{a^2 + b^2 = ?}$

.....[160]

✓ $2a + b = 14$, $a \cdot b = 24$

$\Rightarrow \mathbf{4a^2 + b^2 = ?}$

.....[100]

Örnek / Example 5

$a - b = 5$ ve / and $a \cdot b = 14$

$\Rightarrow \mathbf{a^2 + b^2 = ?}$

A) 47 B) 49 C) 51 D) 53 E) 55

Çözüm / Solution

Yanıt / Answer **D**

$a^2 + b^2 = (a - b)^2 + 2ab$

$= 5^2 + 2 \cdot 14$

$= 25 + 28$

$= 53$

Örnek / Example 6

$a + b = 12$ ve / and $a \cdot b = 10$

$\Rightarrow a^2 + b^2 = ?$

A) 32 B) 48 C) 64 D) 96 E) 124

Çözüm / Solution Yanıt / Answer **E**

$a^2 + b^2 = (a + b)^2 - 2ab$

$a^2 + b^2 = 12^2 - 2 \cdot 10$

$= 144 - 20$

$= 124$

Örnek / Example 7

$$\frac{3(x+2)^2 - 6x(x+2) + 3x^2}{3} = ?$$

A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution Yanıt / Answer **D**

$$\frac{3(x+2)^2 - 6x(x+2) + 3x^2}{3}$$

$$= \frac{3[(x+2)^2 - 2x(x+2) + x^2]}{3}$$

$$= x^2 + 4x + 4 - 2x^2 - 4x + x^2$$

$$= 4$$

Özellik / Property

★ $\left(ax + \frac{b}{x}\right)^2 = a^2x^2 + \frac{b^2}{x^2} + 2ab$

★ $\left(ax - \frac{b}{x}\right)^2 = a^2x^2 + \frac{b^2}{x^2} - 2ab$

Örnek / Example 8

$x - \frac{1}{x} = 3 \Rightarrow x^2 + \frac{1}{x^2} = ?$

A) 8 B) 9 C) 11 D) 13 E) 15

Çözüm / Solution Yanıt / Answer **C**

$x - \frac{1}{x} = 3 \Rightarrow x^2 - 2 + \frac{1}{x^2} = 9$

$$x^2 + \frac{1}{x^2} = 11$$

PEKİŞTİRME / REINFORCE

✓ $\left(x + \frac{1}{x}\right)^2 = ?$

..... $\left[x^2 + 2 + \frac{1}{x^2}\right]$

✓ $\left(x - \frac{1}{x}\right)^2 = ?$

..... $\left[x^2 - 2 + \frac{1}{x^2}\right]$

✓ $\left(x + \frac{3}{x}\right)^2 = ?$

..... $\left[x^2 + 6 + \frac{9}{x^2}\right]$

✓ $\left(x - \frac{2}{x}\right)^2 = ?$

..... $\left[x^2 - 4 + \frac{4}{x^2}\right]$

PEKİŞTİRME / REINFORCE

✓ $x^2 - 2x + 6 = 0 \Rightarrow x^2 + \frac{36}{x^2} = ?$

.....[−8]

✓ $x^2 - x + 1 = 0 \Rightarrow x^2 + \frac{1}{x^2} = ?$

.....[−1]

✓ $x^2 - 8x + 4 = 0 \Rightarrow x^2 + \frac{16}{x^2} = ?$

.....[56]

Örnek / Example 9

$x^2 - 4x + 5 = 0 \Rightarrow x^2 + \frac{25}{x^2} = ?$

A) 5 B) 6 C) 8 D) 9 E) 10

Çözüm / Solution Yanıt / Answer **B**

$$\frac{x^2 - 4x + 5}{x} = \frac{0}{x}$$

$$\frac{x^2}{x} - \frac{4x}{x} + \frac{5}{x} = 0$$

$$x + \frac{5}{x} = 4$$

$$x^2 + \frac{25}{x^2} + 2 \cdot x \cdot \frac{5}{x} = 16$$

$$x^2 + \frac{25}{x^2} = 6$$

PEKİŞTİRME / REINFORCE

✓ $a + b = 15,\ a \cdot b = 50 \Rightarrow a - b = ?$

.....[5]

✓ $a - b = 10,\ a \cdot b = 56 \Rightarrow a + b = ?$

.....[18]

Örnek / Example 10

$a + \frac{1}{a} = 3 \Rightarrow a^2 + \frac{1}{a^2} = ?$

A) 5 B) 6 C) 7 D) 8 E) 9

Çözüm / Solution Yanıt / Answer **C**

$$\left(a + \frac{1}{a}\right)^2 = 3^2$$

$$a^2 + 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2} = 9$$

$$a^2 + \frac{1}{a^2} = 9 - 2$$

$$a^2 + \frac{1}{a^2} = 7$$

Özellik / Property

★ $(a + b)^2 = (a - b)^2 + 4ab$

★ $(a - b)^2 = (a + b)^2 - 4ab$

Örnek / Example 11

$a > b$,

$a + b = 8$, $a \cdot b = 15 \Rightarrow$ **$a - b = ?$**

A) 6 B) 5 C) 4 D) 3 E) 2

Çözüm / Solution

Yanıt / Answer **E**

$(a - b)^2 = (a + b)^2 - 4ab$

$(a - b)^2 = (8)^2 - 4 \cdot 15$

$(a - b)^2 = 64 - 60$

$(a - b)^2 = 4$

$a - b = 2$

Örnek / Example 12

$(3,6 + 2,6)^2 - 4 \cdot (3,6) \cdot (2,6) = ?$

A) 1 B) 4 C) 9 D) 16 E) 25

Çözüm / Solution

Yanıt / Answer **A**

$(a - b)^2 = (a + b)^2 - 4ab$

$(3,6 + 2,6)^2 - 4 \cdot (3,6) \cdot (2,6) = (3,6 - 2,6)^2$

$= 1^2 = 1$

PEKİŞTİRME / REINFORCE

✓ $x = 1,8$, $y = 3,2 \Rightarrow (x - y)^2 + 4xy = ?$

.....[25]

✓ $x = 3,4$, $y = 1,4 \Rightarrow (x + y)^2 - 4xy = ?$

.....[4]

PEKİŞTİRME / REINFORCE

✓ $a + b + c = 10$

$ab + ac + bc = 22$

$\Rightarrow$ $a^2 + b^2 + c^2 = ?$

.....[56]

✓ $a + b - c = 12$

$-ac - bc - ab = 20$

$\Rightarrow$ $a^2 + b^2 + c^2 = ?$

.....[184]

Özellik / Property

★ $(a + b + c)^2 = a^2 + b^2 + c^2 + 2\cdot(a\cdot b + a\cdot c + b\cdot c)$

★ $(a + b - c)^2 = a^2 + b^2 + c^2 + 2\cdot(a\cdot b - a\cdot c - b\cdot c)$

Örnek / Example 13

$a + b + c = 12$, $a^2 + b^2 + c^2 = 56$ $\Rightarrow$ **$ac + bc + ab = ?$**

A) 40 B) 44 C) 48 D) 52 E) 56

Çözüm / Solution Yanıt / Answer **B**

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2\cdot(ab + ac + bc)$

$144 = 56 + 2\cdot(ab + ac + bc)$

$88 = 2\cdot(ab + ac + bc)$

$44 = ab + ac + bc$

Örnek / Example 14

$a + b - c = 6$, $a^2 + b^2 + c^2 = 110$ $\Rightarrow$ **$ac + bc - ab = ?$**

A) 37 B) 35 C) 24 D) 22 E) 19

Çözüm / Solution Yanıt / Answer **A**

$a + b - c = 6$

$(a + b - c)^2 = 6^2$

$\underbrace{a^2 + b^2 + c^2}_{110} + 2\,(ab - ac - bc) = 36$

$110 + 2\,(ab - ac - bc) = 36$

$2\,(ab - ac - bc) = -74$

$ab - ac - bc = -37$

$ac + bc - ab = 37$

Örnek / Example 15

$a, b, c \in Z^+$,

$a^2 + 2bc = 63,\ b^2 + 2ac = 47,\ c^2 + 2ab = 59 \Rightarrow \mathbf{a + b + c = ?}$

A) 11 B) 12 C) 13 D) 14 E) 15

Çözüm / Solution

Yanıt / Answer **C**

$$\begin{aligned} &a^2 + 2bc = 63 \\ &b^2 + 2ac = 47 \\ +\ &c^2 + 2ab = 59 \end{aligned}$$

$a^2 + b^2 + c^2 + 2\cdot(ab + ac + bc) = 169$

$(a + b + c)^2 = 13^2 \Rightarrow a + b + c = 13$

Özellik / Property

★ $a^2 + b^2 = 0 \Rightarrow \mathbf{a = 0,\ b = 0}$

Etkinlikler / Activities

a. $(a - 2)^2 = (b + 3)^2 = 0 \Rightarrow a\cdot b = ?$

$a - 2 = 0 \qquad b + 3 = 0$

$a = 2 \qquad b = -3$

$\Rightarrow a\cdot b = -6$

b. $(a - 5)^2 = (b + 1)^2 + (c - 3)^2 = 0 \Rightarrow a + b + c = ?$

$a - 5 = 0 \qquad b + 1 = 0 \qquad c - 3 = 0$

$a = 5 \qquad b = -1 \qquad c = 3$

$a + b + c = 5 - 1 + 3 = 7$

Örnek / Example 16

$(x - y - 12)^2 + (x - 4)^2 = 0 \Rightarrow \mathbf{y = ?}$

A) −2 B) −4 C) −6 D) −7 E) −8

Çözüm / Solution

Yanıt / Answer **E**

$x - 4 = 0$

$x = 4$

$x - y - 12 = 0$

$4 - y - 12 = 0$

$y = -8$

Örnek / Example 17

$x^2 + y^2 + 2x - 4y + 5 = 0 \Rightarrow x \cdot y = ?$

A) –2 B) – 1 C) 0 D) 1 E) 2

Çözüm / Solution Yanıt / Answer **A**

$x^2 + 2x + 1 + y^2 - 4y + 4 = 0$

$(x + 1)^2 + (y - 2)^2 = 0$

$x + 1 = 0, \quad y - 2 = 0$

$x = -1 \qquad y = 2 \Rightarrow x \cdot y = (-1) \cdot 2 = -2$

ÖNEMLİ

- $x, y, \in Z^+$,

a bir asal sayı olmak üzere.
a is a prime number.

$x^2 - y^2 = a \Rightarrow x + y = a, \; x - y = 1$

İKİ KARE FARKI İFADELER
DIFFERENCE of TWO SQUARES EXPRESSIONS

$x^2 - y^2 = (x + y) \cdot (x - y)$

Etkinlikler / Activities

a. $a^2 - 64 = a^2 - 8^2 = (a - 8) \cdot (a + 8)$

b. $1 - y^2 = 1^2 - y^2 = (1 - y) \cdot (1 + y)$

c. $4a^2 - 25 = (2a)^2 - 5^2 = (2a - 5) \cdot (2a + 5)$

d. $\frac{4}{x^2} - \frac{9}{4y^2} = \left(\frac{2}{x}\right)^2 - \left(\frac{3}{2y}\right)^2 = \left(\frac{2}{x} - \frac{3}{2y}\right) \cdot \left(\frac{2}{x} + \frac{3}{2y}\right)$

PEKİŞTİRME / REINFORCE

✓ $26^2 - 16^2 = ?$

.....[420]

✓ $28^2 - 8^2 = ?$

.....[720]

✓ $\frac{1}{16} - \frac{1}{64} = ?$

.....[$\frac{3}{64}$]

✓ $13^2 - 12^2 = ?$

.....[25]

✓ $\frac{1}{4} - \frac{1}{16} = ?$

.....[$\frac{3}{16}$]

Örnek / Example 18

$x, y \in N^+$

$x^2 - y^2 = 29 \Rightarrow x \cdot y = ?$

A) 200 B) 205 C) 210 D) 220 E) 230

Çözüm / Solution Yanıt / Answer **C**

$x^2 - y^2 = 29$

$(x + y) + (x - y) = 29 \cdot 1$

$\left.\begin{matrix} x + y = 29 \\ x - y = 1 \end{matrix}\right\} \Rightarrow x = 15, \; y = 14$

$x \cdot y = 15 \cdot 14 = 210$

Örnek / Example 19

$37^2 - 9 = ?$

A) 1160 B) 1260 C) 1280 D) 1360 E) 1380

Çözüm / Solution

Yanıt / Answer **D**

$37^2 - 9 = 37^2 - 3^2 = (37 - 3)\ (37 + 3)$

$= 34 \cdot 40$

$= 1360$

Örnek / Example 20

$(y - x)^2 - (x + y)^2 = ?$

A) $-4xy$ B) $-2xy$ C) $2xy$ D) $4xy$ E) $8xy$

Çözüm / Solution

Yanıt / Answer **A**

$(y - x)^2 - (x + y)^2 = (y - x + x + y)\cdot(y - x - x - y)$

$= (2y) \cdot (-2x)$

$= -4xy$

Örnek / Example 21

$x^2 - 5 = 12 \Rightarrow x^4 - 25 = ?$

A) 260 B) 264 C) 268 D) 272 E) 276

Çözüm / Solution

Yanıt / Answer **B**

$x^2 - 5 = 12,\ x^2 + 5 = 22$

$x^4 - 25 = (x^2 - 5)\cdot(x^2 + 5)$

$= 12\cdot22$

$= 264$

İKİ KÜP TOPLAMI ve FARKI
SUM and DIFFERENCE of TWO CUBES

I. $a^3 - b^3 = (a - b) \cdot (a^2 + ab + b^2)$

II. $a^3 + b^3 = (a + b) \cdot (a^2 - ab + b^2)$

Etkinlikler / Activities

a. $x^3 - 27 = x^3 - 3^3 = (x - 3) \cdot (x^2 + 3x + 9)$

b. $1 + y^3 = 1^3 + y^3 = (1 + y) \cdot (1 - y + y^2)$

c. $a^{-3} + b^{-3} = \left(\frac{1}{a}\right)^3 + \left(\frac{1}{b}\right)^3 = \left(\frac{1}{a} + \frac{1}{b}\right) \cdot \left(\frac{1}{a^2} - \frac{1}{ab} + \frac{1}{b^2}\right)$

d. $16a^3 - 250b^3 = 2 \cdot (8a^3 - 125b^3)$

$= 2 \cdot ((2a)^3 - (5b)^3)$

$= 2 \cdot (2a - 5b) \cdot (4a^2 + 10ab + 25b^2)$

e. $\frac{a^3}{8} + \frac{8b^3}{27} = \left(\frac{a}{2}\right)^3 + \left(\frac{2b}{3}\right)^3 = \left(\frac{a}{2} + \frac{2b}{3}\right) \cdot \left(\frac{a^2}{4} - \frac{ab}{3} + \frac{4b^2}{9}\right)$

Örnek / Example 22

$a^3 - b^3 = 48$, $a - b = 4$ $\Rightarrow$ $a^2 + ab + b^2 = ?$

A) 8 B) 10 C) 12 D) 14 E) 16

Çözüm / Solution

Yanıt / Answer **C**

$a^3 - b^3 = (a - b) \cdot (a^2 + ab + b^2)$

$48 = 4 \cdot (a^2 + ab + b^2)$

$12 = a^2 + ab + b^2$

Örnek / Example 23

$a + b = 12$, $ab - a^2 - b^2 = -6$

$\Rightarrow$ $a^3 + b^3 = ?$

A) 64 B) 66 C) 68 D) 70 E) 72

Çözüm / Solution

Yanıt / Answer **E**

$a^3 + b^3 = (a + b) \cdot (a^2 - ab + b^2)$

$a^3 + b^3 = 12 \cdot 6$

$a^3 + b^3 = 72$

TAM KÜP İFADELER / PERFECT CUBE EXPRESSIONS

I. $(a + b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3$

II. $(a - b)^3 = a^3 - 3a^2 b + 3ab^2 - b^3$

Örnek / Example 24

$x = 4$

$\Rightarrow x^3 + 3x^2 + 3x + 10 = ?$

A) 125 B) 128 C) 130 D) 132 E) 134

Çözüm / Solution

Yanıt / Answer **E**

$x^3 + 3x^2 + 3x + 10 = x^3 + 3 \cdot x^2 \cdot 1 + 3x \cdot 1^2 + 1^3 + 9$

$= (x + 1)^3 + 9$

$= (4 + 1)^3 + 9$

$= 125 + 9 = 134$

Özellik / Property

★ $a^3 + b^3 = (a + b)^3 - 3 \cdot a \cdot b(a + b)$

★ $a^3 - b^3 = (a - b)^3 + 3 \cdot a \cdot b(a - b)$

Örnek / Example 25

$a + b = -2$ ve / and $a \cdot b = -15$

$\Rightarrow a^3 + b^3 = ?$

A) −144 B) − 98 C) − 49 D) 16 E) 48

Çözüm / Solution

Yanıt / Answer **B**

$a^3 + b^3 = (a + b)^3 - 3ab\,(a + b)$

$a^3 + b^3 = (-2)^3 - 3 \cdot (-15) \cdot (-2)$

$= -8 - 90$

$= -98$

Örnek / Example 26

$a^3 + b^3 = 27$,

$a + b = 2 \Rightarrow a \cdot b = ?$

A) $-\frac{12}{5}$ B) $-\frac{13}{5}$ C) $-\frac{14}{5}$ D) $-\frac{17}{6}$ E) $-\frac{19}{6}$

Çözüm / Solution

Yanıt / Answer **E**

$a^3 + b^3 = (a + b)^3 - 3 \cdot a \cdot b(a + b)$

$27 = 2^3 - 3 \cdot a \cdot b \cdot 2$

$27 = 8 - 6ab$

$6ab = -19$

$ab = \frac{-19}{6}$

Örnek / Example 27

$x - \frac{4}{x} = -2 \Rightarrow x^3 - \frac{64}{x^3} = ?$

A) 8 B) 4 C) – 16 D) –32 E) – 64

Çözüm / Solution

Yanıt / Answer **D**

$x^3 - \frac{64}{x^3} = \left(x - \frac{4}{x}\right)^3 + 3 \cdot x \cdot \frac{4}{x} \cdot \left(x - \frac{4}{x}\right)$

$x^3 - \frac{64}{x^3} = (-2)^3 + 3 \cdot 4 \cdot (-2)$

$x^3 - \frac{64}{x^3} = -8 - 24$

$x^3 - \frac{64}{x^3} = -32$

ÇARPANLARA AYIRMA / FACTORING

Bir cebirsel ifadeyi, iki veya daha fazla ifadenin çarpımı biçiminde yazmaya ***çarpanlara ayırma*** denir.

The operation of writing an algebraic expression as multiplication of two or more than two expressions is called ***factorization(or factoring).***

ORTAK ÇARPAN PARANTEZİNE ALMA
BRACKETING of COMMON FACTORS

Cebirsel bir ifadenin tüm terimlerinde ortak ifade varsa, bu ortak terim ifadenin çarpanıdır.

If all the terms of an algebraic expression contain a common factor, then that term can be factored out of the expression.

I. $Ax + Ay = A\cdot(x + y)$

Etkinlikler / Activities

a. $a^2 + a\cdot b = a\cdot a + a\cdot b$

$= a\cdot(a + b)$

b. $x^2 + a\cdot x + x = x\cdot x + x\cdot a + x\cdot 1$

$= x\cdot(x + a + 1)$

c. $3x^2\cdot y^2\cdot z - 6x\cdot y^2\cdot z^2 = 3\cdot x\cdot x\cdot y^2\cdot z - 3\cdot 2\cdot x\cdot y^2\cdot z\cdot z$

$= 3xy^2 z\cdot(x - 2z)$

d. $15a^2\cdot b - 20ab^2 - 35\cdot a = 5\cdot 3\cdot a\cdot a\cdot b - 5\cdot 4\cdot a\cdot b^2 - 7\cdot 5\cdot a$

$= 5\cdot a\cdot(3ab - 4b^2 - 7)$

e. $(x + y)\cdot(m + n) - x - y = (x + y)\cdot(m + n) - (x + y)$

$= (x + y)\cdot(m + n - 1)$

f. $\frac{2\cdot a}{x^2} - \frac{4\cdot b}{x} - \frac{8}{x^3} = \frac{2}{x}\cdot\frac{a}{x} - \frac{2}{x}\cdot 2b - \frac{2}{x}\cdot\frac{4}{x^2}$

$= \frac{2}{x}\left(\frac{a}{x} - 2b - \frac{4}{x^2}\right)$

g. $x^2 - bx - ax + ab = x(x - b) - a(x - b)$

$= (x - b)\cdot(x - a)$

PEKİŞTİRME / REINFORCE

✓ $x^2 - 2x = ?$

.....[$x\cdot(x - 2)$]

✓ $3x^2 + 4x + 5x^3 = ?$

.....[$x\cdot(3x + 4 + 5x^2)$]

✓ $x^3 - 2x^2 = ?$

.....[$x^2\cdot(x - 2)$]

✓ $3x^2 + 5x = ?$

.....[$x\cdot(3x + 5)$]

✓ $ax^2 + bx^3 = ?$

.....[$x^2\cdot(a + bx)$]

✓ $2x^2 + 4x^3 = ?$

.....[$2x^2\cdot(1 + 2x)$]

✓ $3x^2 + 9x^3 = ?$

.....[$3x^2\cdot(1 + 3x)$]

PEKİŞTİRME / REINFORCE

✓ $mx + my + nx + ny = ?$

.....[(m + n)·(x + y)]

✓ $x^2 + x + x + 1 = ?$

.....[(x + 1)·(x + 1)]

✓ $x^4 + x^3 + x^2 + x = ?$

.....[$(x^3 + x)$·(x + 1)]

✓ $a^3 + a^2 + a + 1 = ?$

.....[$(a^2 + 1)$·(a + 1)]

GRUPLANDIRMA YOLUYLA ÇARPANLARA AYIRMA
FACTORIZATION by GROUPING

Her terimde ortak olan bir ifade yoksa terimler, ikişer ikişer veya üçer üçer gruplandırılarak ortak çarpan parentezine alınır ve çarpanlarına aylılır.

If there is not one single common expressions, then expressions are grouped to groups of two's or three's and common expression factorization is done.

I. $Ax + By + Bx + Ay = A(x + y) + B(x + y)$

$= (x + y)(A + B)$

Etkinlikler / Activities

a. $a^3 - a^2 + a - 1 = a^2(a - 1) + (a - 1)$

$= (a - 1)(a^2 + 1)$

b. $2ab - 3b + 10a - 15 = b(2a - 3) + 5(2a - 3)$

$= (2a - 3)(b + 5)$

c. $3x - xy + 3y - y^2 = x(3 - y) + y(3 - y)$

$= (3 - y)(x + y)$

Örnek / Example 28

$x + y = 2,\ a - b = \frac{1}{2} \Rightarrow$ **bx + by − ax − ay = ?**

A) −2 B) −1 C) 1 D) 2 E) 3

Çözüm / Solution

Yanıt / Answer **B**

$bx + by - ax - ay = b(x + y) - a(x + y)$

$= (x + y)(b - a)$

$a - b = \frac{1}{2} \Rightarrow b - a = -(a - b) = -\frac{1}{2}$, $bx + by - ax - ay = (x + y)\cdot(b - a)$

$= 2 \cdot \left(-\frac{1}{2}\right) = -1$

Örnek / Example 29

$m + n = 5,$

$x - y = 4 \Rightarrow$ **mx − my + nx − ny = ?**

A) 12 B) 16 C) 20 D) 22 E) 24

Çözüm / Solution

Yanıt / Answer **C**

$mx - my + nx - ny = m(x - y) + n(x - y)$

$= (x - y)(m + n)$

$= 4 \cdot 5 = 20$

$Ax^2 + Bx + C$ ŞEKLİNDEKİ İFADELERİN ÇARPANLARA AYRILMASI
FACTORIZATION of the EXPRESSIONS in $Ax^2 + Bx + C$ FORM

▼ $A = 1$, $m \cdot n = C$, $m + n = B$

$x^2 + Bx + C = (x + m)(x + n)$ dir.

↓	↓	↓
x	m+n	m
x		n

Etkinlikler / Activities

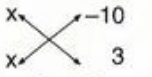
a. $x^2 + 7x + 12$



$3x + 4x = 7x$ olduğundan, $x^2 + 7x + 12 = (x + 4) \cdot (x + 3)$ tür.

Since $3x + 4x = 7x$, $x^2 + 7x + 12 = (x + 4) \cdot (x + 3)$.

b. $x^2 - 7x - 30$



$-10x + 3x = -7x$ olduğundan, $x^2 - 7x - 30 = (x - 10) \cdot (x + 3)$ tür.

Since $-10x + 3x = -7x$, $x^2 - 7x - 30 = (x - 10) \cdot (x + 3)$.

c. $m^4 + 6m^2 + 9$



$3m^2 + 3m^2 = 6m^2$ olduğundan, $m^4 + 6m^2 + 9 = (m^2 + 3) \cdot (m^2 + 3)$ tür.

Since $3m^2 + 3m^2 = 6m^2$, $m^4 + 6m^2 + 9 = (m^2 + 3) \cdot (m^2 + 3) = (m^2 + 3)^2$

d. $(5 - 3x)^2 + 4 \cdot (5 - 3x) - 21$

$5 - 3x = t$

$(5 - 3x)^2 + 4\,(5 - 3x) - 21 = t^2 + 4t - 21$

$= (t + 7)\,(t - 3)$

$= (5 - 3x + 7)\,(5 - 3x - 3)$

$= (12 - 3x)\,(2 - 3x)$

$= 3(4 - x)\,(2 - 3x)$

PEKİŞTİRME / REINFORCE

✓ $x^2 + 3x + 2 = ?$

.....[(x + 1)·(x + 2)]

✓ $x^2 - 5x - 6 = ?$

.....[(x − 6)·(x + 1)]

✓ $x^4 - x^2 - 12 = ?$

.....[$(x^2 - 4)$·$(x^2 + 3)$]

✓ $x^2 - x - 2 = ?$

.....[(x − 2)·(x + 1)]

✓ $x^2 - 5x + 6 = ?$

.....[(x − 3)·(x − 2)]

✓ $x^2 - 4x + 4 = ?$

.....[(x − 2)·(x − 2)]

PEKİŞTİRME / REINFORCE

✓ $2x^2 + xy - 3y^2 = ?$

.....[(2x + 3y)·(x – y)]

✓ $2x^2 + 5x - 3 = ?$

.....[(2x – 1)·(x + 3)]

✓ $3x^2 - 2x - 8 = ?$

.....[(3x + 4)·(x – 2)]

✓ $2x^2 - xy - y^2 = ?$

.....[(2x + y)·(x – y)]

✓ $2x^2 + 5xy + 3y^2 = ?$

.....[(2x + 3y)·(x + y)]

▼ $A = 1,\quad m \cdot n = C,\quad p \cdot q = A,\quad m \cdot q + n \cdot p = B$

$Ax^2 + Bx + C = (px + m) \cdot (qx + n)$

$\downarrow \qquad \downarrow$

$px \qquad m$

$qx \qquad n$

Etkinlikler / Activities

a. $15x^2 - 14x + 3$

$\downarrow \qquad \downarrow$

$5x \qquad -3$

$3x \qquad -1$

$(-9x) + (-5x) = -14x$ [Çapraz çarpım ortadaki terime eşit olmalıdır.]

$15x^2 - 14x + 3 = (5x - 3)(3x - 1)$ [The cross - multiply should equal to the middle term.]

b. $6x^2 - 19x + 15$

$\downarrow \qquad \downarrow$

$3x \qquad -5$

$2x \qquad -3$

$-9x - 10x = -19x$ [Çapraz çarpım ortadaki terime eşit olmalıdır.]

$6x^2 - 19x + 15 = (3x - 5)(2x - 3)$ [The cross - multiply should equal to the middle term.]

c. $2x^2 + 5ax - 3a^2$

$\downarrow \qquad \downarrow$

$2x \qquad -a$

$x \qquad 3a$

$6ax - ax = 5ax$ [Çapraz çarpım ortadaki terime eşit olmalıdır.]

$2x^2 + 5ax - 3a^2 = (2x - a)(x + 3a)$ [The cross - multiply should equal to the middle term.]

TERİM EKLEYİP ÇIKARARAK ÇARPANLARA AYIRMA
FACTORIZATION by TERM ADDING or SUBTRACTING

Verilen bir ifade, terim ekleyip çıkartılarak, bilinen bir özdeşliğin açılımına dönüştürülür.

A given expression can convert to expantion of a known identity by adding and subtracting a certain term.

Etkinlikler / Activities

a. $x^4 - 23x^2 + 1$ ifadesinde,

x^4 ile 1 in yanında $2x^2$ olsaydı ifade tam kare olur.

O halde, $2x^2$ eklenip çıkarılırsa aşağıdaki eşitlik bulunur.

$$x^4 - 23x^2 + 1 = x^4 - 23x^2 + 1 - 2x^2 + 2x^2$$
$$= x^4 + 2x^2 + 1 - 25x^2$$
$$= (x^2 + 1)^2 - (5x)^2$$
$$= (x^2 + 1 - 5x)(x^2 + 1 + 5x)$$

a. In expression $x^4 - 23x^2 + 1$, if there was $2x^2$ next to x^4 and 1, given expression becomes a perfect square expression.

Therefore, if you add and subtract the term $2x^2$,

$$x^4 - 23x^2 + 1 = x^4 - 23x^2 + 1 - 2x^2 + 2x^2$$
$$= x^4 + 2x^2 + 1 - 25x^2$$
$$= (x^2 + 1)^2 - (5x)^2$$
$= (x^2 + 1 - 5x)(x^2 + 1 + 5x)$ is obtained.

b. $x^4 + 4y^4$ ifadesinde,

$4x^2y^2$ olsaydı ifade çarpanlara ayrılabilirdi.

O halde, ifadeye $4x^2y^2$ eklenip çıkarılırsa, aşağıdaki eşitlik bulunur.

$$x^4 + 4y^4 = x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2$$
$$= (x^2 + 2y^2)^2 - (2xy)^2$$
$$= (x^2 + 2y^2 - 2xy)(x^2 + 2y^2 + 2xy)$$

b. In expression $x^4 + 4y^4$, if there was $4x^2y^2$, we could have factorize them.

Therefore, if you add and subtract $4x^2y^2$,

$$x^4 + 4y^4 = x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2$$
$$= (x^2 + 2y^2)^2 - (2xy)^2$$
$= (x^2 + 2y^2 - 2xy)(x^2 + 2y^2 + 2xy)$ is obtained.

PEKİŞTİRME / REINFORCE

✓ $A = x^2 + 6x + 8 \Rightarrow$ **min(A) = ?**

.....[–1]

✓ $A = x^2 + 12x + 30 \Rightarrow$ **min(A) = ?**

.....[–6]

✓ $A = x^2 + y^2 + 2x + 4y \Rightarrow$ **min(A) = ?**

.....[–5]

Örnek / Example 30

$A = x^2 + 8x + 9 \Rightarrow$ **min(A) = ?**

A) –7 B) –6 C) –5 D) –4 E) –3

Çözüm / Solution

Yanıt / Answer **A**

$A = x^2 + 8x + 9 = x^2 + 8x + 9 + 7 - 7$

$= (x + 4)^2 - 7$

$\Rightarrow \min(A) = 0 - 7 = -7$

Örnek / Example 31

$x^2 + y^2 - 4x + 6y = A \Rightarrow$ **min(A) = ?**

A) –15 B) –13 C) –12 D) –10 E) –8

Çözüm / Solution

Yanıt / Answer **B**

$x^2 - 4x + y^2 + 6y = A$

$x^2 - 4x + 4 + y^2 + 6y + 9 = A + 13$

$(x - 2)^2 + (y + 3)^2 = A + 13$

$A = (x - 2)^2 + (y + 3)^2 - 13$

$\min(A) = -13$

Örnek / Example 32

$x^2 + 2x + 4 + \frac{4}{(x+1)^2} = 143 \Rightarrow (x+1) + \frac{2}{(x+1)} = ?$

A) 15 B) 14 C) 13 D) 12 E) 11

Çözüm / Solution

Yanıt / Answer **D**

$x^2 + 2x + 1 + 4 + \frac{4}{(x+1)^2} = 143 + 1$

$(x+1)^2 + 2 \cdot (x+1) \cdot \frac{2}{(x+1)} + \frac{4}{(x+1)^2} = 144$

$\left((x+1) + \frac{2}{(x+1)}\right)^2 = 144$

$\Rightarrow (x+1) + \frac{2}{(x+1)} = 12$

RASYONEL İFADELERDE SADELEŞTİRME
SIMPLIFICATION in RATIONAL EXPRESSIONS

Pay ve paydadaki harfli ifadeler, çarpım durumunda değilse önce çarpanlarına ayrılır, sonra sadeleştirme yapılır.

If the letters in denominator and numerator are not in the multiplication form, first factorization and then simplification are done.

ÖNEMLİ

- $x^{-1} = \frac{1}{x}$
- $x^{-2} = \frac{1}{x^2}$

Örnek / Example 33

$$\frac{a^3b - ab^3}{a^3b - 2a^2b^2 + ab^3} = ?$$

A) $\frac{a-b}{a+b}$ B) $\frac{a+b}{a-b}$ C) $\frac{a}{a+b}$

D) $\frac{b}{a-b}$ E) $a \cdot b$

Çözüm / Solution

Yanıt / Answer **B**

$$\frac{a^3b - ab^3}{a^3b - 2a^2b^2 + ab^3}$$

$$= \frac{ab(a^2 - b^2)}{ab(a^2 - 2ab + b^2)}$$

$$= \frac{ab(a-b)(a+b)}{ab(a-b)(a-b)}$$

$$= \frac{a+b}{a-b}$$

Örnek / Example 34

$$\frac{2a^2 + 4ab + 2b^2}{2a + 2b} = ?$$

A) $2a + b$ B) $a + b$ C) $a - b$

D) $a + 2b$ E) $2a - b$

Çözüm / Solution

Yanıt / Answer **B**

$$\frac{2a^2 + 4ab + 2b^2}{2a + 2b} = \frac{2(a^2 + 2ab + b^2)}{2(a+b)}$$

$$= \frac{(a+b)^2}{a+b}$$

$$= a + b$$

Örnek / Example 35

$$\frac{16a^2b^2 - 4x^2y^2}{4ab - 2xy} = ?$$

A) $xy + 2ab$ B) $2xy + 4ab$ C) $2xy - 4ab$

D) $4xy + 2ab$ E) $2xy$

ÖNEMLİ

- $a^2 - b^2 = (a - b)(a + b)$

[İki kare farkı]

[Difference of two squares]

Çözüm / Solution Yanıt / Answer **B**

$$\frac{16a^2b^2 - 4x^2y^2}{4ab - 2xy} = \frac{(4ab)^2 - (2xy)^2}{4ab - 2xy}$$

$$= \frac{(4ab - 2xy)(4ab + 2xy)}{4ab - 2xy}$$

$$= 4ab + 2xy$$

Örnek / Example 36

$$\frac{x^2 - x^{-1}}{x - x^{-2}} = ?$$

A) $x - 1$ B) $-x$ C) x D) x^2 E) $x + 1$

Çözüm / Solution Yanıt / Answer **C**

$$\frac{x^2 - x^{-1}}{x - x^{-2}} = \frac{x^2 - \frac{1}{x}}{x - \frac{1}{x^2}}$$

$$= \frac{\frac{x^3 - 1}{x}}{\frac{x^3 - 1}{x^2}}$$

$$= \frac{x^3 - 1}{x} \cdot \frac{x^2}{x^3 - 1} = x$$

Örnek / Example 37

$\frac{x^2-3x+2}{x^2-4} \cdot \frac{x^2+3x+2}{x^2-1} = ?$

A) -1 B) $\frac{x-1}{x+1}$ C) 0

D) $\frac{x+1}{x-1}$ E) 1

Çözüm / Solution

Yanıt / Answer **E**

$$\frac{x^2-3x+2}{x^2-4} \cdot \frac{x^2+3x+2}{x^2-1} = \frac{(x-2)\cdot(x-1)}{(x-2)\cdot(x+2)} \cdot \frac{(x+2)\cdot(x+1)}{(x-1)\cdot(x+1)}$$

$$= 1$$

Örnek / Example 38

$\frac{x^2+x+1}{x^3-1} : \frac{x+1}{x^2-1} = ?$

A) 1 B) $\frac{1}{x-1}$ C) $\frac{1}{x+1}$ D) $\frac{x-1}{x+1}$ E) $\frac{x+1}{x-1}$

Çözüm / Solution

Yanıt / Answer **A**

$$\frac{x^2+x+1}{x^3-1} : \frac{x+1}{x^2-1} = \frac{x^2+x+1}{(x-1)\cdot(x^2+x+1)} \cdot \frac{x^2-1}{x+1}$$

$$= \frac{(x-1)\cdot(x+1)}{(x-1)\cdot(x+1)}$$

$$= 1$$

Örnek / Example 39

$a+b=4 \Rightarrow \dfrac{a^2-4b-b^2-4a}{a^2-b^2-8b-16} = ?$

A) $\frac{2}{3}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 4

Çözüm / Solution Yanıt / Answer **B**

$$\frac{(a-b)(a+b)-4(a+b)}{a^2-(b+4)^2} = \frac{(a+b)(a-b-4)}{(a+b+4)(a-b-4)}$$
$$= a+b = 4$$
$$= \frac{4}{4+4} = \frac{1}{2}$$

Örnek / Example 40

$$\frac{\frac{1}{x}+\frac{4}{x^2}+\frac{4}{x^3}}{x^2-4} : \frac{x+2}{x^2-2x} = ?$$

A) x B) 1 C) x^{-1} D) x^{-2} E) x^{-3}

Çözüm / Solution Yanıt / Answer **D**

$$\frac{\frac{1}{x^3}(x^2+4x+4)}{(x-2)\cdot(x+2)} \cdot \frac{x(x-2)}{x+2} = \frac{1}{x^2} = x^{-2}$$

Örnek / Example 41

$$\frac{x^3-y^3}{x^2+xy+y^2} : \frac{x^2-y^2}{x+y} = ?$$

A) 1 B) $x-y$ C) $x+y$ D) $\frac{1}{x-y}$ E) $\frac{1}{x+y}$

Çözüm / Solution

Yanıt / Answer **A**

$$\frac{(x-y)\cdot(x^2+xy+y^2)}{x^2+xy+y^2} \cdot \frac{x+y}{(x-y)(x+y)} = 1$$

Örnek / Example 42

$$\frac{a^3b+2a^2b^2+ab^3}{a+b} \cdot \frac{a-b}{a^2-b^2} = ?$$

A) $a+b$ B) $a-b$ C) ab D) $\frac{1}{ab}$ E) $\frac{1}{a+b}$

Çözüm / Solution

Yanıt / Answer **C**

$$\frac{ab(a^2+2ab+b^2)}{a+b} \cdot \frac{a-b}{(a-b)(a+b)}$$

$$= \frac{ab\cdot(a+b)^2}{a+b} \cdot \frac{1}{a+b}$$

$$= ab$$

1. $\left.\begin{matrix} x+y=5 \\ x\cdot y=3 \end{matrix}\right\} \Rightarrow x^2+y^2+2=?$

A) 15 B) 17 C) 19 D) 21 E)23

2. $\left.\begin{matrix} x+y=4 \\ x\cdot y=2 \end{matrix}\right\} \Rightarrow x^3+y^3=?$

A) 36 B) 40 C) 44 D) 48 E) 52

3. $\frac{3}{a-2}+\frac{2a+4}{a^2-4}=?$

A) $\frac{3}{a+2}$ B) $\frac{2}{a+2}$ C) $\frac{5}{a-2}$ D) $\frac{2}{a-2}$ E) $\frac{3}{a-2}$

4. $\frac{x^2-a^2}{a^2x-ax^2}=?$

A) $\frac{1}{ax}$ B) $\frac{x}{a}$ C) $\frac{x-a}{ax}$ D) $\frac{-x-a}{ax}$ E) $\frac{x+a}{ax}$

5. $\frac{a^3-a^2+a-1}{a^2-a}=?$

A) $\frac{a^2+1}{a}$ B) $\frac{a^2-1}{a}$ C) $\frac{a}{a-1}$ D) $\frac{a}{a+1}$ E) $\frac{a^2+1}{a-1}$

6. $\frac{1-a}{a}+\frac{a}{a+1}=?$

A) $\frac{a-1}{a}$ B) $\frac{a}{a-1}$ C) $\frac{a}{a+1}$ D) $\frac{1}{a\cdot(a-1)}$ E) $\frac{1}{a\cdot(a+1)}$

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7. $\dfrac{x^2-\frac{1}{4}}{x-\frac{1}{2}}-\dfrac{1}{2}=?$

A) x B) $\frac{x}{2}$ C) $\frac{1}{2}$

D) 2x E) –x

8. $\dfrac{x}{\frac{1}{x}+1}+\dfrac{x}{x+1}=?$

A) x – 1 B) x + 1 C) x

D) 1 E) – x

9. $\dfrac{ax-1}{abx^2-(a+b)x+1}=?$

A) $-\dfrac{1}{bx-1}$ B) $\dfrac{1}{ax+1}$ C) $\dfrac{1}{ax-1}$

D) $\dfrac{1}{bx-1}$ E) $\dfrac{1}{bx+1}$

10. $2a+3-\dfrac{2a^2+3a-9}{2a-3}=?$

A) 1 B) a C) –a

D) $\dfrac{a}{3-2a}$ E) $\dfrac{2}{3-2a}$

11. $\left(\dfrac{a+3}{a-2}-\dfrac{3-a}{2-a}\right)\cdot(4-a^2)=?$

A) –6a + 2 B) – 6(a – 2) C) 6(a – 2)

D) – 6(a + 2) E) 6(a + 2)

12. $\dfrac{2ax^3-8a^3x}{3ax^2-6a^2x}=?$

A) $\dfrac{2(x-2a)}{a}$ B) $\dfrac{x+2a}{3x}$ C) $\dfrac{x-2a}{3x-a}$

D) $\dfrac{2(x-2a)}{3(x+a)}$ E) $\dfrac{2(x+2a)}{3}$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
D	B	C	D	A	E	A	C	D	B	D	E

1. $x^2 + y^2 = (x + y)^2 - 2xy$

$x^2 + y^2 = 5^2 - 2 \cdot 3$

$x^2 + y^2 = 19$

$x^2 + y^2 + 2 = 19 + 2 = 21$

Yanıt / The Answer is D

---

2. $x^3 + y^3 = (x + y)^3 - 3xy \cdot (x + y)$

$x^3 + y^3 = 4^3 - 3 \cdot 2 \cdot (4)$

$x^3 + y^3 = 64 - 24 = 40$

Yanıt / The Answer is B

---

3. $\frac{3}{a-2} + \frac{2a+4}{a^2-4} = \frac{3}{a-2} + \frac{2(a+2)}{(a-2)(a+2)}$

$= \frac{3}{a-2} + \frac{2}{a-2}$

$= \frac{5}{a-2}$

Yanıt / The Answer is C

---

4. $\frac{x^2-a^2}{a^2x-ax^2} = \frac{(x-a)(x+a)}{ax \cdot (a-x)}$

$= \frac{-(a-x) \cdot (x+a)}{ax \cdot (a-x)}$

$= \frac{-x-a}{ax}$

Yanıt / The Answer is D

---

5. $\frac{a^3-a^2+a-1}{a^2-a} = \frac{a^2 \cdot (a-1)+(a-1)}{a \cdot (a-1)}$

$= \frac{(a-1) \cdot (a^2+1)}{a \cdot (a-1)}$

$= \frac{a^2+1}{a}$

Yanıt / The Answer is A

---

6. $\frac{1-a}{\underset{(a+1)}{a}} + \frac{a}{\underset{(a)}{a+1}} = \frac{(1-a)(a+1)+a^2}{a \cdot (a+1)}$

$= \frac{1-a^2+a^2}{a \cdot (a+1)}$

$= \frac{1}{a \cdot (a+1)}$

Yanıt / The Answer is E

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7. $\dfrac{x^2-\frac{1}{4}}{x-\frac{1}{2}}-\dfrac{1}{2}=\dfrac{\left(x-\frac{1}{2}\right)\cdot\left(x+\frac{1}{2}\right)}{x-\frac{1}{2}}-\dfrac{1}{2}=x+\dfrac{1}{2}-\dfrac{1}{2}=x$

Yanıt / The Answer is A

---

8. $\dfrac{x}{\frac{1}{x}+1}+\dfrac{x}{x+1}=\dfrac{x}{\frac{1+x}{x}}+\dfrac{x}{x+1}$

$=\dfrac{x^2}{1+x}+\dfrac{x}{x+1}$

$=\dfrac{x^2+x}{x+1}$

$=\dfrac{x\cdot(x+1)}{x+1}$

$=x$

Yanıt / The Answer is C

---

9. $abx^2-(a+b)x+1=(ax-1)(bx-1)$

$\downarrow \qquad \downarrow$

$ax \qquad -1$

$bx \qquad -1$

$\dfrac{ax-1}{abx^2-(a+b)x+1}=\dfrac{ax-1}{(ax-1)(bx-1)}=\dfrac{1}{bx-1}$

Yanıt / The Answer is D

---

10. $2a^2+3a-9=(2a-3)(a+3)$

$\downarrow \qquad \downarrow$

$2a \qquad -3$

$a \qquad +3$

$2a+3-\dfrac{2a^2+3a-9}{2a-3}=2a+3-\dfrac{(2a-3)(a+3)}{2a-3}$

$=2a+3-(a+3)$

$=a$

Yanıt / The Answer is B

---

11. $\left(\dfrac{a+3}{a-2}-\dfrac{3-a}{2-a}\right)\cdot(4-a^2)=\left(\dfrac{a+3}{a-2}+\dfrac{3-a}{a-2}\right)\cdot(4-a^2)$

$=\left(\dfrac{a+3+3-a}{a-2}\right)\cdot(4-a^2)$

$=\dfrac{6}{a-2}\cdot(2-a)(2+a)$

$=\dfrac{-6\cdot(a-2)(a+2)}{a-2}$

$=-6(a+2)$

Yanıt / The Answer is D

---

12. $\dfrac{2ax^3-8a^3x}{3ax^2-6a^2x}=\dfrac{2ax\cdot(x^2-4a^2)}{3ax\cdot(x-2a)}$

$=\dfrac{2\cdot(x-2a)(x+2a)}{3\cdot(x-2a)}$

$=\dfrac{2(x+2a)}{3}$

Yanıt / The Answer is E

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1. $n^2 + n - 11 = 0 \Rightarrow n^3 + 7n^2 = ?$

A) $6n + 65$ B) $7n + 77$ C) $5n + 44$

D) $3n + 45$ E) $5n + 66$

[GAZİANTEP ÜNİVERSİTESİ – YÖS 2020]

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2. $y \neq \mp 1, x \neq 0$

$$\left(\frac{x - \frac{x}{y}}{y - \frac{1}{y}}\right) : \left(\frac{x}{y^2 - 1}\right) + 1 = ?$$

Üstteki ifadenin en sade şekli nedir?

What is the simpliest form of the expression above?

A) y B) -1 C) $1 - y$

D) 1 E) y^2

[KARABÜK ÜNİVERSİTESİ – YÖS 2020]

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3. $x^2 + x + 1 = 0$ **ise** $x^{2013} + x^{2023} = ?$

If $x^2 + x + 1 = 0$, then $x^{2013} + x^{2023} = ?$

A) $x - 1$ B) $x + 1$ C) $x^2 + 1$

D) $x^2 - 1$ E) $x^2 + x + 1$

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2020]

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4. $2^{20} + 2^{11} - 24$ **ifadesi aşağıdakilerden hangisi ile tam bölünür?**

Which one of the following divides $2^{20} + 2^{11} - 24$ without remainder?

A) $2^{10} - 6$ B) $2^{10} - 2$ C) $2^{10} + 2$

D) $2^{10} + 4$ E) $2^{10} + 6$

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2020]

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5. $\left.\begin{matrix} x - 9y = 40 \\ \sqrt{x} - 3\sqrt{y} = 2 \end{matrix}\right\} \Rightarrow x + y = ?$

A) 76 B) 84 C) 96 D) 116 E) 130

[EGE ÜNİVERSİTESİ – YÖS 2019]

---

6. $x \neq y$

$$\frac{x^2}{y} - \frac{y^2}{x} = y - x \Rightarrow \frac{x}{y} + \frac{y}{x} = ?$$

A) -2 B) -1 C) 0 D) 1 E) 4

[İSTANBUL MEDENİYET ÜNİVERSİTESİ – YÖS 2019]

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7. $\frac{x^3 - 1}{x^2 - 1} : \left(\frac{1}{x} + x + 1\right) = ?$

A) $\frac{x^2}{x+1}$ B) $\frac{x}{x^2+x+1}$ C) $\frac{x}{x-1}$

D) $\frac{x}{x+1}$ E) $\frac{x^2}{x-1}$

[DUMLUPINAR ÜNİVERSİTESİ – YÖS 2018]

---

8. $\frac{xy^2 - 4x - 3y^2 + 12}{xy - 3y - 2x + 6} + \frac{xy - 2x - y + 2}{x - 1} = ?$

A) $2y$ B) $2(y - x)$ C) $2y - 4$

D) $2y + 4$ E) $2x - 4$

[İSTANBUL ÜNİVERSİTESİ – YÖS 2018]

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9. $\left(1-\frac{a}{a+b}\right):\left(1-\frac{b}{a+b}\right)=?$

A) $\frac{a+b}{b}$ B) $\frac{a+b}{a}$ C) $\frac{a}{b}$ D) $\frac{b}{a}$ E) 1

[BALIKESİR ÜNİVERSİTESİ – YÖS 2017]

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10. $\left(\frac{1+k}{1-k}-\frac{1-k}{1+k}\right)\cdot\left(k-\frac{1}{k}\right)=?$

A) –2 B) –4 C) 1 D) 2 E) 4

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2017]

---

11. $a = 2013$

$b = 2015$

$\Rightarrow a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5 = ?$

A) –32 B) –21 C) 0 D) 16 E) 20

[ANKARA ÜNİVERSİTESİ – YÖS 2016]

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12. $x > y$

$x^2 + y^2 = \sqrt{5}$

$xy = 1$

$\Rightarrow x^4 - y^4 = ?$

A) 1 B) $\sqrt{3}$ C) $\sqrt{5}$ D) $\sqrt{6}$ E) 3

[İSTANBUL ÜNİVERSİTESİ – YÖS 2015]

---

13. $m - 2n = 2$

$\Rightarrow m^2 + 3m - 4mn + 4n^2 - 6n = ?$

A) 6 B) 8 C) 10 D) 12 E) 14

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2015]

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14. $\frac{x^2-2x-3}{6x^2+7x-3}:\frac{x^2-x-2}{2x^2-x-6}=?$

A) $\frac{x+2}{x-3}$ B) $\frac{x+3}{3x-1}$ C) $\frac{x-2}{3x+1}$ D) $\frac{x-3}{3x-1}$ E) $\frac{x-1}{3x-1}$

[İSTANBUL ÜNİVERSİTESİ – YÖS 2014]

---

15. $\frac{(x+2y)^2-8xy}{(x-y)^2-y^2}=?$

A) $1+\frac{2y}{x}$ B) $1+\frac{x}{2y}$ C) $1-\frac{x}{2y}$ D) $1-\frac{2y}{x}$ E) $x-\frac{1}{y}$

[AKDENİZ ÜNİVERSİTESİ – YÖS 2014]

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16. $x + y = -2$

$a - b = -1$

$\Rightarrow bx + by - ax - ay = ?$

A) 1 B) 2 C) 3 D) –2 E) –3

[AKDENİZ ÜNİVERSİTESİ – YÖS 2013]

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	A	B	E	E	A	D	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	B	A	C	C	D	D	D

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1. $n^2 + n - 11 = 0 \Rightarrow n^2 = 11 - n$

$n^3 + 7n^2 = n^2(n + 7)$

$= (11 - n)\cdot(n + 7)$

$= -n^2 + 4n + 77$

$= n - 11 + 4n + 77$

$= 5n + 66$

Yanıt / The Answer is E

---

2. $\left(\dfrac{x - \frac{x}{y}}{y - \frac{1}{y}}\right) : \left(\dfrac{x}{y^2 - 1}\right) + 1 = \dfrac{\frac{xy - x}{y}}{\frac{y^2 - 1}{y}} \cdot \dfrac{y^2 - 1}{x} + 1$

$= \dfrac{x(y - 1)}{y^2 - 1} \cdot \dfrac{y^2 - 1}{x} + 1$

$= y - 1 + 1 = y$

Yanıt / The Answer is A

---

3. $x^2 + x + 1 = 0$

$(x - 1)\cdot(x^2 + x + 1) = 0$

$x^3 - 1 = 0$

$x^3 = 1$

$x^{2013} + x^{2023} = (x^3)^{671} + (x^3)^{674}\cdot x$

$= 1^{671} + 1^{674}\cdot x$

$= 1 + x$

Yanıt / The Answer is B

---

4. $2^{20} + 2^{11} - 24 = (2^{10})^2 + 2\cdot 2^{10} + 1 - 25$

$= (2^{10} + 1)^2 - 25$

$= (2^{10} + 1)^2 - 5^2$

$= (2^{10} + 1 - 5)\cdot(2^{10} + 1 + 5)$

$= (2^{10} - 4)\cdot(2^{10} + 6)$

Yanıt / The Answer is E

---

5. $\left.\begin{matrix} x - 9y = 40 \\ \sqrt{x} - 3\sqrt{y} \end{matrix}\right\} \Rightarrow x - 9y = \underbrace{(\sqrt{x} - 3\sqrt{y})}_{2}\cdot\underbrace{(\sqrt{x} + 3\sqrt{y})}_{20} = 40$

$\sqrt{x} - 3\sqrt{y} = 2$

$\Rightarrow \underline{+\ \sqrt{x} + 3\sqrt{y} = 20}$

$2\sqrt{x} = 22$

$\Rightarrow \sqrt{x} = 11$

$\Rightarrow x = 121,\ y = 9$

$\Rightarrow x + y = 130$

Yanıt / The Answer is E

---

6. $\underset{(x)}{\dfrac{x^2}{y}} - \underset{(y)}{\dfrac{y^2}{x}} = y - x$

$\dfrac{x^3 - y^3}{xy} = y - x$

$(x - y)\cdot(x^2 + xy + y^2) = xy\cdot(y - x)$

$x^2 + xy + y^2 = -xy \Rightarrow x^2 + y^2 = -2xy$

$\Rightarrow \underset{(x)}{\dfrac{x}{y}} + \underset{(y)}{\dfrac{y}{x}} = \dfrac{x^2 + y^2}{xy} = \dfrac{-2xy}{xy}$

$= -2$

Yanıt / The Answer is A

---

7. $\dfrac{(x-1)(x^2+x+1)}{(x-1)(x+1)} : \dfrac{1+x^2+x}{x} =$

$\dfrac{x^2+x+1}{x+1} \cdot \dfrac{x}{x^2+x+1} = \dfrac{x}{x+1}$

Yanıt / The Answer is D

---

8. $\dfrac{xy^2 - 4x - 3y^2 + 12}{xy - 3y - 2x + 6} + \dfrac{xy - 2x - y + 2}{x - 1}$

$= \dfrac{y^2(x-3) - 4(x-3)}{x(y-2) - 3(y-2)} + \dfrac{y(x-1) - 2(x-1)}{x-1}$

$= \dfrac{(x-3)(y-2)(y+2)}{(y-2)\cdot(x-3)} + \dfrac{(x-1)(y-2)}{x-1}$

$= y + 2 + y - 2 = 2y$

Yanıt / The Answer is A

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9. $\left(1-\frac{a}{a+b}\right):\left(1-\frac{b}{a+b}\right)$

$=\frac{a+b-a}{a+b}\cdot\frac{a+b}{a+b-b}=\frac{b}{a}$

Yanıt / The Answer is D

---

10. $=\frac{k^2+2k+1-(1-2k+k^2)}{(1-k)(1+k)}\cdot\frac{k^2-1}{k}$

$=\frac{4k}{1-k^2}\cdot\frac{k^2-1}{k}=-4$

Yanıt / The Answer is B

---

11. $a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5=(a-b)^5$

$=(2013-2015)^5$

$=(-2)^5$

$=-32$

Yanıt / The Answer is A

---

12. $x^2+y^2=(x+y)^2-2xy$

$\sqrt{5}=(x+y)^2-2$

$\sqrt{5}+2=(x+y)^2 \Rightarrow x+y=\sqrt{\sqrt{5}+2}$

$x^2+y^2=(x-y)^2+2xy$

$\sqrt{5}=(x-y)^2+2$

$\sqrt{5}-2=(x-y)^2 \Rightarrow x-y=\sqrt{\sqrt{5}-2}$

$x^4-y^4=(x^2-y^2)\cdot(x^2+y^2)$

$=\sqrt{\sqrt{5}+2}\cdot\sqrt{\sqrt{5}-2}\cdot\sqrt{5}$

$=\sqrt{5-4}\cdot\sqrt{5}$

$=\sqrt{5}$

Yanıt / The Answer is C

13. $m^2+3m-4mn+4n^2-6n=(m-2n)^2+3\cdot(m-2n)$

$=(m-2n)\cdot(m-2n+3)$

$=2\cdot(2+3)$

$=2\cdot5$

$=10$

Yanıt / The Answer is C

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14. $\frac{x^2-2x-3}{6x^2+7x-3}:\frac{x^2-x-2}{2x^2-x-6}=\frac{(x-3)(x+1)}{(2x+3)(3x-1)}\cdot\frac{(2x+3)(x-2)}{(x-2)(x+1)}$

$=\frac{x-3}{3x-1}$

Yanıt / The Answer is D

---

15. $\frac{(x+2y)^2-8xy}{(x-y)^2-y^2}=\frac{x^2+4xy+4y^2-8xy}{x^2-2xy+y^2-y^2}$

$=\frac{x^2-4xy+4y^2}{x^2-2xy}$

$=\frac{(x-2y)^2}{x^2-2xy}$

$=\frac{x-2y}{x}=1-\frac{2y}{x}$

Yanıt / The Answer D

---

16. $bx+by-ax-ay$

$b(x+y)-a(x+y)$

$(x+y)\cdot(b-a)=$

$(-2)\cdot(1)=-2$

Yanıt / The Answer is D

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1. $x - \frac{1}{x} = 3 \Rightarrow x^2 + \frac{1}{x^2} = ?$

A) 10 B) 11 C) 12 D) 13 E)14

2. $x^2 + y^2 = 74$,
$x + y = 12$
$\Rightarrow x \cdot y = ?$

A) 35 B) 70 C) 109 D) 140 E)147

3. $x > y$,
$x^2 + y^2 = 17$,
$x \cdot y = 4 \Rightarrow x - y = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

4. $b > a,\ b \neq 0$
$\frac{12}{b} = a,\ a^2 + b^2 = 25 \Rightarrow b - a = ?$

A) 6 B) 4 C) 1 D) –1 E) –4

5. $\left.\begin{matrix} a - b = 6 \\ \frac{1}{a} - \frac{1}{b} = 2 \end{matrix}\right\} \Rightarrow a \cdot b = ?$

A) –6 B) –3 C) 0 D) 12 E) 15

6. $\left.\begin{matrix} x + y = 8 \\ x \cdot y = 8 \end{matrix}\right\} \Rightarrow x^2 + y^2 = ?$

A) 18 B) 28 C) 38 D) 48 E)58

7. $a + b = 11$,
$c = 5 \Rightarrow a^2 - c^2 + 2ab + b^2 = ?$

A) 56 B) 69 C) 96 D) 102 E)112

8. $\left.\begin{matrix} a - b = 10 \\ a \cdot b = -15 \end{matrix}\right\} \Rightarrow a^2 + b^2 = ?$

A) 80 B) 70 C) 60 D) 45 E) 35

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9. $\left.\begin{matrix} x-y=17 \\ x^2-y^2=17 \end{matrix}\right\} \Rightarrow \mathbf{x^2+y^2=?}$

A) 145 B) 143 C) 140
D) 124 E) 115

10. $\left.\begin{matrix} x-y=3 \\ x\cdot y=2 \end{matrix}\right\} \Rightarrow \mathbf{x^3-y^3=?}$

A) 5 B) 15 C) 25
D) 35 E) 45

11. $a+2b=5$,
$a\cdot b=2 \Rightarrow \mathbf{a^3+8b^3=?}$

A) 28 B) 36 C) 49
D) 65 E) 82

12. $a+b=11$,
$a-b=6 \Rightarrow \mathbf{a^2-b^2+a+b=?}$

A) 17 B) 33 C) 48
D) 56 E)77

13. $(x+y)^2+x^2+4x+4=0$
$\Rightarrow \mathbf{x\cdot y=?}$

A) –6 B) –4 C) 4
D) 6 E) 8

14. $a^2+b^2=72$
$a+b=10 \Rightarrow \mathbf{a\cdot b=?}$

A) 8 B) 10 C) 12
D) 14 E) 16

15. $x^3-y^3=11$
$xy^2-x^2y=-1 \Rightarrow \mathbf{x-y=?}$

A) –4 B) –2 C) 2
D) 4 E) 6

16. $x^3+y^3=2$
$x+y=2 \Rightarrow \frac{1}{x}+\frac{1}{y}=?$

A) 2 B) 3 C) 4
D) 5 E) 6

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	A	C	C	B	D	C	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	E	D	E	B	D	C	A

1. $61^2 - 60^2 = ?$

A) 121 B) 241 C) 660

D) 1001 E) 3599

2. $(3x^2 - 3)^2 - (2x^2 - 2)^2 = ?$

A) 5 B) $x - 1$ C) $x + 1$

D) $x^2 + 1$ E) $5(x^2 - 1)^2$

3. $x > 0$,

$x^2 + \frac{1}{x^2} = \frac{17}{4} \Rightarrow x + \frac{1}{x} = ?$

A) $\frac{2}{5}$ B) $\frac{5}{2}$ C) $\frac{5}{6}$

D) $\frac{5}{3}$ E) $\frac{13}{6}$

4. $x, y \in Z^+$,

$x^2 - y^2 = 19 \Rightarrow 2x - y = ?$

A) 11 B) 13 C) 17

D) 21 E) 30

5. $x \in R^+$

$$\left.\begin{array}{l} x + y = 4 \\ \frac{1}{x} + \frac{1}{y} = 2 \end{array}\right\} \Rightarrow x \cdot y = ?$$

A) 0 B) 1 C) 2

D) 3 E) 4

6. $x > y$,

$$\left.\begin{array}{l} x^2 - xy = 13 \\ y^2 - xy = 12 \end{array}\right\} \Rightarrow x - y = ?$$

A) 6 B) 5 C) 3

D) 2 E) 1

7. $a > 0$, $a - \frac{1}{a} = \sqrt{3} \Rightarrow a + \frac{1}{a} = ?$

A) $\sqrt{7}$ B) $\sqrt{6}$ C) $2\sqrt{6}$

D) $2\sqrt{7}$ E) $3\sqrt{7}$

8. $a, b \in R^+$,

$$\left.\begin{array}{l} a^2 + ab = 21 \\ ab + b^2 = 15 \end{array}\right\} \Rightarrow a + b = ?$$

A) 2 B) 3 C) 4

D) 5 E) 6

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9. $x > y$,

$$\left.\begin{matrix} x^2 - xy = 3 \\ xy - y^2 = 2 \end{matrix}\right\} \Rightarrow x - y = ?$$

A) 0 B) 1 C) 2

D) 3 E) 4

10. $$\left.\begin{matrix} x^3 - 3x^2y = 65 \\ 3xy^2 - y^3 = 60 \end{matrix}\right\} \Rightarrow x - y = ?$$

A) 3 B) 4 C) 5

D) 6 E) 7

11. $$\left.\begin{matrix} a = x^3 - 3x^2y \\ a = y^3 - 3y^2x \end{matrix}\right\} \Rightarrow x + y = ?$$

A) x B) 2x C) 3x

D) 4x E) 5x

12. $\left(\frac{a}{2} - \frac{2}{a}\right)^2 - \left(\frac{a}{2} + \frac{2}{a}\right)^2 = ?$

A) –1 B) –2 C) –4

D) –8 E) –12

13. $\frac{(99)^2 + 199}{(49)^2 + 99} = ?$

A) 2 B) 4 C) 8

D) 10 E) 50

14. $\frac{(4,78)^2 - (3,22)^2}{0,39} = ?$

A) 8 B) 16 C) 32

D) 48 E) 64

15. $\frac{(102)^2 - (98)^2}{(12)^2 - 8^2} = ?$

A) 10 B) 12 C) 14

D) 16 E) 18

16. $\frac{(99)^3 - (99)^2 - 98}{(99)^2 - 1} = ?$

A) 96 B) 97 C) 98

D) 99 E) 100

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	E	B	A	C	B	A	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	C	B	C	B	C	A	C

1. $\frac{x}{x+y}+\frac{y}{x-y}=?$

A) $\frac{x-y}{x+y}$ B) $\frac{x^2+y^2}{x^2-y^2}$ C) $\frac{x}{y}$

D) $\frac{y}{x}$ E) x^2+y^2

2. $\frac{a^2-b^2}{4a^2+4ab}=?$

A) $\frac{a-b}{4a}$ B) $\frac{a+b}{a-b}$ C) $\frac{a+b}{2(a-b)}$

D) $\frac{a+b}{5a}$ E) $\frac{a+b}{4a}$

3. $20x^2-19x+3=?$

A) (4x + 3) (5x – 1) B) (4x – 3) (5x – 1)

C) (4x + 3) (5x + 1) D) (5x + 3) (4x + 1)

E) (20x + 1) (x + 3)

4. $\left(\frac{2x^2-x-3}{x^2-1}\right)\cdot\left(1-\frac{1}{x}\right)=?$

A) $\frac{x+1}{x}$ B) $2-\frac{3}{x}$ C) $3-\frac{1}{x}$

D) 2x+1 E) 2x+3

5. $\frac{a^2}{(a-b)^2}-\frac{a}{a-b}=?$

A) $\frac{b}{(a-b)^2}$ B) $\frac{a+b}{a-b}$ C) $\frac{a-b}{a+b}$

D) $\frac{ab}{(a-b)^2}$ E) $\frac{a^2}{(a-b)^2}$

6. $\left(\frac{1}{x}+\frac{1}{y}\right):\left(\frac{x^2-y^2}{xy}\right)=?$

A) x – y B) x + y C) $\frac{1}{x-y}$

D) $\frac{1}{x+y}$ E) $\frac{x-y}{xy}$

7. $\frac{a^2}{a-b}+\frac{b^2}{b-a}=?$

A) 2a B) b C) a – b

D) $\frac{a+b}{b-a}$ E) a + b

8. $\left(a-\frac{b^2}{a}\right):\left(1+\frac{b}{a}\right)=?$

A) 1 B) a C) b

D) a – b E) $\frac{a}{b}$

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9. $\left(1-\frac{5}{x}\right):\left(1-\frac{25}{x^2}\right)=?$

A) x B) x – 5 C) $\frac{x}{x+5}$

D) $\frac{x}{x-5}$ E) $\frac{x-5}{x+5}$

10. $\frac{x^2-2x-3}{x-3}=?$

A) x – 1 B) x + 2 C) x +1

D) x – 3 E) x + 4

11. $\frac{2a}{a^2-b^2}+\frac{2b}{b^2-a^2}=?$

A) $\frac{2}{a-b}$ B) $\frac{1}{a-b}$ C) $\frac{1}{b+a}$

D) $\frac{2}{b+a}$ E) $\frac{3}{b+a}$

12. $\frac{x^3y-y^3x}{x^2y-xy^2}=?$

A) x – y B) x C) y

D) x + y E) $\frac{1}{x+y}$

13. $\frac{x+\frac{2x}{x-2}}{1+\frac{4}{x^2-4}}=?$

A) x B) x + 1 C) x + 2

D) $\frac{1}{x}$ E) $\frac{1}{x+1}$

14. $m=1-3^{-1} \Rightarrow \frac{mx}{x-2y}+\frac{2my}{2y-x}=?$

A) $\frac{-1}{3}$ B) $\frac{-2}{3}$ C) $\frac{1}{3}$

D) $\frac{2}{3}$ E) $\frac{4}{3}$

15. $\left(\frac{3}{x}-\frac{6}{x+3}\right)\cdot\left(3+\frac{x^2+9}{x-3}\right)=?$

A) –3 B) –3x C) x – 3

D) 3x E) 3

16. $a=5-b \Rightarrow \frac{a^3+a^2b-ab^2-b^3}{a^2-b^2}=?$

A) –5 B) –3 C) 0

D) 3 E) 5

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	A	B	B	D	C	E	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	C	D	D	C	D	A	E

1. $3x^2y - 6x^2y^2 - 9xy^3 = ?$

A) $3y\,(x^2 - 2x^2y - 3y^2)$

B) $3xy\,(x - 2xy - 3y^2)$

C) $2xy\,(x - 2y - 3y^2)$

D) $3xy\,(x^2 + 2xy + 3x^2)$

E) $3xy\,(x - 2y + y^2)$

2. $(a^2 + 5a - 14) : \dfrac{a^2 - 4}{5a} = ?$

A) $\dfrac{5a(a+7)}{a+2}$ B) $\dfrac{a+2}{5a}$ C) $\dfrac{5a(a+2)}{a-2}$

D) $\dfrac{5a}{a+2}$ E) $\dfrac{a+7}{a+2}$

3. $\dfrac{(2x-1)^2 - x^2}{3x^2 - 4x + 1} = ?$

A) 1 B) $x - 1$ C) $x + 1$

D) $\dfrac{x-1}{3}$ E) $\dfrac{x-1}{x+1}$

4. $\dfrac{4x^2 - y^2 - 4x + 1}{4x^2 - y^2 - 2y - 1} = ?$

A) $\dfrac{2x+y+1}{2x-y-1}$ B) $\dfrac{2x-y-1}{2x+y+1}$ C) $\dfrac{2x+y-1}{2x+y+1}$

D) $\dfrac{2x+y+1}{2x+y-1}$ E) $\dfrac{2x-y-1}{2x+y-1}$

5. $a^2 + 2bc - b^2 - c^2 = ?$

A) $(a - b - c)\,(a - b + c)$

B) $(a - b - c)\,(a + b + c)$

C) $(a - b + c)\,(a + b - c)$

D) $(a + b)\,(a - b + c)$

E) $(a + b)\,(a + b + c)$

6. $\left(\dfrac{x^2+xy}{xy+y^2} - \dfrac{xy-y^2}{x^2-xy}\right) : \left(\dfrac{1}{y} - \dfrac{1}{x}\right) = ?$

A) x B) $x - y$ C) y

D) $x + y$ E) $\dfrac{x-y}{x+y}$

7. $\dfrac{6x^2 - 13x - 5}{4x^2 - 25} = ?$

A) $\dfrac{2x+3}{2x-5}$ B) $3x + 1$ C) $\dfrac{3x-1}{2x-5}$

D) $\dfrac{2x+1}{x-5}$ E) $\dfrac{3x+1}{2x+5}$

8. $\dfrac{x^2 + 2x - 3}{x^2 + 3x} : \dfrac{x^2 - 4x + 3}{x^3 - 9x} = ?$

A) $x + 3$ B) x C) $x - 3$

D) $x - 1$ E) $\dfrac{x+3}{x}$

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9. $\frac{x^3+27}{x^2-9} : \frac{x^2-3x+9}{x^2-3x} = ?$

A) 1 B) x – 3 C) x

D) x + 3 E) $\frac{x}{x+3}$

10. $\frac{a^2+2a-3}{a^3+5a^2+6a} : \frac{a^2-3a+2}{a^3-4a} = ?$

A) 1 B) a + 1 C) –1

D) a^2 E) $\frac{1}{a}$

11. $\frac{a^3-a^2}{3(a+1)} : \frac{a^2-1}{(a^2+a)^2} = ?$

A) $\frac{1}{3}$ B) $\frac{a^2}{3}$ C) $\frac{a^4}{3}$

D) $\frac{a}{3}$ E) $\frac{1}{a}$

12. $\left(\frac{x+1}{x-1} - \frac{x-1}{x+1}\right) \cdot \left(x - \frac{1}{x}\right) = ?$

A) x B) $\frac{1}{x+1}$ C) $\frac{4}{x+1}$

D) 4 E) 8

13. $\frac{x^2-x-12}{x^2-16} : \frac{x+3}{x+4} = ?$

A) x – 2 B) x – 1 C) 1

D) x + 1 E) x + 2

14. $\frac{4x^2+14x+6}{x+3} : \frac{4x+2}{x-1} = ?$

A) 2(x – 1) B) x – 1 C) 1

D) x E) 2x

15. $\frac{4x^3+16x^2}{4x^2+12x} : \frac{x^3-16x}{x^2-x-12} = ?$

A) –2 B) x – 1 C) –1

D) x + 1 E) 1

16. $\left(\frac{a}{a+b} + \frac{a-b}{a}\right) : \left(\frac{a}{a-b} + \frac{a+b}{a}\right) = ?$

A) a + b B) a – b C) $\frac{a+b}{a-b}$

D) $\frac{a-b}{a+b}$ E) 1

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	A	A	C	C	D	E	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	A	C	D	C	B	E	D

1. $\dfrac{(x+y)^2-4(x+y)}{(x+y)^2-16} = ?$

A) $\frac{1}{2}$ B) $\frac{x+y}{x+y+2}$ C) $\frac{3}{7}$

D) $\frac{x+y}{x+y+4}$ E) $\frac{6}{7}$

2. $a+b=1 \Rightarrow \dfrac{a^2-3a+2}{a+ab-b-1} = ?$

A) -1 B) $a-b$ C) $2b$

D) 1 E) 2

3. $a+c=3$, $b+2=0$

$\Rightarrow \dfrac{a+b-c}{a+b+c} : (a^2-b^2-c^2+2bc) = ?$

A) 6 B) -3 C) 1

D) $\frac{1}{3}$ E) $\frac{1}{5}$

4. $\dfrac{a^3-a^2b+b^3-ab^2}{a^2-2ab+b^2} = ?$

A) $\frac{a}{b}$ B) b C) a

D) $a-b$ E) $a+b$

5. $\dfrac{(x-2)^3}{x^3-8} : \dfrac{x^2-4x+4}{x^2+2x+4} = ?$

A) 1 B) $x-2$ C) x^2-4

D) $(x-2)^2$ E) $x+2$

6. $\dfrac{x^2+x-6}{x^3-8} : \dfrac{2x^2+6x}{x^2+2x+4} = ?$

A) $2x$ B) $\frac{1}{2x}$ C) $x+3$

D) $x(x+2)$ E) x^2-2x

7. $\dfrac{a^3-9a-a^2b+9b}{a^2-ab-3a+3b} = ?$

A) $a+3$ B) $a-3$ C) $3-a$

D) $a-9$ E) $a+9$

8. $\dfrac{a^4b-ab^4}{a^3b+a^2b^2+ab^3} : \dfrac{a^2-3ab+2b^2}{4b^2-a^2} = ?$

A) $a+b$ B) $2a-b$ C) $-a-2b$

D) $a-3b$ E) $a-b$

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9. $\frac{m^3+m^2n+mn^2}{m^3+mn^2} : \frac{m^3-n^3}{m^4-n^4} = ?$

A) $m-n$ B) m^2+n^2 C) n

D) n E) $m+n$

10. $\frac{a^3-16a-a^2b+16b}{a^2-ab-4a+4b} = ?$

A) $a+4$ B) $a-4$ C) $16-a$

D) $a+16$ E) a^2-16

11. $\frac{a^3-b^3}{a^2+ab+b^2} : \frac{a^2+ab-2b^2}{a^2+2ab} = ?$

A) 1 B) a C) b

D) $a+b$ E) $\frac{a-b}{a}$

12. $\frac{x^6-y^6}{x^4+x^2y^2+y^4} = ?$

A) x^2+y^2 B) x^2-y^2 C) x^3-y^3

D) $\frac{x^3+y^2}{x-y}$ E) x^2y^2

13. $\left.\begin{matrix} a+2b=1 \\ 2a-b=2 \end{matrix}\right\} \Rightarrow \frac{2a^2+3ab-2b^2}{\frac{a}{2}+b} = ?$

A) $\frac{3}{2}$ B) $\frac{5}{2}$ C) $\frac{7}{2}$

D) 4 E) 5

14. $\left.\begin{matrix} a+b=3 \\ x+y=2 \end{matrix}\right\} \Rightarrow ax+by+bx+ay = ?$

A) 4 B) 5 C) 6

D) 7 E) 9

15. $\frac{a^3+b^3}{(a+b)^2-3ab} \cdot \frac{a-b}{a^2-b^2} = ?$

A) $a-b$ B) $a+b$ C) $a \cdot b$

D) 1 E) 2

16. $a+b=\frac{1}{6}$

$\Rightarrow \frac{(a-b)^2+ab}{3a^3+3b^3} = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	A	E	E	A	B	A	C
9	10	11	12	13	14	15	16
E	A	B	B	D	C	D	B

1. $\left.\begin{array}{l} A = 3 - 3^x \\ B = 3 + 3^x \\ A \cdot B = 6 \end{array}\right\} \Rightarrow x = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1

D) 2 E)3

2. $2^x - 2^{-x} = 5\sqrt{5} \Rightarrow 4^x + 4^{-x} = ?$

A) 120 B) 121 C) 127

D) 123 E) 124

3. $\left.\begin{array}{l} a = \sqrt{3} - 1 \\ b = \sqrt{3} + 1 \end{array}\right\} \Rightarrow \frac{a}{b} + \frac{b}{a} = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

4. $\left.\begin{array}{l} \frac{1}{a} + \frac{1}{b} = \frac{6}{5} \\ a \cdot b = 10 \end{array}\right\} \Rightarrow a^2 + b^2 = ?$

A) 110 B) 117 C) 120

D) 121 E) 124

5. $x - \frac{1}{x} = 5 \Rightarrow 2x^2 + \frac{2}{x^2} = ?$

A) 90 B) 51 C) 52

D) 53 E) 54

6. $a - b = \frac{1}{5} \Rightarrow \frac{(a+b)^2 - ab}{a^3 - b^3} = ?$

A) $\frac{1}{5}$ B) $\frac{1}{3}$ C) 1

D) 3 E) 5

7. $\left(\frac{2x}{5} - \frac{5}{4}\right)^2 = ?$

A) $\frac{4x^2}{25} - x + \frac{25}{16}$ B) $\frac{2x^2}{25} - 2x + \frac{5}{16}$

C) $\frac{4x^2}{25} - 2x + \frac{25}{16}$ D) $\frac{4x^2}{5} - 4x + \frac{25}{4}$

E) $\frac{4x^2}{25} - 4x + \frac{25}{16}$

8. $x = 6{,}541$

$y = 3{,}459$

$\Rightarrow x^3 + 3x^2y + 3xy^2 + y^3 = ?$

A) 1 B) 10 C) 10^2

D) 10^3 E) 10^4

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9. $2x^2 + x - 1 = (ax - 1)(x + b)$

$\Rightarrow$ **a + b = ?**

A) 6 B) 5 C) 4

D) 3 E) 2

10. $\dfrac{2x^2 - xy - y^2}{x^2 - y^2} = ?$

A) $\dfrac{2x-y}{x-y}$ B) $\dfrac{2x+y}{x+y}$ C) $\dfrac{x+y}{x-y}$

D) $\dfrac{2x+y}{x-y}$ E) $\dfrac{2x+y}{x+2y}$

11. $\dfrac{x^2 + 2xy + y^2}{x^2 - y^2} : \dfrac{x+y}{4x-4y} = ?$

A) 1 B) $\dfrac{x+y}{x-y}$ C) $\dfrac{x-y}{x+y}$

D) 4 E) $\dfrac{2x+y}{x+2y}$

12. $\dfrac{3x^2 + xy - 2y^2}{2x^2 + 4xy + 2y^2} \cdot \dfrac{1}{2}(x+y) = ?$

A) 3x – 2y B) x – 2y C) $\dfrac{3x-2y}{2}$

D) $\dfrac{3x-2y}{4}$ E) $\dfrac{x+y}{4}$

13. $\dfrac{2a^2 + 3a - 2}{a^2 + 3a + 2} \cdot \dfrac{a+1}{1-2a} = ?$

A) –2 B) –1 C) a + 1

D) 1 E) 2

14. $(2a + b - 12)^2 + (a - b - 3)^2 = 0$

$\Rightarrow$ **a·b = ?**

A) 4 B) 6 C) 8

D) 10 E) 12

15. $(a - 3b + 4)^4 + (2a + b - 6)^6 = 0$

$\Rightarrow$ **a = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

16. $x^2 + y^2 + 4x + 4y + 8 = 0 \Rightarrow \dfrac{x}{y} = ?$

A) –2 B) –1 C) 1

D) 2 E) $\dfrac{1}{2}$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	C	D	E	E	E	A	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	B	D	D	B	D	B	C

1. $\frac{a}{b}+\frac{b}{a}=4 \Rightarrow \frac{a^2+b^2}{2ab}=?$

A) 1 B) 2 C) 3

D) 4 E) 5

2. $\frac{2a^5-2a^4-2a^3-2a^2}{a^4-a^3-a^2-a}=?$

A) a – 1 B) a C) 2a

D) a + 1 E) 2a + 1

3. $t^3-1=x \Rightarrow t^6-1=?$

A) x^2+x B) x^2-2x+1 C) x^2+2x

D) $2x^2+x$ E) x^2+2x+1

4. $\frac{4x^2-4y^2+x-y}{4x+4y+1}=?$

A) x^2+xy+y^2 B) x^2-y^2 C) $x-y$

D) $x+y$ E) x^2+y^2

5. $\left.\begin{matrix}2x+3y=8\\ z-t=5\end{matrix}\right\} \Rightarrow 2xz-2xt+3yz-3yt=?$

A) 37 B) 38 C) 39

D) 40 E) 41

6. $\frac{x^3}{5}+\frac{y^3}{5}=2x^2-2xy+2y^2 \Rightarrow x+y=?$

A) 4 B) 6 C) 8

D) 10 E) 12

7. $3x-\frac{1}{2x}=6 \Rightarrow 9x^2+\frac{1}{4x^2}=?$

A) 36 B) 37 C) 38

D) 39 E) 40

8. $\frac{4x-7}{x-3}+\frac{3x-4}{3-x}=?$

A) 1 B) 2 C) 3

D) 4 E) 5

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9. $\frac{a^3b^2c-a^2bc}{a^2b^2-ab}=4 \Rightarrow a=?$

A) $\frac{1}{c}$ B) $\frac{2}{c}$ C) $2c$

D) c E) $\frac{4}{c}$

10. $a>1$,

$\left(\frac{a}{a+1}-\frac{1}{a-1}\right)\frac{1}{a^2-2a-1}=\frac{1}{8} \Rightarrow a=?$

A) 2 B) 3 C) 4

D) 5 E) 6

11. $\frac{x^2+(a+3)x+3a}{3x+3a}=3 \Rightarrow x=?$

A) 2 B) 3 C) 4

D) 5 E) 6

12. $(4{,}132)^2-2(4{,}132)\cdot(3{,}132)+(3{,}132)^2=?$

A) 1 B) 2 C) 3

D) 4 E) 5

13. $\frac{x^2-x-12}{x^2+5x+6}\cdot\frac{x^2+3x+2}{x^2-16}=\frac{2}{5} \Rightarrow x=?$

A) 1 B) 2 C) 3

D) 4 E) 5

14. $\frac{x-y}{2x}=\frac{y}{x+y} \Rightarrow x^2-2xy-y^2=?$

A) -2 B) -1 C) 0

D) 1 E) 2

15. $x^2-6x+10=A \Rightarrow \min(A)=?$

A) 1 B) 2 C) 3

D) 4 E) 5

16. $x^2+8x+8=A \Rightarrow \min(A)=?$

A) -10 B) -8 C) -6

D) -4 E) -2

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	C	C	C	D	D	D	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	B	E	A	A	C	A	B

1. $\dfrac{(4x+5y)^2-(4x-5y)^2}{5xy} = ?$

A) 4 B) 5 C) 9
D) 12 E) 16

2. $4x^2 + 9y^2 + 12x - 6y + 10 = 0 \Rightarrow x \cdot y = ?$

A) $-\frac{1}{3}$ B) $-\frac{1}{2}$ C) -1
D) $\frac{1}{2}$ E) 1

3. $a + \dfrac{1}{a-1} = 7 \Rightarrow (a-1)^2 + \dfrac{1}{(a-1)^2} = ?$

A) 30 B) 31 C) 32
D) 33 E) 34

4. $\left.\begin{matrix} A = 3\sqrt{3}-3 \\ B = 3\sqrt{3}+3 \end{matrix}\right\} \Rightarrow \dfrac{(A-B)^2+4AB}{A \cdot B} = ?$

A) 3 B) 6 C) 9
D) 18 E) 27

5. $\dfrac{(x-y)^2+2xy}{x^6+y^6} : \dfrac{1}{x^4-x^2y^2+y^4} = ?$

A) 1 B) $x + y$ C) $x^2 + y^2$
D) $(x + y)^2$ E) $(x - y)^2$

6. $A = 1002 \Rightarrow A^2 - 4A = ?$

A) $10^6 - 4$ B) $10^6 - 2$ C) $10^8 - 1$
D) $10^4 - 4$ E) $10^4 + 4$

7. $x^2 + ax - 18 = (x + b)(x - 2) \Rightarrow b - a = ?$

A) 1 B) 2 C) 3
D) 4 E) 5

8. $x > 0$,

$\left(\dfrac{2}{x-y} - \dfrac{x+y}{x^2-xy}\right) \cdot x^{-1} = 9^{-1} \Rightarrow x = ?$

A) 1 B) 2 C) 3
D) 4 E) 5

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9. $\frac{x^3y^2 - x^2y}{xy} + x = ?$

A) x B) xy C) x^2y

D) xy^2 E) x^2y^2

10. $\frac{a}{4b} + \frac{b}{9a} = \frac{1}{3} \Rightarrow \frac{a}{b} = ?$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1

D) $\frac{4}{3}$ E) $\frac{5}{3}$

11. $\frac{4x-1}{x-1} + \frac{5x+4}{2x-3} + \frac{3x}{1-x} + \frac{3x+7}{3-2x} = ?$

A) –2 B) –1 C) 0

D) 1 E) 2

12. $x \neq y$

$3x^2 + 2xy - y^2 = 0,$

$3x - y = 4 \Rightarrow y = ?$

A) –2 B) –1 C) 0

D) 1 E) 2

13. $x^2 - 4 = 2a$

$\Rightarrow (x^2 - 2a + 1)^2 = ?$

A) 4 B) 9 C) 16

D) 25 E) 100

14. $\frac{x-y}{x+y} + \frac{x+y}{x-y} = 6 \Rightarrow \frac{x^2}{y^2} = ?$

A) 1 B) $\sqrt{2}$ C) $\sqrt{3}$

D) 2 E) 3

15. $x, y \in N$

$x^2 - 9a^2 = 36$

$4y^2 - 9a^2 = 11$

$x - 2y = 5 \Rightarrow x = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

16. $x + \frac{1}{x} = 6 \Rightarrow \frac{x^6 + x^2}{x^4} = ?$

A) 30 B) 31 C) 32

D) 33 E) 34

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	B	E	B	A	A	B	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	B	E	B	D	D	E	E

1. $\frac{x^2+3x-4}{x^2+x-12}\cdot\frac{x^2-2x-3}{x^2-1}\cdot\frac{x^2-2x}{x-2}=?$

A) $-x+1$ B) x C) $x+1$

D) $x+2$ E) $x-2$

2. $\left(\frac{a^2-a-9}{a-1}-1\right)\cdot\left(\frac{a^2-1}{a^2-2a-8}\right)=?$

A) $-a+1$ B) $a-1$ C) $a+1$

D) $a-4$ E) $a+2$

3. $x^2+\frac{1}{x^2}=10 \Rightarrow \left|x+\frac{1}{x}\right|=?$

A) $\sqrt{3}$ B) $2\sqrt{3}$ C) $3\sqrt{3}$

D) $4\sqrt{3}$ E) 12

4. $\frac{x^2-y^2+3x+3y}{x^2-y^2+6x+9}=\frac{x+y}{10} \Rightarrow x+y=?$

A) 4 B) 5 C) 6

D) 7 E) 8

5. $x^{32}=A$

$\Rightarrow (x^{16}+1)(x^8+1)(x^4+1)(x^2+1)(x+1)(x-1)=?$

A) A B) $A-1$ C) $1-A$

D) $A+1$ E) $2A$

6. $x^2-x+1=0 \Rightarrow x^6-1=?$

A) -2 B) -1 C) 0

D) 1 E) 2

7. $x^2=x-1 \Rightarrow \frac{x^3-2}{x^2+1}=?$

A) $-\frac{3}{x}$ B) $-\frac{1}{x}$ C) $-x$

D) $\frac{1}{x}$ E) $\frac{3}{x}$

8. $2^{\frac{1}{8}}=x \Rightarrow \frac{\left(2^{\frac{1}{16}}-1\right).\left(2^{\frac{1}{16}}+1\right)}{2^{\frac{1}{4}}-1}=?$

A) $\frac{1}{x-1}$ B) $\frac{1}{x+1}$ C) x

D) $x-1$ E) $x+1$

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9. $\frac{x^3+x^2-2x}{x^2-3x+2}\cdot\frac{x^2-2x}{x^2+2x}=4 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

10. $\frac{x}{5}=\frac{3}{2y} \Rightarrow$ **4xy = ?**

A) 10 B) 15 C) 20

D) 25 E) 30

11. $\frac{4x^2-25y^2}{2x^2-5xy}\cdot\frac{8x^2}{2x+5y}=16 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

12. $4x^2+y^2-12x+2y+10=0$

$\Rightarrow$ **x + y = ?**

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1

D) 2 E) 3

13. $A=4x^2-12x+10 \Rightarrow$ **min(A) = ?**

A) –2 B) –1 C) 0

D) 1 E) 2

14. $\frac{x^2-ax+b}{x^2-6x+8}=\frac{x-5}{x-4} \Rightarrow \frac{b+4}{a}=?$

A) 1 B) 2 C) 3

D) 4 E) 5

15. $a^3+3a^2b=70$

$b^3+3ab^2=55$

$\Rightarrow$ **a + b = ?**

A) 3 B) 4 C) 5

D) 6 E) 7

16. $x>0$,

$\frac{x^3-4x}{x+2}:\frac{x-2}{x^2}=8 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	C	B	D	B	C	A	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	E	B	B	D	B	C	B

BÖLÜM CHAPTER 6

TARİHSEL NOT / HISTORICAL NOTE

Boole, George [1815 – 1864]

İngiliz matematikçi, matematiksel mantığın babalarından birisi olarak tanınır. İlk büyük çalışması, 1854'de yayımlanan "Düşünme kurallarının incelenmesi (Investigation of the Laws of Thought)"dir. Boolean cebirleri olarak bilinen sembolik çıkarım türünü geliştirmiştir. Onun De Morgan ve diğerleri ile olan çalışması, modern cebirin gelişmesini sağlamıştır.

British mathematician who was one of the founding fathers of mathematical logic. His major work, published in 1854, is his Investigation of the Laws of Thought. The kind of Symbolic argument that he developed led to the study of so-called Boolean algebras, which are of current significance in computing and algebra. His work, together with that of De Morgan and others, helped to pave the way for the development of modern formal algebra.

ORAN ve ORANTI
RATIOS and PROPORTIONS

Bu bölüm, 23 çözümlü örnek, 12 çözümlü test sorusu, 16 çözümlü YÖS sorusu, 64 test sorusu içermektedir.

This chapter includes 23 examples with solutions, 12 tests questions with solutions, 16 YÖS questions with solutions and 64 test questions.

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BÖLÜM
06
CHAPTER

ORAN ve ORANTI
RATIOS and PROPORTIONS

ORAN ve ORANTI / RATIOS and PROPORTIONS

Tanım: Birimleri aynı olan iki niceliğin karşılaştırılmasına *oran* denir.
a'nın b'ye oranı a : b veya $\frac{a}{b}$ şeklinde yazılır.

Definition: Comparing two quantities with the same units is called ***ratio.***
The ratio of a to b is written as a : b or $\frac{a}{b}$.

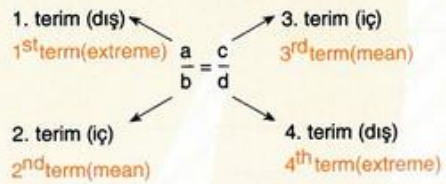
$$\frac{3kg}{8kg} = \frac{3}{8}\ ,\ \frac{7cm}{5cm} = \frac{7}{5}\ ,\ \frac{3\,lt}{12lt} = \frac{1}{4}$$

Tanım: Oran katsayıları aynı olan iki veya daha fazla orandan oluşan eşitliklere ***orantı*** denir.

Definition: A proportion is an equation consisting of two or more expressions with the same ratio.

$$\frac{a}{b} = k,\ \frac{c}{d} = k \Rightarrow \frac{a}{b} = \frac{c}{d} = k$$

Bu orantıda, a ve b sayıları, c ve d sayıları ile orantılıdır.
a, b, c, d sayılarına ***orantının terimleri*** denir k ye **orantı sabiti** denir.
In this proportion the numbers a and b are proportional to the numbers c and d.
a, b, c, d numbers are called ***terms of proportion*** k is called **proportionality constant.**



$\frac{a}{b} = \frac{c}{d}$ orantısı (proportion) $a : b = c : d$ biçiminde gösterilmiştir (is shown by).

dışlar (extremes): a, d
içler (means): b, c

İç terimlerin çarpımı dış terimlerin çarpımına eşittir.
The product of the means equals the product of the extremes.

$$a \cdot d = b \cdot c$$

Özellik / Property

★ $\frac{a}{c}=\frac{b}{d} \Leftrightarrow a:b=c:d$

$\frac{a}{d}=\frac{b}{e}=\frac{c}{f} \Leftrightarrow a:b:c=d:e:f$

★ $\frac{a}{b}=\frac{c}{d} \Rightarrow$

I. $a \cdot d = b \cdot c$

II. $\frac{a}{c}=\frac{b}{d}$

III. $\frac{d}{b}=\frac{c}{a}$

IV. $\frac{b}{a}=\frac{d}{c}$

★ $\frac{a}{b}=\frac{c}{d}=k \Rightarrow$

I. $\frac{a+c}{b+d}=k$

II. $\frac{m\cdot a}{m\cdot b}=\frac{t\cdot c}{t\cdot d}=\frac{m\cdot a+t\cdot c}{m\cdot b+t\cdot d}=k$

★ $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow \frac{a\cdot c}{b\cdot d}=k^2$

★ $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow \frac{a^n}{b^n}=\frac{c^n}{d^n}=k^n$

Örnek / Example 1

$\left.\begin{array}{l}\frac{x}{3}=\frac{4}{y}\\ x\cdot y\cdot z=36\end{array}\right\} \Rightarrow z=?$

A) 2 B) 3 C) 4 D) 6 E) 12

Çözüm / Solution

Yanıt / Answer **B**

$\frac{x}{3}=\frac{4}{y} \Rightarrow x\cdot y=12$

$x\cdot y\cdot z=36$

$12\cdot z=36$

$z=3$

PEKİŞTİRME / REINFORCE

✓ $\frac{a}{2}=\frac{b}{5}=\frac{c}{7}$,
$a+b+c=42 \Rightarrow b=?$
..... [15]

✓ $\frac{a}{3}=\frac{b}{4}=\frac{c}{5}$,
$a+b+c=48 \Rightarrow c=?$
..... [20]

Örnek / Example 2

$\frac{3a+b}{a-b}=\frac{7}{2} \Rightarrow \frac{a}{b}=?$

A) 2 B) 5 C) 7 D) 9 E) 11

Çözüm / Solution

Yanıt / Answer **D**

$\frac{3a+b}{a-b}=\frac{7}{2}$

$6a+2b=7a-7b$

$9b=a$

$\Rightarrow \frac{a}{b}=\frac{9b}{b}=9$

Örnek / Example 3

$\frac{a}{b}=\frac{3}{7}, \frac{b}{c}=\frac{7}{4}$

$a+b+c=56 \Rightarrow c=?$

A) 12 B) 16 C) 20 D) 24 E) 28

Çözüm / Solution

Yanıt / Answer **B**

$\frac{a}{b}=\frac{3}{7} \Rightarrow a=3k$

$b=7k$

$\frac{b}{c}=\frac{7}{4} \Rightarrow b=7k$

$c=4k$

$a+b+c=3k+7k+4k=56$

$14k=56$

$k=4$

$c=4\cdot k=16$

Örnek / Example 4

$\frac{a}{b}=\frac{3}{4}, \frac{b}{c}=\frac{5}{7}$

$a+b+c=126 \Rightarrow a=?$

A) 30 B) 40 C) 56 D) 66 E) 76

Çözüm / Solution

Yanıt / Answer **A**

$\frac{a}{b}=\frac{3}{4}, \frac{b}{c}=\frac{5}{7}$

$\Rightarrow \frac{a}{b}=\frac{15}{20}, \frac{b}{c}=\frac{20}{28}$

$a=15k,\ b=20k,\ c=28k$

$a+b+c=126$

$63k=126$

$k=2$

$a=15\cdot 2=30$

Örnek / Example 5

$3x = 4y = 7z$

$x + y + z = 183 \Rightarrow$ **x = ?**

A) 80 B) 82 C) 84 D) 86 E) 88

Çözüm / Solution

Yanıt / Answer **C**

$3x = 4y = 7z \Rightarrow x = 28k,\ y = 21k,\ z = 12k$

$x + y + z = 183$

$61k = 183$

$k = 3$

$x = 28 \cdot 3 = 84$

Örnek / Example 6

$2xy = 4yz = 5xz$

$x - y + 2z = 18 \Rightarrow$ **z = ?**

A) 12 B) 13 C) 14 D) 15 E) 16

Çözüm / Solution

Yanıt / Answer **A**

$2xy = 4yz = 5xz$

$x = 4k,\ y = 5k,\ z = 2k$

$x - y + 2z = 18$

$4k - 5k + 4k = 18$

$3k = 18$

$k = 6$

$z = 2 \cdot k = 2 \cdot 6 = 12$

Örnek / Example 7

$\frac{a}{3} = \frac{b}{4} = \frac{c}{5},\ 2a + c = 33 \Rightarrow$ **b = ?**

A) 11 B) 12 C) 13 D) 14 E) 15

Çözüm / Solution

Yanıt / Answer **B**

$\frac{a}{3} = \frac{b}{4} = \frac{c}{5} = k \Rightarrow \begin{cases} a = 3k \\ b = 4k \\ c = 5k \end{cases}$

$2a + c = 33$

$2 \cdot 3k + 5k = 33$

$11k = 33$

$k = 3$

$b = 4k \Rightarrow b = 4 \cdot 3 = 12$

PEKİŞTİRME / REINFORCE

✓ $\frac{a}{2} = \frac{b}{5} = \frac{c}{7}$,
$3a - 2b + c = 24 \Rightarrow a + b = ?$
..... [56]

✓ $\frac{a}{4} = \frac{b}{7} = \frac{c}{11}$,
$2a + 3b + c = 80 \Rightarrow a = ?$
..... [8]

✓ $\frac{a}{2} = \frac{b}{3} = \frac{c}{5}$,
$2a - b + 3c = 64 \Rightarrow c = ?$
..... [20]

PEKİŞTİRME / REINFORCE

✓ $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=4$,

$2a+3c-e=56$

$f-3d=10 \Rightarrow$ **b = ?**

..... [12]

✓ $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=3$,

$3a+c-2e=27$

$-d+2f=-6 \Rightarrow$ **a = ?**

..... [3]

Örnek / Example 8

$$\left.\begin{array}{l}\frac{a-1}{3}=\frac{b+2}{4}=\frac{c-2}{5}\\ 5a-2c=36\end{array}\right\} \Rightarrow \textbf{b = ?}$$

A) 29 B) 28 C) 27 D) 26 E) 25

Çözüm / Solution

Yanıt / Answer **D**

$$\frac{a-1}{3}=\frac{b+2}{4}=\frac{c-2}{5}=k \Rightarrow \begin{cases}a=3k+1\\ b=4k-2\\ c=5k+2\end{cases}$$

$5a-2c=36 \Rightarrow 5\cdot(3k+1)-2\cdot(5k+2)=36$

$15k+5-10k-4=36$

$5k+1=36$

$5k=35$

$k=7$

$b=4k-2=4\cdot 7-2=28-2=26$

Örnek / Example 9

$$\left.\begin{array}{l}\frac{a-b}{x}=\frac{a-c}{y}=\frac{b+c}{z}\\ x+y+z=8\\ a-c=\frac{1}{2}\end{array}\right\} \Rightarrow \textbf{y = ?}$$

A) 2a B) $\frac{a}{2}$ C) $\frac{2}{a}$ D) a E) $\frac{4}{a}$

Çözüm / Solution

Yanıt / Answer **C**

$\frac{a-b+a-c+b+c}{x+y+z}=k$

$\frac{2a}{8}=k$

$a=4k$

$\frac{a}{4}=k$

$\frac{a-c}{y}=k$

$\frac{\frac{1}{2}}{y}=k$

$\frac{\frac{1}{2}}{y}=\frac{a}{4}$

$y=\frac{2}{a}$

Örnek / Example 10

$a, b, c \in Z^-$

$\frac{a}{0,1} = \frac{b}{0,3} = \frac{c}{2} \Rightarrow ? > ? > ?$

A) $a > b > c$ B) $a > c > b$ C) $c > b > a$
D) $b > c > a$ E) $c > a > b$

Çözüm / Solution

Yanıt / Answer **A**

$\frac{a}{0,1} = \frac{b}{0,3} = \frac{c}{2} = k$

$\left.\begin{array}{l} a = 0,1k \\ b = 0,3k \\ c = 2k \end{array}\right\} \Rightarrow a > b > c$

Örnek / Example 11

$\frac{a}{2} = \frac{b}{3} = \frac{c}{5}$ ve / and $3a + 2b - 4c = -24 \Rightarrow \mathbf{a = ?}$

A) 2 B) 4 C) 6 D) 8 E) 10

Çözüm / Solution

Yanıt / Answer **C**

$\frac{a}{2} = \frac{b}{3} = \frac{c}{5} = k$

$\frac{3a + 2b - 4c}{3 \cdot 2 + 2 \cdot 3 - 4 \cdot 5} = k$

$\frac{-24}{-8} = k$

$3 = k$

$a = 2 \cdot k = 2 \cdot (3) = 6$

Örnek / Example 12

$\frac{x}{3} = \frac{y}{4} = \frac{z}{7} = \frac{4x - 5y + kz}{13} \Rightarrow \mathbf{k = ?}$

A) 5 B) 4 C) 3 D) 2 E) 1

Çözüm / Solution

Yanıt / Answer **C**

$\frac{4x}{3 \cdot 4} = \frac{-5y}{-5 \cdot 4} = \frac{zk}{7k} = \frac{4x - 5y + kz}{13}$

$\frac{4x - 5y + zk}{12 - 20 + 7k} = \frac{4x - 5y + zk}{13}$

$7k - 8 = 13$

$7k = 21$

$k = 3$

Örnek / Example 13

$$\left.\begin{array}{l}\frac{a}{x}=\frac{b}{y}=\frac{c}{z}=\frac{1}{3}\\ a-2b+3c=2\\ 2y-3z=-1\end{array}\right\} \Rightarrow x=?$$

A) 4 B) 5 C) 6 D) 8 E) 10

Çözüm / Solution

Yanıt / Answer **B**

$$\frac{a}{x}=\frac{b}{y}=\frac{c}{z}=\frac{1}{3}$$

$$\frac{a-2b+3c}{x-2y+3z}=\frac{1}{3}$$

$$2y-3z=-1$$

$$\Rightarrow -2y+3z=1$$

$$\frac{2}{x+1}=\frac{1}{3}$$

$$x+1=6$$

$$x=5$$

Örnek / Example 14

$\frac{a}{2}=\frac{b}{3}=\frac{c}{4}=k$ ve / and $a\cdot b\cdot c=192 \Rightarrow a=?$

A) 10 B) 8 C) 6 D) 4 E) 2

Çözüm / Solution

Yanıt / Answer **D**

$$\frac{a}{2}=\frac{b}{3}=\frac{c}{4}=k \Rightarrow \frac{a\cdot b\cdot c}{2\cdot 3\cdot 4}=k^3$$

$$\frac{192}{24}=k^3$$

$$8=k^3$$

$$2=k$$

$k=2 \Rightarrow a=4$

ORANTI ÇEŞİTLERİ / PROPORTION TYPES

DOĞRU ORANTI / DIRECT PROPORTION

Orantılı iki çokluktan biri artarken diğeri aynı oranda artıyorsa ya da biri azalırken diğeri de aynı oranda azalıyorsa bu iki çokluk doğru orantılıdır.

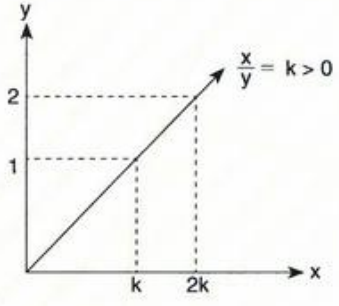
$\frac{x}{y} = k$, $k \in R^+$ ifadesine doğru orantının denklemi denir.

x ile y çokluklarının doğru orantılı olduğu grafik aşağıdaki gibidir.

If in two proportional quantities increasing the value of one of them results in increasing the value of the other one with the same ratio, or decreasing the value of one of them results in decreasing the other one with the same ratio, these two quantities are called directly proportional quantities.

$\frac{x}{y} = k$, $k \in R^+$ is called the equation of the direct proportion.

The graph of directly proportional quantities x and y is like the figure shown below.



TERS ORANTI / INVERSE PROPORTION

Orantılı iki çokluktan biri artarken diğeri aynı oranda azalıyorsa ya da biri azalırken diğeri aynı oranda artıyorsa bu iki çokluk ters orantılıdır.

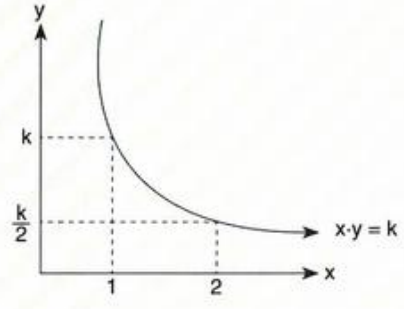
$x \cdot y = k$, $k \in R^+$ ifadesine ters orantının denklemi denir.

x ile y çokluklarının ters orantılı olduğu grafik aşağıdaki gibidir.

If in two proportional quantities increasing the value of one of them results in decreasing the value of the other one with the same ratio, or decreasing the value of one of them results in increasing the other one with the same ratio, these two quantities are called inversely proportional quantities.

$x \cdot y = k$, $k \in R^+$ is called the equation of the inverse proportion.

The graph of inversely proportional quantities x and y is like the figure shown below.



Örnek / Example 15

$k = \frac{x+4}{y-2}$

$x = 5 \Rightarrow y = 5$

$x = 14 \Rightarrow \mathbf{y = ?}$

A) 1 B) 6 C) 8 D) 10 E) 14

Çözüm / Solution

Yanıt / Answer **C**

$x = 5, y = 5 \Rightarrow k = \frac{5+4}{5-2} = 3$

$k = 3, x = 14 \Rightarrow 3 = \frac{14+4}{y-2}$

$3y - 6 = 18$

$3y = 24$

$y = 8$

Örnek / Example 16

$(x + 3)\cdot(y - 2) = k$

$x = 1 \Rightarrow y = 8$

$x = 3 \Rightarrow \mathbf{y = ?}$

A) 6 B) 7 C) 8 D) 9 E) 10

Çözüm / Solution

Yanıt / Answer **A**

$x = 1, \ y = 8 \Rightarrow k = (1 + 3)\cdot(8 - 2) = 24$

$x = 3, \ k = 24 \Rightarrow 24 = (3 + 3)\cdot(y - 2)$

$24 = 6\cdot(y - 2)$

$4 = y - 2$

$y = 6$

Örnek / Example 17

12 işçinin günde 6 saat çalışıp 10 günde bitirebildiği bir işi 6 içşi günde 8 saat çalışarak kaç günde bitirebilir?

A task finished in 10 days by 12 workers working 6 hours a day will take how many days to finish by 6 workers working 8 hours a day?

A) 4 B) 10 C) 12 D) 15 E) 20

Çözüm / Solution

Yanıt / Answer **D**

$12\cdot6\cdot10 = 6\cdot8\cdot x$

$x = 15$

Örnek / Example 18

Bir baba elindeki 280 cevizi 3, 4 ve 7 yaşlarındaki çocuklarına yaşlarıyla doğru orantılı olacak şekilde dağıtıyor. Buna göre en çok cevizi alan çocuk kaç ceviz almıştır?

A father splits 280 nuts among his children of 3, 4 and 7 years old directly proportional to their ages. Accordingly, how many nuts did the kid who got the most number of nuts get?

A) 120 B) 140 C) 150 D) 155 E) 160

Çözüm / Solution

Yanıt / Answer **B**

$3k + 4k + 7k = 280$

$14k = 280$

$k = 20$

$\Rightarrow 7 \cdot k = 140$

Örnek / Example 19

490 bilye 3 ve 5 ile doğru orantılı 6 ile ters orantılı olacak şekilde parçalara ayrılıyor. Bilye sayısının en az olduğu parçada kaç tane bilye vardır?

490 marbles split up to parts directly proportional to 3 and 5, and inversely proportional to 6, the part with the least number of marbles have how many marbles?

A) 10 B) 14 C) 18 D) 22 E) 26

Çözüm / Solution

Yanıt / Answer **A**

$3k + 5k + \frac{k}{6} = 490$

$\frac{18k + 30k + k}{6} = 490$

$49k = 490 \cdot 6$

$k = 60$

$\Rightarrow \frac{k}{6} = \frac{60}{6} = 10$

ORTALAMA ÇEŞİTLERİ / TYPES of MEAN

ARİTMETİK ORTALAMA / ARITHMETIC MEAN

n tane sayının aritmetik ortalaması bu n sayının toplamının n' ye bölünmesidir. Buna göre $a_1, a_2, a_3, a_4, ..., a_n$ sayılarının aritmetik ortalaması $\frac{a_1+a_2+a_3+...+a_n}{n}$ dir

The arithmetic mean of n numbers is division of the sum of these numbers into n. Accordingly, the arithmetic mean of numbers $a_1, a_2, a_3, a_4, ... a_n$ is $\frac{a_1+a_2+a_3+...+a_n}{n}$.

GEOMETRİK ORTALAMA / GEOMETRIC MEAN

n tane sayının geometrik ortalaması bu n sayının n. dereceden kök içerisinde çarpılmasıdır. Buna göre $a_1, a_2, a_3, a_4, ..., a_n$ sayılarının geometrik ortalaması $\sqrt[n]{a_1 \cdot a_2 \cdot a_3 \cdot ... \cdot a_n}$ dir.

The geometric mean of n numbers is the n'th root of the product of these numbers. Accordingly, the geometric mean of numbers $a_1, a_2, a_3, a_4, ..., a_n$ is $\sqrt[n]{a_1 \cdot a_2 \cdot a_3 \cdot ... \cdot a_n}$

Örnek / Example 20

Aritmetik ortalaması 34 olan ardışık 5 çift sayının en büyüğünün değeri kaçtır?

What is the biggest one of 5 consecutive even numbers with arithmetic mean of 34?

A) 30 B) 32 C) 34 D) 36 E) 38

Çözüm / Solution Yanıt / Answer **C**

$$\frac{2x+(2x+2)+(2x+4)+(2x+6)+(2x+8)}{5} = 34$$

$$\frac{10x+20}{5} = 34$$

$$2x+4 = 34 \Rightarrow x = 15$$

$$\Rightarrow 2x+8 = 2 \cdot 15 + 8 = 38$$

Örnek / Example 21

Geometrik ortalaması 12 olan iki sayıdan biri 16 ise diğer sayı kaçtır?

Two numbers have geometric mean of 12; if one of them is 16, then what is the other one?

A) 4 B) 7 C) 9 D) 12 E) 14

Çözüm / Solution Yanıt / Answer **C**

$$\sqrt{16 \cdot x} = 12$$

$$16 \cdot x = 144$$

$$x = 9$$

Örnek / Example 22

$a, b \in R,\ a \cdot b \in Z$

$a^2 + 3ab + 9b^2 = 48 \Rightarrow$ **max(a·b) = ?**

A) 5 B) 6 C) 7 D) 8 E) 9

Çözüm / Solution

Yanıt / Answer **A**

Aritmetik ortalama ≥ Geometrik ortalama

Arithmetic average- geometric average

$$\frac{a^2 + 3ab + 9b^2}{3} \geq \sqrt[3]{a^2 \cdot 3ab \cdot 9b^2}$$

$$\frac{48}{3} \geq \sqrt[3]{27a^3b^3}$$

$$16 \geq 3ab$$

$$\frac{16}{3} \geq a \cdot b \Rightarrow \max(a \cdot b) = 5$$

HARMONİK ORTALAMA / HARMONIC AVERAGE

$a_1, a_2, a_3, \ldots a_n$ gibi n tane sayının harmonik ortalaması;

Harmonic average of numbers as $a_1, a_2, a_3, \ldots a_n$ in amount of n

$$x = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_n}} \text{ dir.}$$

Örnek / Example 23

$x, y, z \in R^+$

$5x + 6y = 9xy$

$2z + 4x = 8zx$

$4y + 3z = 7yz$

olduğuna göre x, y ve z sayılarının harmonik ortalaması kaçtır?

Accordingly, what is the harmonic average of x, y and z?

A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution

Yanıt / Answer **A**

$$5x + 6y = 9xy \Rightarrow \frac{5}{y} + \frac{6}{x} = 9$$

$$2z + 4x = 8zx \Rightarrow \frac{2}{x} + \frac{4}{z} = 8$$

$$4y + 3z = 7yz \Rightarrow \frac{4}{z} + \frac{3}{y} = 7$$

$$\frac{8}{y} + \frac{8}{x} + \frac{8}{z} = 24$$

$$8\left(\frac{1}{y} + \frac{1}{x} + \frac{1}{z}\right) = 24$$

$$\frac{1}{y} + \frac{1}{x} + \frac{1}{z} = 3$$

Harmonik ortalama / Harmonic average: $\frac{3}{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}} = \frac{3}{3} = 1$

1. $\frac{a}{b}=\frac{2}{3}$, $2a+b=84 \Rightarrow \mathbf{b=?}$

A) 14 B) 28 C) 27 D) 30 E) 36

2. $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}$,
$3a-b+2c=66 \Rightarrow \mathbf{b=?}$

A) 12 B) 18 C) 22 D) 33 E) 44

3. $\frac{a}{b}=\frac{3}{4} \Rightarrow \frac{\mathbf{a+b}}{\mathbf{a}}=\mathbf{?}$

A) $\frac{7}{3}$ B) $\frac{7}{4}$ C) $\frac{14}{3}$ D) $\frac{3}{4}$ E) $\frac{3}{7}$

4. $\frac{x}{y}=\frac{2}{3} \Rightarrow \frac{\mathbf{3x-4y}}{\mathbf{x-y}}=\mathbf{?}$

A) –6 B) $-\frac{1}{5}$ C) $\frac{5}{6}$ D) –1 E) 6

5. $a:b:c=2:3:4$,
$3a+4b-c=-28$
$\Rightarrow \mathbf{b=?}$

A) 6 B) 5 C) 4 D) – 4 E) – 6

6. $a, b, c \in R$,
$\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=2 \Rightarrow \frac{\mathbf{a}}{\mathbf{d}}=\mathbf{?}$

A) 2 B) 4 C) 6 D) 8 E) 16

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7. $\left.\begin{array}{l}\frac{a}{b}=\frac{2}{5},\\ \frac{b}{4}=c,\\ a+b=21\end{array}\right\} \Rightarrow c=?$

A) $\frac{13}{5}$ B) $\frac{15}{4}$ C) 6

D) 28 E) 60

8. $\frac{1}{3a}=\frac{1}{4b}=\frac{1}{6c}$,

$a+b+c=27 \Rightarrow a-c=?$

A) 3 B) 4 C) 6

D) 8 E) 12

9. $\left.\begin{array}{l}\frac{a}{-2}=\frac{b}{3}=3\\ a+b+c=1\end{array}\right\} \Rightarrow c=?$

A) 2 B) 1 C) –2

D) –1 E)–4

10. $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}$,

$2a-3b+c=5 \Rightarrow a=?$

A) –15 B) –10 C) 5

D) 10 E) 15

11. **Bir baba elindeki 320₺ parayı yaşları 3, 5 ve 8 olan çocuklarına yaşlarıyla doğru orantılı olacak şekilde dağıtıyor. En az parayı alan çocuk kaç ₺ almıştır?**

A father splits 320₺ among his children of ages 3, 5, and 8 directly proportional to their ages.how much will the kid who gets the least amount of money will get?

A) 60 B) 55 C) 50

D) 45 E) 40

12. **Yaşlarının aritmetik ortalaması 12 olan 5 kişilik basketbol takımına yaşları toplamı 28 olan 3 kişi daha katıldığında yeni oluşan takımdaki oyuncuların yaşlarının aritmetik ortalaması kaçtır?**

If three players with sum of ages of 28 years join a basketball team of 5 people in which their ages arithmetic mean is 12, what will be the arithmetic mean of ages of the new formed team?

A) 11 B) 12 C) 13

D) 14 E) 15

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
E	B	A	E	E	D	B	C	C	B	A	A

1. $\frac{a}{b}=\frac{2}{3} \Rightarrow a=2k,\ b=3k$

$2a+b=84$

$2\cdot 2k+3k=84$

$7k=84 \Rightarrow k=12$

$b=3k=3\cdot 12=36$

Yanıt / The Answer is E

---

2. $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}=k \Rightarrow \frac{3a}{6}=\frac{-b}{-3}=\frac{2c}{8}=k$

$\frac{3a-b+2c}{6-3+8}=\frac{66}{11}=k$

$k=6$

$\Rightarrow b=3\cdot 6=18$

Yanıt / The Answer is B

---

3. $\frac{a}{b}=\frac{3}{4} \Rightarrow a=3k, b=4k$

$\frac{a+b}{a}=\frac{3k+4k}{3k}=\frac{7k}{3k}=\frac{7}{3}$

Yanıt / The Answer is A

---

4. $\frac{x}{y}=\frac{2}{3} \Rightarrow x=2k,\ y=3k$

$\frac{3x-4y}{x-y}=\frac{3\cdot 2k-4\cdot 3k}{2k-3k}=\frac{-6k}{-k}=6$

Yanıt / The Answer is E

---

5. $a:b:c=2:3:4$

$\frac{a}{2}=\frac{b}{3}=\frac{c}{4}=k \Rightarrow a=2k,\ b=3k,\ c=4k$

$3a+4b-c=3\cdot 2k+4\cdot 3k-4k$

$=6k+12k-4k$

$14k=-28$

$k=-2$

$b=3\cdot(-2)=-6$

Yanıt / The Answer is E

---

6. $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=2 \Rightarrow \frac{a\cdot b\cdot c}{b\cdot c\cdot d}=2^3$

$\frac{a}{d}=8$

Yanıt / The Answer is D

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7. $\frac{a}{b}=\frac{2}{5} \Rightarrow 5a=2b \Rightarrow a=\frac{2b}{5}$

$a+b=\frac{2b}{5}+b=21$

$\frac{7b}{5}=21 \Rightarrow b=15$

$\frac{b}{4}=c \Rightarrow \frac{15}{4}=c$

Yanıt / The Answer is B

8. $\frac{1}{3a}=\frac{1}{4b}=\frac{1}{6c} \Rightarrow 3a=4b=6c$

$a=4k$

$b=3k$

$c=2k$

$\Rightarrow a+b+c=9k=27$

$k=3$

$a-c=2k=2\cdot 3=6$

Yanıt / The Answer is C

9. $\frac{a}{-2}=\frac{b}{3}=3 \Rightarrow$

$a=-6\,,\ b=9$

$a+b+c=1 \Rightarrow -6+9+c=1$

$c=-2$

Yanıt / The Answer is C

10. $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}=k$

$a=2k\,,\ b=3k,\ c=4k$

$2a-3b+c=2\cdot 2k-3\cdot 3k+4k=5$

$4k-9k+4k=5$

$-k=5 \Rightarrow k=-5$

$a=2k=2\cdot(-5)=-10$

Yanıt / The Answer is B

11. $3k+5k+8k=320$

$16k=320$

$k=20$

$3\cdot k=3\cdot 20=60$

Yanıt / The Answer is A

12. $\frac{x_1+x_2+\ldots+x_5}{5}=12$

$x_1+x_2+\ldots+x_5=60$

$x_1+x_2+\ldots+x_8=60+28=88$

$\Rightarrow \frac{x_1+x_2+\ldots+x_8}{8}=\frac{88}{8}=11$

Yanıt / The Answer is A

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1. $\frac{2p+r}{t+3} = \frac{3t+p}{r+4} = \frac{2r+3}{p-6} = k$

$\Rightarrow$ **2r − 3p = ?**

A) −21 B) −14 C) −28 D) 7 E) −7

[GAZİANTEP ÜNİVERSİTESİ – YÖS 2020]

2. $\frac{x}{y} = \frac{y}{t} = \frac{t}{z} = 3 \Rightarrow \frac{x-y+t-z}{y-t} = ?$

A) 1 B) $\frac{5}{2}$ C) 3 D) $\frac{10}{3}$ E) 4

[HARRAN ÜNİVERSİTESİ – YÖS 2020]

3. $\frac{x+y}{9} = \frac{y+z}{15} = \frac{x+z}{12} = \frac{4}{3}$

$\Rightarrow$ **x+y+z = ?**

A) 18 B) 24 C) 30 D) 36 E) 54

[ANKARA ÜNİVERSİTESİ – YÖS 2019]

4. $\frac{x}{z+5} = \frac{y}{x+1} = \frac{z}{y-2} = \frac{3}{5}$

$\Rightarrow$ **x+y+z = ?**

A) 6 B) 10 C) 12 D) 16 E) 20

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2018]

5. $\frac{a}{b} = -2 \Rightarrow \frac{2b}{a} - \frac{3a}{b} = ?$

A) −5 B) −4 C) −3 D) 4 E) 5

[İNÖNÜ ÜNİVERSİTESİ – YÖS 2018]

6. $\left.\begin{array}{l} \frac{x-y}{2} = \frac{y+z}{3} = \frac{z}{4} \\ 2x-3y-z = 7 \end{array}\right\} \Rightarrow x = ?$

A) 3 B) 4 C) 5 D) 7 E) 9

[İSTANBUL ÜNİVERSİTESİ – YÖS 2018]

7. $\frac{x}{y} = \frac{3}{4}, \frac{y}{z} = \frac{3}{2} \Rightarrow \frac{y-x}{x-z} = ?$

A) $\frac{1}{6}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 2 E) 3

[İSTANBUL ÜNİVERSİTESİ – YÖS 2017]

8. $\left.\begin{array}{l} a-\frac{1}{b} = 5 \\ b-\frac{1}{a} = 15 \end{array}\right\} \Rightarrow \frac{a-b}{a} = ?$

A) −2 B) 2 C) 1 D) −1 E) 0

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2017]

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $\left.\begin{array}{l} x+\frac{1}{y}=2 \\ y+\frac{1}{x}=5 \end{array}\right\} \Rightarrow \frac{x+2y}{x-2y}=?$

A) –1,5 B) –1 C) 0 D) 1 E) 1,5

[KARADENİZ TEKNİK ÜNİVERSİTESİ – YÖS 2017]

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10. $\left.\begin{array}{l} A=\frac{20B}{100} \\ B=\frac{30C}{100} \\ A=\frac{k\cdot C}{100} \end{array}\right\} \Rightarrow k=?$

A) 3 B) 4 C) 5 D) 6 E) 7

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

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11. $\left.\begin{array}{l} \frac{x}{y}=\frac{7}{4} \\ x-y=\frac{3}{2} \end{array}\right\} \Rightarrow y=?$

A) –4 B) –1 C) 2 D) 6 E) 8

[ATATÜRK ÜNİVERSİTESİ – YÖS 2016]

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12. $a<0, b<0, c<0$

$\frac{a}{2}=\frac{b}{3}=4c$

Aşağıdakilerden hangisi doğrudur?

Which of the followings is true?

A) $c<b<a$ B) $b=c<a$

C) $a>c>b$ D) $c<a<b$

E) $c>a>b$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2015]

13. $x+\frac{1}{y}=2$

$y+\frac{1}{x}=5 \Rightarrow \frac{x+2y}{x-2y}=?$

A) –1,5 B) –1 C) 0 D) 1 E) 1,5

[KARADENİZ TEKNİK ÜNİVERSİTESİ – YÖS 2015]

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14. $2a=3c$

$c=2b$

$\frac{3a+b+2c}{c}=?$

A) $\frac{15}{2}$ B) $\frac{16}{3}$ C) $\frac{17}{4}$ D) 4 E) 7

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2014]

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15. $\frac{K}{L}=\frac{1}{7}$, $\frac{L}{M}=\frac{1}{4}$, $K+L+M=72 \Rightarrow L=?$

A) 2 B) 12 C) 14 D) 36 E) 56

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2014]

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16. $x, y, z \in Z$,

$\left.\begin{array}{l} \frac{x}{y}=\frac{2}{5} \\ \frac{z}{x}=\frac{4}{3} \end{array}\right\} \Rightarrow \frac{3x+2y}{z}=?$

A) 4 B) 5 C) 6 D) 7 E) 8

[SÜLEYMAN DEMİREL ÜNİVERSİTESİ – YÖS 2014]

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	D	B	A	E	D	E	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	D	C	E	A	E	C	C

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1. $\frac{2p + r + 3t + p + 2r + 3}{t + 3 + r + 4 + p - 6} = k$

$\frac{3(p + r + t + 1)}{p + r + t + 1} = k \Rightarrow k = 3$

$\frac{2r + 3}{p - 6} = 3 \Rightarrow 2r + 3 = 3p - 18$

$= 2r - 3p = -21$

Yanıt / The Answer is A

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2. $\frac{x}{y} = \frac{y}{t} = \frac{t}{z} = 3 \Rightarrow x = 3y,\ y = 3t,\ t = 3z$

$\frac{x - y + t - z}{y - t} = \frac{3y - y + 3z - z}{3t - t}$

$= \frac{2y + 2z}{2t} = \frac{y + z}{t}$

$= \frac{9z + z}{3z}$

$= \frac{10}{3}$

Yanıt / The Answer is D

---

3. $\frac{x+y}{9} = \frac{y+z}{15} = \frac{x+z}{12} = \frac{4}{3}$

$\frac{x+y+y+z+x+z}{36} = \frac{4}{3}$

$\frac{2(x+y+z)}{36} = \frac{4}{3}$

$x + y + z = 24$

Yanıt / The Answer is B

---

4. $\frac{x}{z+5} = \frac{y}{x+1} = \frac{z}{y-2} = \frac{3}{5}$

$\frac{x+y+z}{x+y+z+4} = \frac{3}{5}$

$5(x+y+z) = 3(x+y+z) + 12$

$x + y + z = 6$

Yanıt / The Answer is A

---

5. $\frac{a}{b} = -2$

$a = -2b$

$\frac{2b}{-2b} - \frac{3(-2b)}{b} =$

$-1 - (-6) = 5$

Yanıt / The Answer is E

---

6. $\frac{x-y}{2} = \frac{y+z}{3} = \frac{z}{4}$

$(2) \quad (-1)$

$\frac{2x - 2y - y - z}{4 - 3} = \frac{z}{4}$

$7 = \frac{z}{4}$

$28 = z$

$\frac{y + 28}{3} = \frac{z}{4} = 7$

$y = -7$

$\Rightarrow \frac{x - (-7)}{2} = \frac{z}{4}$

$\frac{x + 7}{2} = 7$

$x = 7$

Yanıt / The Answer is D

---

7. $\frac{x}{y} = \frac{3}{4},\ \frac{y}{z} = \frac{3}{2}$

$(3) \quad (4)$

$x = 9k$

$y = 12k$

$z = 8k$

$\Rightarrow \frac{12k - 9k}{9k - 8k} = 3$

Yanıt / The Answer is E

---

8. $a - \frac{1}{b} = 5 \Rightarrow ab - 1 = 5b, \quad b = 3k$

$b - \frac{1}{a} = 15 \Rightarrow ab - 1 = 15a, \quad a = k$

$\frac{a-b}{a} = \frac{k - 3k}{k} = -2$

Yanıt / The Answer is A

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9. $x+\frac{1}{y}=2 \Rightarrow xy+1=2y, \quad x=2k$

$y+\frac{1}{x}=5 \Rightarrow xy+1=5x \quad y=5k$

$\frac{x+2y}{x-2y}=\frac{2k+2\cdot5k}{2k-2\cdot5k}=\frac{12}{-8}=\frac{-3}{2}=-1,5$

Yanıt / The Answer is A

---

10. $A=\frac{20B}{100} \Rightarrow 10A=2B$

$B=\frac{30C}{100} \Rightarrow 10B=3C$

$A=\frac{k\cdot C}{100} \Rightarrow 100A=k\cdot C$

$100\cdot6=k\cdot100$

$6=k$

$B=10t, \; A=2t, \; B=3t, \; C=10t$ (÷3, ÷10)

$B=30, \; C=100, \; A=6$

Yanıt / The Answer is D

---

11. $\frac{x}{y}=\frac{7}{4} \Rightarrow x=7k$

$y=4k \Rightarrow y=4\cdot\frac{1}{2}$

$x-y=\frac{3}{2}$

$3k=\frac{3}{2}$

$k=\frac{1}{2}$

Yanıt / The Answer is C

---

12. $\frac{a}{2}=\frac{b}{3}=4c \Rightarrow a=8c, \; b=12c$

$a<0, \; b<0, \; c<0 \Rightarrow \mathbf{c>a>b}$

Yanıt / The Answer is E

---

13. $\left.\begin{array}{l} x+\frac{1}{y}=2 \Rightarrow xy+1=2y \\ y+\frac{1}{x}=5 \Rightarrow xy+1=5x \end{array}\right\} \Rightarrow 2y=5x$

$2y=5x \Rightarrow y=5k, \; x=2k$

$\frac{x+2y}{x-2y}=\frac{2k+10k}{2k-10k}=\frac{12}{-8}=-1,5$

Yanıt / The Answer is A

---

14. $\left.\begin{array}{l} 2a=3c \\ c=2b \end{array}\right\} \Rightarrow \begin{array}{l} a=3k \\ b=k \\ c=2k \end{array}$

$\Rightarrow \frac{3a+b+2c}{c}=\frac{9k+k+4k}{2k}$

$=\frac{14k}{2k}=7$

Yanıt / The Answer is E

---

15. $\left.\begin{array}{l} \frac{K}{L}=\frac{1}{7} \\ \frac{L}{M}=\frac{1}{4} \end{array}\right\} \Rightarrow \begin{array}{l} K=k \\ L=7k \\ M=28k \end{array}$

$\Rightarrow K+L+M=k+7k+28k=72$

$36k=72$

$k=2$

$L=7\cdot k=7\cdot2=14$

Yanıt / The Answer is C

---

16. $\left.\begin{array}{l} \frac{x}{y}=\frac{2}{5} \\ \frac{z}{x}=\frac{4}{3} \end{array}\right\} \Rightarrow \begin{array}{l} x=6k \\ y=15k \\ z=8k \end{array}$

$\Rightarrow \frac{3x+2y}{z}=\frac{18k+30k}{8k}=\frac{48k}{8k}=6$

Yanıt / The Answer is C

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1. $\frac{a+3b}{4a-b}=\frac{3}{4} \Rightarrow \frac{b}{a}=?$

A) $\frac{7}{12}$ B) $\frac{8}{15}$ C) $\frac{9}{17}$ D) $\frac{4}{3}$ E) $\frac{9}{2}$

2. $\frac{a}{b}=\frac{c}{d}=\frac{4}{5} \Rightarrow \frac{a+b}{b}+\frac{c+d}{d}=?$

A) $\frac{9}{5}$ B) $\frac{9}{10}$ C) $\frac{4}{5}$ D) $\frac{18}{5}$ E) $\frac{13}{5}$

3. $\frac{a+b}{b}=\frac{7}{3}$ ve / and $\frac{b}{c}=6 \Rightarrow \frac{a}{c}=?$

A) $\frac{9}{2}$ B) 8 C) $\frac{6}{5}$ D) 12 E) $\frac{16}{5}$

4. $\frac{a}{b}=\frac{c}{d}=\frac{2}{3} \Rightarrow \left(\frac{a-b}{b}\right)\cdot\left(\frac{c-d}{d}\right)=?$

A) $\frac{3}{8}$ B) $\frac{8}{9}$ C) $\frac{4}{9}$ D) $\frac{1}{9}$ E) $\frac{5}{6}$

5. $\frac{a+2b}{5}=a-2b \Rightarrow \frac{a}{b}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

6. $a:b:c=3:4:5 \Rightarrow \left(\frac{a+b}{b}\right)\cdot\left(\frac{b+c}{c}\right)=?$

A) 3 B) $\frac{63}{20}$ C) 15 D) $\frac{9}{5}$ E) $\frac{16}{5}$

7. $\frac{a+b}{2b}=3 \Rightarrow \frac{2a-b}{13b}=?$

A) $\frac{12}{13}$ B) $\frac{11}{13}$ C) $\frac{10}{13}$ D) $\frac{9}{13}$ E) $\frac{8}{13}$

8. $\frac{x}{y}=\frac{z}{t}=\frac{3}{2} \Rightarrow \left(\frac{x+y}{x}\right)\left(\frac{z+t}{t}\right)=?$

A) $\frac{3}{2}$ B) $\frac{25}{6}$ C) $\frac{5}{2}$ D) $\frac{25}{16}$ E) $\frac{16}{3}$

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9. $\frac{a}{3}=\frac{b}{5}=\frac{c}{9}$

$5a+4b-2c=51 \Rightarrow$ **a + b + c = ?**

A) 48 B) 51 C) 60

D) 63 E) 72

10. $\frac{x}{y}=\frac{2}{3}$, $\frac{y}{z}=\frac{2}{3}$

$x+y+z=152 \Rightarrow$ **z = ?**

A) 72 B) 64 C) 48

D) 40 E) 32

11. $\frac{a-2b}{b}=3 \Rightarrow \frac{a+4b}{2a+5b}=?$

A) $\frac{4}{9}$ B) $\frac{4}{5}$ C) $\frac{3}{5}$

D) $\frac{2}{7}$ E) $\frac{1}{3}$

12. $\frac{a}{8}=\frac{b}{5}=\frac{c}{4}$,

$c=a-6 \Rightarrow$ **b = ?**

A) 16 B) 9 C) 7

D) $\frac{27}{8}$ E) $\frac{15}{2}$

13. $\left.\begin{array}{l}\frac{x}{3}=\frac{2y}{5}=\frac{z}{4}\\ 2x-6y+z=5\end{array}\right\} \Rightarrow$ **x = ?**

A) 12 B) 9 C) 7,5

D) 3 E) –3

14. $\left.\begin{array}{l}a:b=2:5\\ c:b=3:7\\ a+b=(a-c)\cdot k\end{array}\right\} \Rightarrow$ **k = ?**

A) –49 B) –21 C) 21

D) 35 E) 49

15. $\left.\begin{array}{l}\frac{3}{x}=\frac{2}{y}=\frac{4}{z}\\ 2x-3y+z=2\end{array}\right\} \Rightarrow$ **z = ?**

A) 2 B) 3 C) 4

D) 5 E) 6

16. $x:y:(-4)=(-3):5:z$

$x+y=z^2 \Rightarrow$ **z = ?**

A) –4 B) –2 C) 2

D) 4 E) 6

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	D	B	D	C	B	D	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	A	C	E	E	A	A	B

1. $\frac{x}{y}=\frac{2}{3} \Rightarrow \frac{2x+y}{x+y}=?$

A) $\frac{12}{5}$ B) $\frac{7}{5}$ C) $\frac{25}{18}$ D) $\frac{9}{7}$ E) 5

2. $x:y:z=11:9:7 \Rightarrow \frac{x-y-z}{x+y}=?$

A) $-\frac{1}{2}$ B) $-\frac{1}{3}$ C) $-\frac{1}{4}$ D) $-\frac{1}{5}$ E) $-\frac{1}{6}$

3. $\frac{a}{b}=\frac{c}{d}=\frac{2}{5} \Rightarrow \left(\frac{a+c}{c}\right)\cdot\left(\frac{c}{d+b}\right)=?$

A) $\frac{5}{8}$ B) $\frac{2}{5}$ C) $\frac{16}{7}$ D) $\frac{3}{7}$ E) $\frac{7}{3}$

4. $\frac{a}{b}=\frac{c}{d}=\frac{d}{e}=\frac{4}{5} \Rightarrow \frac{b\cdot c\cdot d}{a\cdot d\cdot e}=?$

A) $\frac{4}{5}$ B) $\frac{5}{6}$ C) $\frac{64}{25}$ D) $\frac{3}{7}$ E) $\frac{7}{3}$

5. $\frac{a}{b}=\frac{3}{5}$, $\frac{b}{c}=\frac{5}{6} \Rightarrow \frac{c}{a}=?$

A) $\frac{1}{2}$ B) 2 C) $\frac{15}{4}$ D) 5 E) $\frac{5}{2}$

6. $\frac{a}{b}=\frac{5}{6}$ ve / and $\frac{b}{c}=\frac{3}{4} \Rightarrow a:b:c=?:?:?$

A) 5 : 12 : 8
B) 8 : 12 : 15
C) 5 : 6 : 8
D) 6 : 8 : 15
E) 3 : 4 : 6

7. $10:8:x=5:y:3 \Rightarrow (x+y)^2=?$

A) 36 B) 49 C) 64 D) 81 E) 100

8. $\frac{x}{y}=k$,
$x=18 \Rightarrow y=2$,
$y=6 \Rightarrow x=?$

A) 54 B) 162 C) 172 D) 180 E) 196

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9. $a,b,c \in Z^+$, $\frac{a}{6}=\frac{b}{5}$, $\frac{b}{c}=\frac{4}{3}$ $\Rightarrow$ **?** < **?** < **?**

A) a < b < c

B) b < c < a

C) b < a < c

D) a < c < b

E) c < b < a

10. $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$ $\Rightarrow$ $\frac{2x+3y}{4y-2z}$ = **?**

A) 5 B) 4 C) 3

D) 2 E) 1

11. $\frac{a}{2}=\frac{b}{4}=\frac{c}{3}$, $3a-2b+c=3$ $\Rightarrow$ **c = ?**

A) 18 B) 15 C) 12

D) 9 E)6

12. $\frac{x}{5}=\frac{y}{6}=\frac{z}{3}$, $x-3y+5z=8$ $\Rightarrow$ **z = ?**

A) 4 B) 6 C) 8

D) 10 E) 12

13. $\frac{3x+y}{z}=\frac{3y+z}{x}=\frac{3z+x}{y}=k$

$\Rightarrow$ **k = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

14. $\frac{a}{b}=\frac{c}{d}=\frac{4}{5}$

$\Rightarrow \left(\frac{a-b}{a+b}\right)\cdot\left(\frac{c+d}{c-d}\right)$ = **?**

A) 1 B) 4 C) 5

D) 9 E) 81

15. $\left.\begin{array}{l}\frac{x}{y}=2\\ \frac{y}{z}=3\\ x+y+z=20\end{array}\right\}$ $\Rightarrow$ **x = ?**

A) 5 B) 6 C) 9

D) 10 E) 12

16. $\left.\begin{array}{l}a=2b=3c\\ a+b+c=33\end{array}\right\}$ $\Rightarrow$ **b = ?**

A) 3 B) 6 C) 9

D) 12 E) 15

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	C	B	A	B	C	E	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	C	D	E	D	A	E	C

1. $\frac{a}{b}=\frac{7}{3} \Rightarrow \frac{a}{a+b}=?$

A) $\frac{1}{3}$ B) $\frac{4}{3}$ C) $\frac{3}{4}$ D) $\frac{3}{10}$ E) $\frac{7}{10}$

2. $a, b \in Z^+$,
$\frac{a}{b}=\frac{2}{5}$, $b^2-a^2=84 \Rightarrow a \cdot b=?$

A) 10 B) 15 C) 20 D) 40 E) 42

3. $\frac{2x-3}{5}=\frac{x}{3} \Rightarrow x=?$

A) 6 B) 7 C) 8 D) 9 E) 12

4. $\frac{a}{x}=\frac{b}{y}=\frac{c}{z}=\frac{4}{9} \Rightarrow \frac{x+y+z}{a+b+c}=?$

A) $\frac{8}{27}$ B) $\frac{2}{3}$ C) $\frac{9}{4}$ D) $\frac{14}{9}$ E) $\frac{4}{3}$

5. $\frac{x+y}{y}=\frac{5}{2} \Rightarrow \frac{3x-y}{x+2y}=?$

A) 1 B) 4 C) $\frac{3}{5}$ D) $\frac{5}{7}$ E) $\frac{11}{9}$

6. $\frac{x+y}{y}=4 \Rightarrow \frac{x}{x+y}=?$

A) $\frac{3}{4}$ B) $\frac{4}{3}$ C) $\frac{4}{5}$ D) $\frac{5}{4}$ E) 3

7. $a \in N$, $4:5=a^2:20 \Rightarrow a=?$

A) 1 B) 2 C) 3 D) 4 E) 5

8. $\frac{x}{7}=\frac{y}{4}$, $x-y=12 \Rightarrow x+y=?$

A) 24 B) 34 C) 44 D) 54 E) 64

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9. $\frac{a}{b}=\frac{2}{5}$, $\frac{b}{c}=\frac{5}{8}$, $a+c=40 \Rightarrow$ **b = ?**

A) 8 B) 16 C) 20

D) 24 E) 32

10. $\frac{3x-5}{2}=\frac{2x+5}{3} \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

11. $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z-1}{4}$, $x+y+z=38 \Rightarrow$ **z = ?**

A) 11 B) 13 C) 15

D) 17 E) 19

12. $\frac{x+1}{2}=\frac{y-2}{3}=\frac{z+3}{4}$, $3x-2y+z=12 \Rightarrow$ **x = ?**

A) 6 B) 8 C) 10

D) 12 E) 16

13. $\left.\begin{array}{l}5a=12b\\3a=8c\\a+b+c=86\end{array}\right\} \Rightarrow$ **a = ?**

A) 16 B) 24 C) 32

D) 36 E) 48

14. $\left.\begin{array}{l}\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{4}{3}\\3a+2c-e=32\\2d+3b=14\end{array}\right\} \Rightarrow$ **f = ?**

A) –12 B) –10 C) –8

D) 8 E) 10

15. $a:3:b=4:2:3$

$\Rightarrow$ **a + 2b = ?**

A) 15 B) 16 C) 17

D) 18 E) 19

16. $\frac{a}{3}=\frac{b}{5}=\frac{c}{7}=\frac{2a-b+3c}{2x+2} \Rightarrow$ **x = ?**

A) 6 B) 7 C) 8

D) 9 E) 10

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	D	D	C	A	A	D	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	E	D	C	E	B	A	E

1. $\frac{a}{4}=\frac{b}{5} \Rightarrow \frac{2a-b}{a+b}=?$

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$

D) 1 E) 2

2. $\left.\begin{array}{l}\frac{a}{10}=\frac{b}{20}=\frac{c}{30}\\ a+b+c=36\end{array}\right\} \Rightarrow b+c-a=?$

A) 6 B) 12 C) 18

D) 20 E) 24

3. $\left.\begin{array}{l}\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{2}{5}\\ a-2c+3e=28\\ 3f-2d=10\end{array}\right\} \Rightarrow b=?$

A) 60 B) 50 C) 40

D) 30 E) 20

4. $\frac{a}{b}=\frac{c}{d}=k,\ \frac{2a-x}{b+d}=2k \Rightarrow x=?$

A) $-2c$ B) $-\frac{2}{c}$ C) $-c$

D) $\frac{c}{2}$ E) $2c$

5. $\left.\begin{array}{l}\frac{x}{a}=\frac{y}{b}=\frac{5}{4}\\ x^2+y^2=125\end{array}\right\} \Rightarrow a^2+b^2=?$

A) 40 B) 50 C) 60

D) 70 E) 80

6. $\left.\begin{array}{l}ax=by=cz=\frac{1}{2}\\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=16\end{array}\right\} \Rightarrow x+y+z=?$

A) $\frac{1}{16}$ B) $\frac{1}{8}$ C) 1

D) 8 E) 16

7. $\left.\begin{array}{l}a\cdot b=4c\\ b\cdot c=9a\end{array}\right\} \Rightarrow \frac{a}{c}=?$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1

D) $\frac{4}{3}$ E) $\frac{5}{3}$

8. $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=\frac{4}{9} \Rightarrow \sqrt{\frac{a}{d}}=?$

A) $\frac{2}{3}$ B) $\frac{4}{9}$ C) $\frac{8}{27}$

D) $\frac{16}{81}$ E) $\frac{32}{243}$

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9. $\frac{x}{0,3} = \frac{3}{5}$, $3y + 10x = 5 \Rightarrow$ **y = ?**

A) 1 B) 2 C) 3
D) 4 E) 5

10. $\left.\begin{array}{l} ax = by = cz = \frac{3}{5} \\ xy + xz + yz = 120xyz \end{array}\right\} \Rightarrow$ **a + b + c = ?**

A) 18 B) 36 C) 48
D) 72 E) 96

11. a, b Z^+
$\frac{a}{3} = \frac{b}{5} = k \Rightarrow \sqrt{3a} + \sqrt{5b} =$ **?**

A) $\sqrt{k}$ B) $2\sqrt{k}$ C) $4\sqrt{k}$
D) $6\sqrt{k}$ E) $8\sqrt{k}$

12. $\left.\begin{array}{l} \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{6}{35} \\ ax = by = cz = 70 \end{array}\right\} \Rightarrow$ **x + y + z = ?**

A) 12 B) 14 C) 27
D) 36 E) 72

13. 6 işçinin günde 5 saat çalışarak 10 günde bitirebildiği bir işi, 4 işçi günde 3 saat çalışarak kaç günde bitirebilir?
A job which takes 10 days to finish by 6 workers working 5 hours a day will take how many days to finish if done by 4 workers working 3 hours a day

A) 10 B) 15 C) 20
D) 25 E) 30

14. x ile y sayıları sırasıyla 4 ve 5 ile ters orantılı, z sayısı 3 ile doğru orantılıdır?
The numbers x and y are inversely proportional to 4 and 5 respectively and the number z is directly proportional to 3.
$x + y + \frac{z}{4} = 12 \Rightarrow$ **z = ?**

A) 22 B) 24 C) 26
D) 28 E) 30

15. Aritmetik ortalaması 8,5 olan 14 sayının toplamı kaçtır?
What is the sum of 14 numbers with arithmetic mean of 8.5?

A) 109 B) 112 C) 115
D) 118 E) 119

16. Beş kişilik bir basketbol takımının yaş ortalaması 16 dır. Bu takıma 3 kişi daha katılırsa oluşan yeni takımın yaş ortalaması 22 olduğuna göre takıma sonradan katılan 3 oyuncunun yaşlarının toplamı kaçtır?
The average age of a basketball team of 5 people is 16. Three more basketballists attended to the team and made the new average age of the team 22. Accordingly, what is the sum of the ages of the new comers?

A) 49 B) 58 C) 74
D) 38 E) 96

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	E	A	A	E	D	B	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	D	E	A	D	E	E	E

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BÖLÜM CHAPTER 7

TARİHSEL NOT / HISTORICAL NOTE

Abel, Niels Henrik [1802 – 1829]

Norveçli matematikçi 19 yaşında dörtten büyük dereceli denklemlerin çözülemeyeceğini göstermiştir. Başka bir deyişle, bu denklemlerin ikinci dereceden denklemler gibi formülünün olmadığını kanıtlamıştır. Aynı zamada cebirsel fonksiyonlar teorisindeki temel gelişmeleri yanıtlamıştır.

Norwegian mathematician who at the age of 19, proved that the general equation of degree greater than 4 cannot be solved algebraically. In other words, there can be no formula for the roots of such an equation similar to the familiar formula for the quadratic equation. He was also responsible for fundamental developments in theory of algebraic functions.

BİRİNCİ DERECEDEN DENKLEMLER
FIRST DEGREE EQUATIONS

Bu bölüm, 23 çözümlü örnek, 12 çözümlü test sorusu, 16 çözümlü YÖS sorusu, 64 test sorusu içermektedir.

This chapter includes 23 examples with solutions, 12 tests questions with solutions, 16 YÖS questions with solutions and 64 test questions.

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BÖLÜM
07
CHAPTER

BİRİNCİ DERECEDEN DENKLEMLER
FIRST DEGREE EQUATIONS

ÖNEMLİ / IMPORTANT

- $ax + b = cx + d$

  I. $a = c,\ b = d \Rightarrow$ ÇK(SS) = R

  II. $a = c,\ b \neq d \Rightarrow$ ÇK(SS) = Ø

BİRİNCİ DERECEDEN BİR BİLİNMEYENLİ DENKLEMLER
FIRST DEGREE EQUATIONS in ONE VARIABLE

Tanım: a ve b reel sayı ve $a \neq 0$ olmak Üzere,

$ax + b = 0$ ifadesine *birinci dereceden bir bilinmeyenli denklem* denir. Bu denklemi sağlayan, x reel sayısına da *denklemin kökü* denir. Denklemi sağlayan x değerlerinin kümesine *çözüm kümesi* denir ve *ÇK* ile gösterilir.

Definition: A first degree equation in one variable x is an equation that can be written in the form $ax + b = 0$, where a and b are real numbers, and $a \neq 0$. The real number x which satisfy the equation is called the *root of this equation*. The set of the values of x which satisfy the equation is called *solution set* and showen by *SS*.

$$ax + b = 0 \Rightarrow ax = -b$$

$$x = -\frac{b}{a} \Rightarrow \text{ÇK(SS)} = \left\{-\frac{b}{a}\right\}$$

$ax + b = 0$

I. $a = 0,\ b = 0 \Rightarrow$ ÇK(SS) = R

II. $a = 0,\ b \neq 0 \Rightarrow$ ÇK(SS) = Ø

ÖNEMLİ / IMPORTANT

- Birinci dereceden bir bilinmeyenli denklemlerin çözümü yapılırken bilinmeyen yok edildiğinde kalan sayısal değerler birbirine eşit ise çözüm kümesi reel sayılar eşit değil ise çözüm kümesi boş kümedir.
- When solving first degree equations with one unknown, if the numerical values after dissipating the unknown are equal, the solution set is all real numbers and if not, the solution set is empty set.

Özellik / Property

★ $a, b, c \in R$

$a = b \Rightarrow a \mp c = b \mp c$

Özellik / Property

★ $a, b, c \in R$

$a = b \Rightarrow a \cdot c = b \cdot c$

Özellik / Property

★ $a, b, c \in R,\ c \neq 0$

$a = b \Rightarrow a : c = b : c$

Örnek / Example 1

$2x + 6 = 10 \Rightarrow$ ÇK(SS) = ?

A) {1} B) {2} C) {3} D) {4} E) {5}

Çözüm / Solution

Yanıt / Answer **B**

$$2x + 6 = 10$$
$$2x + 6 - 6 = 10 - 6$$
$$2x = 4$$
$$x = 2$$
$$ÇK(SS) = \{2\}$$

Örnek / Example 2

$3(x - 1) + 8 = 2(x - 5) \Rightarrow$ ÇK(SS) = ?

A) {–5} B) {–7} C) {–13} D) {–15} E) {–18}

Çözüm / Solution

Yanıt / Answer **D**

$$3(x - 1) + 8 = 2(x - 5)$$
$$3x - 3 + 8 = 2x - 10$$
$$3x + 5 = 2x - 10$$
$$3x - 2x = -10 - 5$$
$$x = -15 \Rightarrow ÇK(SS) = \{-15\}$$

Örnek / Example 3

$3x - 2 - [2x - 3(4 - x)] + 4 = x - 4 \Rightarrow$ ÇK(SS) = ?

A) {3} B) {4} C) {6} D) {8} E) {9}

Çözüm / Solution

Yanıt / Answer **C**

$$3x - 2 - [2x - 12 + 3x] + 4 = x - 4$$
$$3x - 2 - [5x - 12] = x - 4 - 4$$
$$3x - 2 - 5x + 12 = x - 8$$
$$10 - 2x = x - 8$$
$$10 + 8 = 3x$$
$$18 = 3x$$
$$x = 6$$
$$ÇK(SS) = \{6\}$$

PEKİŞTİRME / REINFORCE

- ✓ $x - 2 = 6 \Rightarrow$ ÇK(SS) = ? ..... [{8}]
- ✓ $3x - 4 = 8 \Rightarrow$ ÇK(SS) = ? ..... [{4}]
- ✓ $3(x - 1) = 9 \Rightarrow$ ÇK(SS) = ? ..... [{4}]
- ✓ $2x + 4 = x - 8 \Rightarrow$ ÇK(SS) = ? ..... [{–12}]
- ✓ $3x - 7 = 2x - 5 \Rightarrow$ ÇK(SS) = ? ..... [{2}]

PEKİŞTİRME / REINFORCE

✓ $3 \cdot (2x - 1) - 2(5x + 4) = 29 \Rightarrow x = ?$

..... [–10]

Örnek / Example 4

$3x - 2 + 2 \cdot (x - 2) = 4 \cdot (x - 3) \Rightarrow x = ?$

A) –8 B) –6 C) –4 D) –2 E) –1

Çözüm / Solution Yanıt / Answer **B**

$3x - 2 + 2 \cdot (x - 2) = 4 \cdot (x - 3)$

$3x - 2 + 2x - 4 = 4x - 12$

$5x - 4x = -12 + 6$

$x = -6$

PEKİŞTİRME / REINFORCE

✓ $\frac{4x}{3} - 15 - \frac{3x}{2} + 10 = 20 \Rightarrow x = ?$

..... [–150]

Örnek / Example 5

$\frac{x-1}{2} - \frac{x-2}{3} = \frac{2}{3} \Rightarrow ÇK(SS) = ?$

A) {5} B) {3} C) $\{\frac{2}{3}\}$ D) $\{\frac{4}{3}\}$ E) $\{\frac{5}{3}\}$

Çözüm / Solution Yanıt / Answer **B**

$\frac{x-1}{2} - \frac{x-2}{3} = \frac{2}{3} \Rightarrow 3(x-1) - 2(x-2) = 4$

$3x - 3 - 2x + 4 = 4$

$x + 1 = 4$

$x + 1 - 1 = 4 - 1$

$x = 3$

$ÇK(SS) = \{3\}$

PEKİŞTİRME / REINFORCE

✓ $\frac{3x-2}{4} - \frac{2x+1}{3} = -\frac{5}{12} \Rightarrow x = ?$

..... [5]

Örnek / Example 6

$\frac{x}{5} - 1 = \frac{x}{2} - 4 \Rightarrow x = ?$

A) 5 B) 7 C) 8 D) 9 E) 10

Çözüm / Solution Yanıt / Answer **E**

$\frac{x}{5} - \frac{1}{1} = \frac{x}{2} - \frac{4}{1} \Rightarrow \frac{2x}{10} - 1 + 4 = \frac{5x}{10}$

$3 = \frac{5x}{10} - \frac{2x}{10}$

$3 = \frac{3x}{10}$

$x = 10$

Örnek / Example 7

$\frac{0,1}{x} - \frac{4}{x} = 3 \Rightarrow$ ÇK(SS) = ?

A) $\left\{-\frac{13}{10}\right\}$ B) $\left\{-\frac{17}{10}\right\}$ C) $\left\{-\frac{19}{10}\right\}$ D) $\left\{-\frac{21}{10}\right\}$ E) $\left\{-\frac{23}{10}\right\}$

Çözüm / Solution

Yanıt / Answer **A**

$$\frac{0,1}{x} - \frac{4}{x} = 3 \Rightarrow \frac{-3,9}{x} = 3$$
$$x = \frac{-3,9}{3}$$
$$x = -1,3 = -\frac{13}{10}$$

$\Rightarrow$ ÇK(SS) = $\left\{-\frac{13}{10}\right\}$

Örnek / Example 8

$\frac{2,\overline{4}}{x-1} = 0,\overline{4} \Rightarrow$ ÇK(SS) = ?

A) $\left\{\frac{9}{2}\right\}$ B) $\left\{\frac{13}{2}\right\}$ C) $\left\{\frac{15}{2}\right\}$ D) $\left\{\frac{17}{2}\right\}$ E) $\left\{\frac{19}{2}\right\}$

Çözüm / Solution

Yanıt / Answer **B**

$$\frac{2,\overline{4}}{x-1} = 0,\overline{4} \Rightarrow \frac{\frac{22}{9}}{x-1} = \frac{4}{9} \Rightarrow \frac{22}{9(x-1)} = \frac{4}{9}$$
$$\Rightarrow 22 = 4x - 4$$
$$\Rightarrow 26 = 4x$$
$$\Rightarrow x = \frac{26}{4} = \frac{13}{2}$$
$$\Rightarrow \text{Ç·K(SS)} = \left\{\frac{13}{2}\right\}$$

Örnek / Example 9

$\frac{2x-1}{4} + \frac{5}{6} = \frac{x}{3} + \frac{1}{3} \Rightarrow x = ?$

A) $-\frac{5}{2}$ B) -2 C) $-\frac{3}{2}$ D) -1 E) $-\frac{1}{2}$

Çözüm / Solution

Yanıt / Answer **C**

$$\underset{(3)}{\frac{2x-1}{4}} + \underset{(2)}{\frac{5}{6}} = \underset{(4)}{\frac{x}{3}} + \underset{(4)}{\frac{1}{3}} \Rightarrow \frac{6x-3+10}{12} = \frac{4x+4}{12}$$
$$6x + 7 = 4x + 4$$
$$6x - 4x = 4 - 7$$
$$2x = -3$$
$$x = -\frac{3}{2}$$

PEKİŞTİRME / REINFORCE

✓ $\frac{2}{x} + \frac{3}{2x} + \frac{4}{3x} = \frac{29}{12} \Rightarrow x = ?$

..... [2]

Örnek / Example 10

$\frac{x}{\frac{1}{2}+1} + 1 = \frac{3x+1}{3} \Rightarrow$ **ÇK(SS) = ?**

A) {–2} B) {–1} C) {0} D) {1} E) {2}

Çözüm / Solution Yanıt / Answer **E**

$$\frac{x}{\frac{1}{2}+1} + 1 = \frac{3x+1}{3}$$

$$\frac{x}{\frac{3}{2}} + 1 = \frac{3x}{3} + \frac{1}{3}$$

$$\frac{2x}{3} - \frac{3x}{3} = -1 + \frac{1}{3}$$

$$\frac{-x}{3} = \frac{-2}{3}$$

$x = 2 \Rightarrow$ ÇK(SS) = {2}

Örnek / Example 11

$2x - \frac{3}{x+4} = -8 - \frac{3}{x+4} \Rightarrow$ **ÇK(SS) = ?**

A) {–4} B) {–2} C) {0} D) ∅ E) R

Çözüm / Solution Yanıt / Answer **D**

$$2x - \frac{3}{x+4} = -8 - \frac{3}{x+4}$$

$$2x = -8$$

$$x = -4$$

$x = -4 \Rightarrow x + 4 = 0 \Rightarrow$ ÇK(SS) = ∅

PEKİŞTİRME / REINFORCE

✓ $a \neq b \neq 0$

$\frac{ax}{b} - \frac{bx}{a} = 1 - \left(\frac{a}{b}\right)^2 \Rightarrow x = ?$

..... $\left[-\frac{a}{b}\right]$

Örnek / Example 12

$3x + 12 + 2\cdot(4 - x) = 2(x + 1) - (x - 18) \Rightarrow$ **ÇK(SS) = ?**

A) 6 B) 12 C) 18 D) ∅ E) R

Çözüm / Solution Yanıt / Answer **E**

$3x + 12 + 8 - 2x = 2x + 2 - x + 18$

$x + 20 = x + 20$

$\Rightarrow$ ÇK(SS) = R

Örnek / Example 13

$\frac{x+2}{x-3}-\frac{4-x}{3-x}=-6 \Rightarrow$ ÇK(SS) = ?

A) {1} B) {2} C) {3} D) {4} E) {5}

Çözüm / Solution

Yanıt / Answer **B**

$$\frac{x+2}{x-3}-\frac{4-x}{3-x}=-6$$

$$\frac{x+2}{x-3}+\underbrace{\frac{-4+x}{3-x}}_{(-1)}=-6$$

$$\frac{x+2+4-x}{x-3}=-6$$

$$\frac{6}{x-3}=-6$$

$$x-3=-1$$

$$x=2 \Rightarrow \text{ÇK(SS)} = \{2\}$$

Örnek / Example 14

$\frac{3x-1}{x-2}+\frac{4}{x}=\frac{5}{x-2}+\frac{2x-8}{4-x} \Rightarrow x = ?$

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $-\frac{4}{5}$ E) $-\frac{5}{6}$

Çözüm / Solution

Yanıt / Answer **D**

$$\frac{3x-1}{x-2}+\frac{4}{x}=\frac{5}{x-2}+\frac{2(x-4)}{4-x}$$

$$\frac{3x-1}{x-2}+\frac{4}{x}+\frac{-5}{x-2}=-2$$

$$\frac{3(x-2)}{(x-2)}=-2-\frac{4}{x}$$

$$3=\frac{-2x-4}{x}$$

$$5x=-4$$

$$x=-\frac{4}{5}$$

BİRİNCİ DERECEDEN İKİ BİLİNMEYENLİ DENKLEM SİSTEMLERİ
SYSTEMS of FIRST DEGREE EQUATIONS with TWO UNKNOWNS

Tanım: a, b, c ∈ R ve a ≠ 0 ve b ≠ 0 olmak üzere, ax + by + c = 0 biçiminde verilen denklemlere *birinci dereceden iki bilinmeyenli denklem* denir.

Definition: Provided that a, b, c ∈ R, a ≠ 0, b ≠ 0, the equations in the ax + by + c = 0 form are called first degree equations with two unknowns.

Tanım: Denklemi sağlayan x ve y değerlerine *denklemin kökleri* denir. Birinci dereceden iki bilinmeyenli denklemlerin çözüm kümesi ikililerden oluşur. Birinci dereceden iki bilinmeyenli denklem, koordinat sisteminde bir doğru belirtir.

$a \neq 0, b \neq 0, a, b, c, d, e, f \in R \Rightarrow$

$ax + by = c$

$dx + ey = f$

denklemlerine *birinci dereceden iki bilinmeyenli denklem sistemi* denir.

Definition: x and y values which satisfy the equation are called the *roots of equation*. The equations in that form have infinite roots and define a line in coordinate system.

$a \neq 0, b \neq 0, a, b, c, d, e, f \in R \Rightarrow$

$ax + by = c$

$dx + ey = f$

System in a form as shown above are called *system of first degree equations with two unknowns*.

Bu denklem sisteminde,

1. $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$ ise çözüm kümesi reel sayılardır.

2. $\frac{a}{d} = \frac{b}{e} \neq \frac{c}{f}$ ise çözüm kümesi boş kümedir.

3. $\frac{a}{d} \neq \frac{b}{e}$ ise çözüm kümesi bir elemanlıdır.

In this equation system,

1. If $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$, then the solution set is real numbers.

2. If $\frac{a}{d} = \frac{b}{e} \neq \frac{c}{f}$, then the solution set is an empty set.

3. If $\frac{a}{d} \neq \frac{b}{e}$, then the solution set has one element.

Etkinlikler / Activities

a. $\left.\begin{matrix} 5x+3y=2 \\ 10x+6y=4 \end{matrix}\right\}$

Denklemlerin katsayılarının oranı $\frac{5}{10}=\frac{3}{6}=\frac{2}{4}$ olduğundan bütün (x,y) reel sayı ikilileri denklemi sağlar.

The ratio of coefficients of equations is $\frac{5}{10}=\frac{3}{6}=\frac{2}{4}$. Therefore, all real number ordered pairs (x,y) satify the equation.

b. $\left.\begin{matrix} 4x+5y=12 \\ \frac{12}{5}x+3y=\frac{36}{5} \end{matrix}\right\} \Rightarrow$ **ÇK(SS) = ?**

denklemlerin katsayılarının oranı, $\frac{4}{\frac{12}{5}}=\frac{5}{3}=\frac{12}{\frac{36}{5}}$ olduğundan bütün (x,y) reel sayı ikilileri denklemi sağlar.

As the ratio of coefficients of equations is $\frac{4}{\frac{12}{5}}=\frac{5}{3}=\frac{12}{\frac{36}{5}}$, all real number ordered pairs (x,y) satify the equation.

c. $\left.\begin{matrix} 3x-4y=-7 \\ -6x+8y=5 \end{matrix}\right\}$

denklemlerin katsayılarının oranı, $\frac{3}{-6}=\frac{-4}{8}\neq\frac{-7}{5}$ olduğundan sistemin çözüm kümesi boş kümedir.

As ratio of coefficients of equations is $\frac{3}{-6}=\frac{-4}{8}\neq\frac{-7}{5}$, the solution set is an empty set.

d. $\left.\begin{matrix} 4x-3y=m \\ -8x+6y=10 \end{matrix}\right\}$ ÇK(SS) = R $\Rightarrow$ **m = ?**

$\frac{4}{-8}=\frac{-3}{6}=\frac{m}{10}$

$6m=-30$

$m=-5$

e. $\left.\begin{matrix} mx+3y=5 \\ 4x+12y=7 \end{matrix}\right\}$ ÇK(SS) = $\varnothing \Rightarrow$ **m = ?**

$\frac{m}{4}=\frac{3}{12}\neq\frac{5}{7}$

$12m=12$

$m=1$

BİRİNCİ DERECEDEN İKİ BİLİNMEYENLİ DENKLEM SİSTEMLERİNİN ÇÖZÜMÜ
SOLUTION of SYSTEMS of FIRST DEGREE EQUATIONS with TWO UNKNOWNS

YOK ETME YÖNTEMİ / THE ELIMINATION METHOD

Bu yöntemde değişkenlerden biri, diğer denklemde yer alan aynı değişkenin aynı büyüklükte katsayılı, fakat zıt işaretli değeri ile çarpılarak değişkenlerden biri yok edilir.

- Bir değişkeni yok etmek için verilen denklemleri toplayın veya çıkarın.
- Kalan değişken için elde edilen denklemi çözün.
- Bilinmeyen değişkenin değerini bulmak için, ana denklemlerden herhangi birinde bulunan değişkenin değerini yazın.

In the method of elimination, we eliminate (remove) one of the variables by making the coefficients of one of variables the same size but opposite in sign in two equations and then adding the equations.

- Add or subtract the given equations to eliminate one variable.
- Solve the resulting equation for the remaining variable.
- Replace the value of the known variable in one of the original equations to find the value of the other unknown variable.

PEKİŞTİRME / REINFORCE

✓ $3x - 2y = 16$,
$x + y = 2$
$\Rightarrow (x,y) = (?, ?)$
..... [(4,–2)]

Örnek / Example 15

$\left.\begin{matrix} 2x - y = -1 \\ x - 2y = 4 \end{matrix}\right\} \Rightarrow$ ÇK(SS) = ?

A) {(–3, 2)} B) {(3, –2)} C) {(3, 2)}
D) {(–2, –2)} E) {(–2, –3)}

Çözüm / Solution Yanıt / Answer **E**

$$\left.\begin{matrix} -2 \,/\, 2x - y = -1 \\ x - 2y = 4 \end{matrix}\right\} \Rightarrow \begin{matrix} -4x + 2y = 2 \\ + \quad x - 2y = 4 \\ \hline -3x = 6 \Rightarrow x = -2 \end{matrix}$$

$x = -2 \Rightarrow 2 \cdot (-2) - y = -1$

$y = -4 + 1 = -3$

$\Rightarrow$ ÇK(SS) = {(–2, –3)}

Örnek / Example 16

$\left.\begin{matrix} x+2y=11 \\ 2x-y=7 \end{matrix}\right\} \Rightarrow$ ÇK(SS) = ?

A) {(3, 3)} B) {(5, 1)} C) {(5, 3)} D) {(3, –5)} E) {(5, –3)}

Çözüm / Solution

Yanıt / Answer **C**

$$\begin{array}{r} x+2y=11 \\ 2/\ 2x-y=7 \\ \hline \end{array} \Rightarrow \begin{array}{r} x+2y=11 \\ +\ 4x-2y=14 \\ \hline 5x=25 \Rightarrow x=5 \end{array}$$

$x = 5 \Rightarrow 5 + 2y = 11$

$2y = 11 - 5$

$2y = 6 \Rightarrow y = 3$

$\Rightarrow$ ÇK(SS) = {(5, 3)}

Örnek / Example 17

$\left.\begin{matrix} 3x+5y=29 \\ 2x-3y=-6 \end{matrix}\right\} \Rightarrow$ ÇK(SS) = ?

A) {(1, 2)} B) {(2, 3)} C) {(3, 4)} D) {(2, 4)} E) {(1, 3)}

Çözüm / Solution

Yanıt / Answer **C**

$$\begin{array}{r} 3 / 3x+5y=29 \\ 5 / 2x-3y=-6 \\ \hline 9x+15y=87 \\ +\ 10x-15y=-30 \\ \hline 19x=57 \\ x=3 \end{array}$$

$x = 3 \Rightarrow 2 \cdot 3 - 3y = -6$

$-3y = -12$

$y = 4$

$\Rightarrow$ ÇK(SS) = {(3, 4)}

PEKİŞTİRME / REINFORCE

✓ $ax + by = 6$,
$bx - ay = 3$,
ÇK = (2, –1) $\Rightarrow$ **a = ?**

..... [3]

YERİNE KOYMA YÖNTEMİ / THE SUBSTITUTION METHOD

Bu yöntemde değişkenlerden biri diğer değişken cinsinden yazılır. Seçilen değişken diğer denklemde yazılarak bir bilinmeyenli denklem haline getirilir.

- İki denklemden birinde, bir değişkeni diğeri cinsinden yazınız.
- Bu değişkeni, diğer denklemde yerine yazınız.
- Bir değişkenin değerini bu denklemden bulunuz.
- İkinci adımda bulunan denklemdeki sonuca, bulunan değeri yazın ve diğer değişkeni bulunuz.

In this method one of the variables is written in terms of the other variable. The chosen variable gets written in the other equation and form an equation with one unknown.

- In either equation, solve for one variable in terms of the other.
- Substitute for that variable in the other equation.
- Solve the resulting equation for the value of one variable.
- Substitute the result from step 2 in either equation. Solve for the other variable.

Örnek / Example 18

$\left.\begin{matrix} 3x - 4y = 7 \\ 2x + y = 12 \end{matrix}\right\} \Rightarrow$ ÇK(SS) = ?

A) {(2, 5)} B) {(4, 4)} C) {(3, 6)}
D) {(1, 10)} E) {(5, 2)}

Çözüm / Solution

Yanıt / Answer **E**

$2x + y = 12 \Rightarrow y = 12 - 2x$

y'nin bu değeri $3x - 4y = 7$ denkleminde yerine konulursa,

Putting the value of y in equation $3x - 4y = 7$,

$$3x - 4(12 - 2x) = 7$$
$$3x - 48 + 8x = 7$$
$$11x = 55$$
$$x = 5 \text{ ve } y = 12 - 2 \cdot 5 = 2$$

ÇK(SS) = {(5,2)} bulunur / is found.

ÖZEL DENKLEMLER / SPECIAL EQUATIONS

Birkaç değişkenden oluşan bir denklem sistemini çözmek için, toplama veya çıkarma ile değişkenlerden biri yok etmeye çalışılır. Eğer iki denklemde de değişkenlerin kat sayıları birbirine eşitse, bu değişkenler toplama veya çıkarma yoluyla yok edilir.

Üç bilinmeyenli üç denklemi çözmek için aşağıdaki adımlar izlenir:

- İki denklem kullanılarak, bir değişken yok edilir. Elde edilen denklem, iki değişken içeren yeni bir denklemdir.
- Verilen denklemlerin kombinasyonları kullanılarak aynı değişkenler yok edilir. Sonuçta iki değişkenden oluşan yeni denklem elde edilir.
- 1. ve 2. adımda elde edilen iki yeni denklem çözülür.
- Değişkenlerden birinin değeri bulunup diğerlerinde yerine yazılarak diğer değişkenler elde dilir.

In solving a system of equations containing several unknowns, we try to eliminate one unknown by addition or subtraction. If the coefficients of one variable are numerically equal in two equations, then that variable may be eliminated by adding or subtracting the two equations.

To solve a system of three equations with three unknowns, we have the following steps:

- Eliminate one variable using two of the equations. The result is a new equation containing, at most, two variables.
- Eliminate the same variable using another set or combination of the given equations. The result is another equation containing, at most, two variables.
- Solve the two new equations obtained in Steps 1 and 2.
- When the value of one variable has been found, work backward through the equations to find the other variables.

Örnek / Example 19

$$\left.\begin{array}{l} x+y=11 \\ y+z=8 \\ x+z=9 \end{array}\right\} \Rightarrow x \cdot y \cdot z = ?$$

A) 45 B) 60 C) 72 D) 84 E) 90

Çözüm / Solution

Yanıt / Answer **E**

$$\begin{array}{r} x+y=11 \\ y+z=8 \\ +\ x+z=9 \\ \hline 2x+2y+2z=28 \end{array} \Rightarrow x+y+z=14$$

$$11+z=14 \Rightarrow z=14-11=3$$

$$x+8=14 \Rightarrow x=14-8=6$$

$$9+y=14 \Rightarrow y=14-9=5$$

$$\Rightarrow x \cdot y \cdot z = 3 \cdot 6 \cdot 5 = 90$$

PEKİŞTİRME / REINFORCE

✓ $x \cdot y = 12$,
$y \cdot z = 6$,
$x \cdot z = 2 \Rightarrow x+y+z = ?$

..... [9]

PEKİŞTİRME / REINFORCE

✓ $x + y = 2z$,
$x + y - z = 10$,
$x - y + z = 12 \Rightarrow x = ?$

..... [11]

Örnek / Example 20

$$\left.\begin{array}{l} 3x + 4y + 3z = 29 \\ x + 3y + z = 13 \end{array}\right\} \Rightarrow y = ?$$

A) 2 B) 3 C) 4 D) 5 E) 6

Çözüm / Solution Yanıt / Answer **A**

$$\begin{array}{rl} & 3x + 4y + 3z = 29 \\ -3/ & x + 3y + z = 13 \\ \hline & 3x + 4y + 3z = 29 \\ + & -3x - 9y - 3z = -39 \\ \hline & -5y = -10 \end{array}$$

$$y = \frac{-10}{-5} = 2$$

Örnek / Example 21

$$\left.\begin{array}{l} \frac{1}{x} + \frac{1}{y} = \frac{5}{6} \\ \frac{1}{x} + \frac{1}{z} = \frac{3}{4} \\ \frac{1}{y} + \frac{1}{z} = \frac{7}{12} \end{array}\right\} \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = ?$$

A) $\frac{13}{12}$ B) $\frac{11}{12}$ C) $\frac{15}{12}$ D) $\frac{17}{12}$ E) $\frac{19}{12}$

Çözüm / Solution Yanıt / Answer **A**

$$\begin{array}{l} \frac{1}{x} + \frac{1}{y} = \frac{5}{6} \\ \frac{1}{x} + \frac{1}{z} = \frac{3}{4} \\ + \ \frac{1}{y} + \frac{1}{z} + \frac{7}{12} \\ \hline \end{array}$$

$$\frac{2}{x} + \frac{2}{y} + \frac{2}{z} = \underset{(2)}{\frac{5}{6}} + \underset{(3)}{\frac{3}{4}} + \frac{7}{12}$$

$$2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = \frac{10 + 9 + 7}{12} = \frac{26}{12}$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{13}{12}$$

Örnek / Example 22

$$\left.\begin{array}{l} 7a-3b+6c=19 \\ 3a-4b+5c=10 \\ 5a+2b+2c=3 \end{array}\right\} \Rightarrow \mathbf{a+b+c=?}$$

A) –7 B) –6 C) 3 D) 6 E) 7

Çözüm / Solution

Yanıt / Answer **B**

$$\begin{array}{rl} -1/\ 7a-3b+6c=19 \Rightarrow & -7a+3b-6c=-19 \\ 3a-4b+5c=10 \Rightarrow & 3a-4b+5c=10 \\ 5a+2b+2c=3 \Rightarrow & +\ 5a+2b+2c=3 \\ \hline & a+b+c=-6 \end{array}$$

Örnek / Example 23

$$\left.\begin{array}{l} x+2y+3z=15 \\ 2x+y+z=10 \\ 3x+3y+2z=29 \end{array}\right\} \Rightarrow \mathbf{x+y+z=?}$$

A) 7 B) 8 C) 9 D) 10 E) 11

Çözüm / Solution

Yanıt / Answer **C**

$$\begin{array}{rl} & x+2y+3z=15 \\ & 2x+y+z=10 \\ + & 3x+3y+2z=29 \\ \hline & 6x+6y+6z=54 \\ & x+y+z=9 \end{array}$$

1. $2x - 6 - x = 6x + 8 - 24 \Rightarrow x = ?$

A) 0 B) 2 C) 4

D) 6 E) 8

2. $6 \cdot (x - 2) - 8 \cdot (x - 3) = 2 \cdot (4 + x) \Rightarrow x = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

3. $\frac{2x}{3} + 4 = \frac{3x}{4} + 3 \Rightarrow x = ?$

A) 12 B) 16 C) 18

D) 20 E) 40

4. $x - (2 - x) \cdot (4 + x) = -(2 - x) \cdot (2 + x) \Rightarrow x = ?$

A) -4 B) -3 C) $\frac{1}{4}$

D) $\frac{4}{3}$ E) 2

5. $3 \cdot (x - 1) = x - 3 + 2x \Rightarrow$ ÇK (SS) = ?

A) Ø B) {0} C) {1}

D) {2} E) R

6. $\frac{x+1}{2} + \frac{x-2}{4} = \frac{x}{4} - 1 \Rightarrow x = ?$

A) -4 B) -3 C) -2

D) 2 E) 3

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7. $y - 2\{y - 2 \cdot [y - 2 \cdot (y - 2)]\} = 21 \Rightarrow y = ?$

A) -2 B) -1 C) 0
D) 1 E) 2

8. $3ax - b = bx - 3a \Rightarrow x = ?$

A) $-a$ B) -1 C) 1
D) $\frac{3}{2}$ E) a

9. $\frac{x-1}{2} + \frac{x+1}{4} = 2x - \frac{x-1}{4} \Rightarrow x = ?$

A) $-\frac{1}{2}$ B) $-\frac{1}{3}$ C) $\frac{1}{4}$
D) $\frac{1}{3}$ E) $\frac{1}{2}$

10. $\left.\begin{array}{l} 2x = 3y \\ \frac{x}{3} + 2y = 10 \end{array}\right\} \Rightarrow y = ?$

A) 1 B) 2 C) 4
D) 6 E) 8

11. $\left.\begin{array}{l} 3x - 5y + 2z = -11 \\ 6x - 2y + 5z = 7 \end{array}\right\} \Rightarrow x + y + z = ?$

A) 1 B) 2 C) 3
D) 4 E) 6

12. $\left.\begin{array}{l} a + b = 14 \\ b + c = 12 \\ a + c = 16 \end{array}\right\} \Rightarrow 2a + b - c = ?$

A) 12 B) 13 C) 14
D) 15 E) 16

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
B	A	A	D	E	C	B	B	A	C	E	E

1. $2x-6-x=6x+8-24$

$x-6=6x-16$

$x-6x=6-16$

$-5x=-10$

$x=2$

Yanıt / The Answer is B

2. $6\cdot(x-2)-8\cdot(x-3)=2\cdot(4+x)$

$6x-12-8x+24=8+2x$

$-2x+12=8+2x$

$-2x-2x=8-12$

$-4x=-4$

$x=1$

Yanıt / The Answer is A

3. $\frac{2x}{3}+4=\frac{3x}{4}+3$

$\underset{(4)}{\frac{2x}{3}}-\underset{(3)}{\frac{3x}{4}}=3-4$

$\frac{8x}{12}-\frac{9x}{12}=-1$

$-\frac{x}{12}=-1$

$x=12$

Yanıt / The Answer is A

4. $x-(2-x)\cdot(4+x)=-(2-x)\cdot(2+x)$

$x-(8+2x-4x-x^2)=-(4+2x-2x-x^2)$

$x-8+2x+x^2=-4+x^2$

$3x-8=-4$

$3x=4$

$x=\frac{4}{3}$

Yanıt / The Answer is D

5. $3\cdot(x-1)=x-3+2x$

$3x-3=x-3+2x$

$3x-3=3x-3$

$0=0$

$\Rightarrow$ ÇK(SS) = R

Yanıt / The Answer is E

6. $\underset{(2)}{\frac{x+1}{2}}+\underset{(1)}{\frac{x-2}{4}}=\underset{(1)}{\frac{x}{4}}-\underset{(4)}{\frac{1}{1}}$

$\frac{2x+2+x-2}{4}=\frac{x-4}{4}$

$3x=x-4$

$2x=-4$

$x=-2$

Yanıt / The Answer is C

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7. $y - 2\{y - 2\cdot[y - 2\cdot(y-2)]\} = 21$

$y - 2\cdot\{y - 2\cdot[y - 2y + 4]\} = 21$

$y - 2\cdot\{y - 2\cdot[-y + 4]\} = 21$

$y - 2\cdot\{y + 2y - 8\} = 21$

$y - 2\cdot[3y - 8] = 21$

$y - 6y + 16 = 21$

$-5y = 21 - 16$

$-5y = 5 \Rightarrow y = -1$

Yanıt / The Answer is B

8. $3ax - b = bx - 3a$

$3ax - bx = b - 3a \Rightarrow (3a - b)x = b - 3a$

$= \dfrac{-(3a-b)}{3a-b}$

$\Rightarrow x = -1$

Yanıt / The Answer is B

9. $\underset{(2)}{\dfrac{x-1}{2}} + \underset{(1)}{\dfrac{x+1}{4}} = \underset{(4)}{\dfrac{2x}{1}} - \underset{(1)}{\dfrac{x-1}{4}}$

$\dfrac{2x-2+x+1}{4} = \dfrac{8x-x+1}{4}$

$\dfrac{3x-1}{4} = \dfrac{7x+1}{4}$

$3x - 7x = 1 + 1$

$-4x = 2$

$x = -\dfrac{2}{4} = -\dfrac{1}{2}$

Yanıt / The Answer is A

10. $2x = 3y \Rightarrow \begin{cases} x = 3k \\ y = 2k \end{cases}$

$\dfrac{x}{3} + 2y = \dfrac{3k}{3} + 2\cdot 2k = 10$

$k = 2$

$y = 2k = 4$

Yanıt / The Answer is C

11. $-1 / 3x - 5y + 2z = -11$

$6x - 2y + 5z = 7$

$-3x + 5y - 2z = 11$

$+ \quad 6x - 2y + 5z = 7$

$3x + 3y + 3z = 18$

$3\cdot(x + y + z) = 18$

$x + y + z = 6$

Yanıt / The Answer is E

12. $a + b = 14$

$b + c = 12$

$-1/a + c = 16$

$2b = 10$

$b = 5 \Rightarrow a = 9 \ , \ c = 7$

$\Rightarrow 2\cdot(9) + 5 - 7 = 16$

Yanıt / The Answer is E

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1. $3a + b - 5c = 8$

$7a + 3b - 9c = 22$

$\Rightarrow$ **a + b + c = ?**

A) 6 B) 5 C) 4 D) 3 E) 2

[PAMUKKALE ÜNİVERSİTESİ – YÖS 2019]

2. $\left.\begin{matrix} x - 2y - 4z = 2 \\ 2x - 3y - 6z = -1 \end{matrix}\right\} \Rightarrow$ **x = ?**

A) 4 B) 3 C) –2 D) –8 E) –9

[İSTANBUL ÜNİVERSİTESİ – YÖS 2018]

3. $\dfrac{4}{1+\dfrac{3}{1+\dfrac{2}{x-2}}} = 2 \Rightarrow$ **x = ?**

A) 3 B) 4 C) 5 D) 6 E) 7

[İNÖNÜ ÜNİVERSİTESİ – YÖS 2018]

4. $a - b = 1$

$a + c = 7$

$b - c = 2$

$3a + c = ?$

A) 8 B) 11 C) 15 D) 17 E) 20

[İSTANBUL ÜNİVERSİTESİ – YÖS 2017]

5. $0{,}36x - 0{,}9 = 0{,}12x + 0{,}3$

$x = ?$

A) 0,2 B) 0,5 C) 0,75 D) 4 E) 5

[ONDOKUZ MAYIS ÜNİVERSİTESİ – 2017]

6. $\dfrac{3}{1-\dfrac{2}{a-1}} = 1 \Rightarrow$ **a = ?**

A) –2 B) –1 C) 0 D) 1 E) 2

[KARADENİZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

7. $\dfrac{\dfrac{1}{x}}{1-\dfrac{3}{x}} = \dfrac{2}{3} \Rightarrow$ **x = ?**

A) $\frac{7}{3}$ B) $\frac{9}{2}$ C) $\frac{3}{8}$ D) $\frac{4}{5}$ E) $\frac{2}{7}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

8. $\left.\begin{matrix} a + 2b - 2c = 6 \\ 3a - 2b - 2c = 2 \end{matrix}\right\} \Rightarrow$ **c – a = ?**

A) –2 B) –1 C) 0 D) 2 E) 3

[İSTANBUL ÜNİVERSİTESİ – YÖS 2015]

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $\frac{1}{x-5}+\frac{1}{3-2y}=0 \Rightarrow x-2y=?$

A) 1 B) 2 C) 3 D) 4 E) 5

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2015]

10. $\frac{3}{x}+\frac{2}{y}=2$

$\frac{1}{x}-\frac{1}{y}=4$

$\Rightarrow x=?$

A) 2 B) $\frac{3}{2}$ C) 1 D) $\frac{1}{2}$ E) $-\frac{1}{4}$

[KARADENİZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

11. $\frac{-3}{x-1}+\frac{1}{x}+\frac{2}{x+2}=0 \Rightarrow x=?$

A) $\frac{-2}{9}$ B) $\frac{-1}{7}$ C) $\frac{-2}{7}$ D) $\frac{1}{7}$ E) $\frac{2}{7}$

[KARADENİZ TEKNİK ÜNİVERSİTESİ – YÖS 2015]

12. $3x-7y=7$

$2y-x=-4$

$\Rightarrow x+y=?$

A) 10 B) 11 C) 13 D) 17 E) 19

[İSTANBUL ÜNİVERSİTESİ – YÖS 2015]

13. $x+4=y$

$y+5=z$

$z+x=2k$

$x+y+z=21 \Rightarrow k=?$

A) –12 B) –6 C) $\frac{43}{6}$ D) 7 E) $\frac{51}{7}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2015]

14. $3(4-x)=10+2x$ ise / if $x=?$

A) $\frac{1}{5}$ B) $\frac{3}{5}$ C) $\frac{2}{3}$ D) $\frac{1}{4}$ E) $\frac{2}{5}$

[AKDENİZ ÜNİVERSİTESİ – YÖS 2014]

15. $1-\frac{5}{1-\frac{3}{x}}=2 \Rightarrow x=?$

A) 2 B) $\frac{4}{5}$ C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{3}{2}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2013]

16. $\left.\begin{array}{l} x+y=9 \\ y+z=12 \\ x+z=17 \end{array}\right\} \Rightarrow x=?$

A) –4 B) 7 C) 10 D) 12 E) 13

[MARMARA ÜNİVERSİTESİ – YÖS 2013]

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	D	A	D	E	C	B	A
9	10	11	12	13	14	15	16
B	D	C	E	C	E	C	B

1. $-2/\ 3a+b-5c=8$

$+\ 7a+3b-9c=22$

$a+b+c=6$

Yanıt / The Answer is **A**

---

2. $-3/\ x-2y-4z=2$

$2/\ 2x-3y-6z=-1$

$+$

$-3x+6y+12z=-6$

$4x-6y-12z=-2$

$+$

$x=-8$

Yanıt / The Answer is **D**

---

3. $\dfrac{4}{1+\dfrac{3}{1+\dfrac{2}{x-2}}}=2$

$\dfrac{3}{1+\dfrac{2}{x-2}}=1$

$\dfrac{2}{x-2}=2 \Rightarrow x-2=1$

$x=3$

Yanıt / The Answer is **A**

---

4. $a-b=1$

$a+c=7$

$+\ b-c=2$

$2a=10 \Rightarrow a=5,\ b=4,\ c=2$

$3a+c=3\cdot5+2=17$

Yanıt / The Answer is **D**

---

5. $0{,}36x-0{,}9=0{,}12x+0{,}3$

$0{,}24x=1{,}2$

$x=\dfrac{1{,}2}{0{,}24}=5$

Yanıt / The Answer is **E**

---

6. $\dfrac{3}{1-\dfrac{2}{a-1}}=1 \Rightarrow \dfrac{3}{\dfrac{a-1-2}{a-1}}=1$

$=\dfrac{3a-3}{a-3}=1$

$3a-3=a-3$

$a=0$

Yanıt / The Answer is **C**

---

7. $\dfrac{\dfrac{1}{x}}{1-\dfrac{3}{x}}=\dfrac{2}{3} \Rightarrow \dfrac{1}{x}\cdot\dfrac{x}{x-3}=\dfrac{2}{3}$

$2x-6=3$

$x=\dfrac{9}{2}$

Yanıt / The Answer is **B**

---

8. $a+2b-2c=6$

$3a-2b-2c=2$

$+$

$4a-4c=8$

$a-c=2$

$\Rightarrow\ c-a=-2$

Yanıt / The Answer is **A**

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9. $\frac{1}{x-5}+\frac{1}{3-2y}=0$

$\frac{1}{x-5}=\frac{1}{2y-3}$

$x-5=2y-3$

$x-2y=2$

Yanıt / The Answer is B

---

10. $\left.\begin{array}{l}\frac{3}{x}+\frac{2}{y}=2\\ \frac{2}{x}-\frac{2}{y}=8\end{array}\right\} \Rightarrow \frac{5}{x}=10 \Rightarrow x=\frac{1}{2}$

Yanıt / The Answer is D

---

11. $\underset{x(x+2)}{\frac{-3}{x-1}}+\underset{(x-1)(x+2)}{\frac{1}{x}}+\underset{x(x-1)}{\frac{2}{x+2}}=0$

$-3x(x+2)+x^2+x-2+2(x^2-x)=0$

$-3x^2-6x+x^2+x-2+2x^2-2x=0$

$x=\frac{-2}{7}$

Yanıt / The Answer is C

---

12. $3x-7y=7 \Rightarrow 3x-35=7$

$3\,/\,2y-x=-4 \qquad 3x=42$

$x=14$

$3x-7y=7$

$+\ 6y-3x=-12$

$y=5$

$x+y=14+5=19$

Yanıt / The Answer E

---

13. $x-y=-4$

$y-z=-5$

$+\ z+x=2k$

$2x=2k-9 \Rightarrow x=\frac{2k-9}{2}$

$z+x=2k \Rightarrow z+\frac{2k-9}{2}=2k$

$2z=2k+9$

$z=\frac{2k+9}{2}$

$y-z=-5 \Rightarrow y-\frac{2k+9}{2}=-5$

$y=2k-1$

$2k-9+2k-1+2k+9=42$

$\Rightarrow k=\frac{43}{6}$

Yanıt / The Answer is C

---

14. $3\cdot(4-x)=10+2x$

$12-3x=10+2x$

$12-10=2x+3x$

$2=5x$

$\frac{2}{5}=x$

Yanıt / The Answer is E

---

15. $1-\frac{5}{1-\frac{3}{x}}=2 \Rightarrow \frac{5}{1-\frac{3}{x}}=-1$

$\frac{3}{x}-1=5$

$\frac{3}{x}=6$

$x=\frac{3}{6}=\frac{1}{2}$

Yanıt / The Answer is C

---

16. $x+y=9$

$y+z=12$

$+\ x+z=17$

$2(x+y+z)=38$

$x+y+z=19$

$x+12=19 \Rightarrow x=7$

Yanıt / The Answer is B

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1. $2-[2-(2-x)]=x \Rightarrow x=?$

A) -2 B) -1 C) 0 D) 1 E) 2

2. $\frac{3x+2}{4}-\frac{4x-3}{4}=2 \Rightarrow x=?$

A) -3 B) $\frac{1}{3}$ C) 3 D) 1 E) $\frac{1}{2}$

3. $\frac{2x+3}{1-x}=-2\frac{1}{2} \Rightarrow x=?$

A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) 4 D) 9 E) 11

4. $0,\overline{2}x+3=x+\frac{1}{3} \Rightarrow x=?$

A) -2 B) 0 C) 2 D) $\frac{24}{7}$ E) $\frac{13}{3}$

5. $\frac{2x-1}{3}+\frac{x-1}{2}=x \Rightarrow x=?$

A) 1 B) 2 C) 3 D) 4 E) 5

6. $\frac{0,2x-0,1}{x-0,8}=0,1 \Rightarrow x=?$

A) $-\frac{5}{3}$ B) $-\frac{3}{5}$ C) 0 D) $\frac{1}{5}$ E) $\frac{3}{5}$

7. $\frac{x+1}{2}-\frac{x-3}{3}=\frac{11}{6} \Rightarrow x=?$

A) 5 B) 4 C) 3 D) 2 E) 1

8. $\frac{x}{\frac{2x-3}{4}}-\frac{4}{\frac{3-2x}{3}}=4 \Rightarrow x=?$

A) 1 B) 2 C) 4 D) 5 E) 6

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9. $\frac{\frac{1}{2}-x}{1-x}=0,1 \Rightarrow x=?$

A) $\frac{1}{2}$ B) $\frac{4}{9}$ C) 1
D) 2 E) $\frac{9}{4}$

10. $\frac{4,8}{0,x}+\frac{1,2}{20}=4,06 \Rightarrow x=?$

A) 16 B) 14 C) 12
D) 8 E) 4

11. $1+\frac{1}{1-\frac{x}{x+1}}=4 \Rightarrow x=?$

A) 2 B) 3 C) 4
D) 5 E) 6

12. $\left.\begin{array}{l}\frac{5}{x}=\frac{7}{y}\\ \frac{5y}{3}=\frac{7x}{3}+z-2\end{array}\right\} \Rightarrow z=?$

A) 0 B) 1 C) 2
D) 3 E) 4

13. $\frac{5-x}{3}-\frac{x-4}{4}=\frac{-5}{6} \Rightarrow x=?$

A) 3 B) 4 C) 5
D) 6 E) 7

14. $\frac{x-2}{4}-\frac{3x-1}{6}=\frac{x-60}{12} \Rightarrow x=?$

A) –20 B) –14 C) –12
D) 14 E) 20

15. $\frac{4+\frac{1}{x}}{4-\frac{1}{x}}=-2 \Rightarrow x=?$

A) $\frac{1}{12}$ B) $\frac{1}{6}$ C) $\frac{1}{3}$
D) 6 E) 12

16. $\frac{10}{3+\frac{2}{x-1}}=2 \Rightarrow x=?$

A) 1 B) 2 C) 3
D) 4 E) 5

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	A	E	D	E	D	D	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	C	A	C	D	D	A	B

1. $0{,}42 - 0{,}03t = 3{,}33 - t \Rightarrow$ **t = ?**

A) 0,3 B) 0,7 C) 1

D) 2 E) 3

2. $\frac{x+1}{3} - \frac{3x-1}{5} = x - 2 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

3. $a - \frac{4a-3}{2} = 4 - \frac{3a-2}{4} \Rightarrow$ **a = ?**

A) – 15 B) – 12 C) 12

D) 15 E) 17

4. $\frac{y}{2} + \frac{y}{3} + \frac{y}{4} = y - 1 \Rightarrow$ **y = ?**

A) –24 B) –18 C) – 15

D) – 14 E) –12

5. $\left.\begin{array}{l} 2x + 4 = 0 \\ 3x - y + 1 = 0 \end{array}\right\} \Rightarrow$ **y = ?**

A) – 7 B) –5 C) 0

D) 1 E) 2

6. $\frac{1 + \frac{1}{x-1}}{1 - \frac{1}{x}} = 1 \Rightarrow$ **x = ?**

A) $\frac{1}{2}$ B) – 2 C) $-\frac{1}{3}$

D) $\frac{2}{3}$ E) 4

7. $1 - \frac{1}{1 + \frac{1}{x-1}} = 2 \Rightarrow$ **x = ?**

A) – 2 B) – 1 C) $\frac{1}{2}$

D) $\frac{3}{4}$ E) 4

8. $\left.\begin{array}{l} 3x + 2y + 5z = 18 \\ 4x + 3y + 6z = 28 \end{array}\right\} \Rightarrow$ **x + y + z = ?**

A) 5 B) 10 C) 15

D) 20 E) 30

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9. $\left.\begin{array}{l} 5a - 8b + 2c = 9 \\ a + 4b - c = 6 \end{array}\right\} \Rightarrow \mathbf{a = ?}$

A) –5 B) –2 C) –1

D) 2 E) 3

10. $\left.\begin{array}{l} 3x - 4y - 5z = 13 \\ 4x - y + 4z = 24 \\ 2x - 10y - 6z = -13 \end{array}\right\} \Rightarrow \mathbf{x + y + z = ?}$

A) 10 B) 12 C) 16

D) 18 E) 24

11. $(2x - y - 6)^2 + (3x + 6y - 34)^2 = 0 \Rightarrow \mathbf{x + y = ?}$

A) 3 B) $\frac{17}{5}$ C) 4

D) $\frac{24}{5}$ E) 8

12. $ax + b^2 = a^2 + bx \Rightarrow \mathbf{x = ?}$

A) a – b B) a C) $\frac{2b}{a}$

D) $\frac{a}{a-b}$ E) a + b

13. $(m - 2)x + 2m + n = x + 3$

ÇK(SS) = R $\Rightarrow$ **m + n = ?**

A) –3 B) –2 C) 0

D) 2 E) 3

14. $(2a - 1)x + (b + 1)y = 3$

$x + y = 1$

ÇK(SS) = R $\Rightarrow$ **a·b = ?**

A) 4 B) 6 C) 8

D) 9 E) 10

15. $\left.\begin{array}{l} a+3b=8 \\ b+3c=10 \\ c+3a=14 \end{array}\right\} \Rightarrow \mathbf{a+b+c = ?}$

A) 6 B) 7 C) 8

D) 9 E) 10

16. $\left.\begin{array}{l} \frac{1}{x}-\frac{2}{y}=\frac{1}{3} \\ \frac{2}{x}+\frac{1}{y}=\frac{1}{6} \end{array}\right\} \Rightarrow \mathbf{y = ?}$

A) –4 B) –6 C) –8

D) –10 E) –12

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	B	B	E	B	A	C	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	A	E	E	C	A	C	D

1. $\frac{2-x}{3}+\frac{x+1}{2}-\frac{x}{4}=\frac{1}{2} \Rightarrow$ **x = ?**

A) –6 B) –4 C) –2
D) 0 E) 8

2. $\frac{2x+3}{1-2x}+\frac{4}{2x-1}+\frac{x}{2}=1 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3
D) 4 E) 5

3. $\frac{x-1}{x+1}+\frac{x}{x+1}+\frac{1}{x+1}=2 \Rightarrow$ **ÇK(SS) = ?**

A) {–2} B) {0} C) {1}
D) Ø E) R

4. $6x - a = 2x + ax - 4$
ÇK(SS) = R $\Rightarrow$ **a = ?**

A) 1 B) 2 C) 3
D) 4 E) 5

5. $\frac{a(x-b)}{3}-\frac{b(x-a)}{4}=\frac{a\cdot b}{12} \Rightarrow$ **x = ?**

A) $\frac{ab}{2b+a}$ B) $\frac{ab}{2a+3b}$ C) $\frac{2ab}{4a-3b}$
D) $\frac{3ab}{2a+3b}$ E) $\frac{4ab}{3a+2b}$

6. a, b, c Z^+,
$a^2 + b^2 - c^2 = 13 + 2ab$
$\Rightarrow$ **min(c) = ?**

A) 7 B) 6 C) 5
D) 4 E) 3

7. $\left.\begin{array}{l} x+2y+5z=25 \\ 2x++y-2z=56 \end{array}\right\} \Rightarrow$ **x + y + z = ?**

A) 3 B) 9 C) 8
D) 27 E) 36

8. $\left.\begin{array}{l} ax-x-y=-1 \\ ax-2y=-a \end{array}\right\} \Rightarrow$ **x = ?**

A) –2 B) –1 C) 0
D) 1 E) 2

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9. $\left.\begin{matrix}4x+3y=8\\3x+4y=5\end{matrix}\right\} \Rightarrow x-y=?$

A) 1 B) 2 C) 3
D) 4 E) 5

10. $\left.\begin{matrix}2x+y+4z=40\\-x-2y+z=10\end{matrix}\right\} \Rightarrow x+y+z=?$

A) 5 B) 8 C) 9
D) 10 E) 15

11. $(x+y-6)^4+(x-y-2)^4=0 \Rightarrow x^2-y^2=?$

A) 6 B) 8 C) 10
D) 12 E) 16

12. $\left.\begin{matrix}2x-y+3z=16\\x-3y+z=14\\3x-2y+2z=36\end{matrix}\right\} \Rightarrow x-y+z=?$

A) 3 B) 5 C) 6
D) 11 E) 12

13. $\frac{1}{x-a}+\frac{1}{x-1}+\frac{1}{x-2}=1,$
$ÇK(SS)=\{3\} \Rightarrow a=?$

A) 1 B) 2 C) 3
D) 4 E) 5

14. $\left.\begin{matrix}x+y=12\\x+z=8\\y+z=10\end{matrix}\right\} \Rightarrow z=?$

A) 1 B) 2 C) 3
D) 4 E) 5

15. $\left.\begin{matrix}x^2-y^2=20\\y^2-z^2=24\\x-z=4\end{matrix}\right\} \Rightarrow x+z=?$

A) 4 B) 6 C) 8
D) 11 E) 13

16. $\left.\begin{matrix}a+2b+3c=20\\a+b+2c=15\\2a+b+2c=12\end{matrix}\right\} \Rightarrow a+b+c=?$

A) 2 B) 5 C) 8
D) 12 E) 15

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	D	D	D	C	B	D	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	D	D	D	E	C	D	A

1. $A = 3xy + 12$

$B = 4xy + 8$

$\Rightarrow$ **A = ?**

A) $\frac{B+20}{5}$ B) $\frac{4B+25}{4}$ C) $\frac{3B+24}{4}$

D) $\frac{3B-24}{5}$ E) $\frac{4B+25}{4}$

2. $\frac{12-3x}{2} - \frac{2x-1}{3} = 2 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

3. $\left.\begin{matrix} 3x-4y=7 \\ 6x-8y=14 \end{matrix}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) {2, 2} B) {–2, 1} C) {3, 4}

D) Ø E) R

4. $3ax - 4b = 12x - 20$

$\forall x \in R$ için / for **a + b = ?**

A) 1 B) 3 C) 5

D) 7 E) 9

5. $\left.\begin{matrix} \frac{x}{2} - \frac{2y}{3} = 2 \\ x + 2y = 9 \end{matrix}\right\} \Rightarrow$ **x·y = ?**

A) 1 B) 3 C) 6

D) 9 E) 12

6. $\left.\begin{matrix} 2a+b-c=16 \\ a+3b+2c=12 \\ a+2b-c=18 \end{matrix}\right\} \Rightarrow$ **a + b + c = ?**

A) 5 B) 9 C) 11

D) 14 E) 16

7. $4x + 2ay = 7$

$3x + 6y = 8$

ÇK(SS) = Ø

$\Rightarrow$ **a = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

8. $2(2x - 1) - 3(x + 1) = 3$

$\Rightarrow$ **x = ?**

A) 2 B) 4 C) 6

D) 8 E) 10

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $7-\dfrac{24}{5+\dfrac{12}{x-1}}=4 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3
D) 4 E) 5

10. $3x-\dfrac{5(x-1)}{3}=\dfrac{25}{3} \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3
D) 4 E) 5

11. $(a+b-6)^2+(a-b-4)^2=0 \Rightarrow$ **a + b = ?**

A) 6 B) 8 C) 12
D) 18 E) 24

12. $\left.\begin{array}{l}\dfrac{1}{x}+\dfrac{3}{y}=14\\ \dfrac{2}{x}-\dfrac{1}{y}=7\end{array}\right\} \Rightarrow$ **x = ?**

A) $\frac{1}{2}$ B) $\frac{1}{5}$ C) $\frac{1}{7}$
D) $\frac{1}{8}$ E) $\frac{1}{9}$

13. $\left.\begin{array}{l}x=a+3\\ y=3a-1\\ 2x=10-y\end{array}\right\} \Rightarrow$ **y − x = ?**

A) −4 B) −2 C) 0
D) 2 E) 4

14. $\left.\begin{array}{l}\dfrac{x}{2}-\dfrac{y}{3}=1\\ x-y=1\end{array}\right\} \Rightarrow$ **x + y = ?**

A) 7 B) 10 C) 12
D) 14 E) 16

15. $\dfrac{2x-5}{x+3}+\dfrac{x-1}{x+1}+\dfrac{11}{x+3}=\dfrac{2}{3}$
$\Rightarrow$ **x = ?**

A) $\frac{-5}{7}$ B) $\frac{-4}{7}$ C) $\frac{-3}{7}$
D) $\frac{-2}{7}$ E) $\frac{-1}{7}$

16. $\left.\begin{array}{l}x+y+z=8\\ xy+xz=16\end{array}\right\} \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3
D) 4 E) 5

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	B	E	E	D	A	D	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	E	A	B	B	A	E	D

BÖLÜM CHAPTER 8

TARİHSEL NOT / HISTORICAL NOTE

Aristarchus of Samos [about 270 BC]

Yunan gök bilimci, Dünya'nın Güneş etrafındaki dönüşünü açıklayan ilk astronomlardan biridir. Güneş ve Ay'ın Dünya'ya olan uzaklıklarını bulmak için geometrik yöntemler kullanmıştır.

Greek astronomer, noted for being the first to affirm that the Earth rotates and travels around the Sun. He treated astronomy mathematically and used geometrical methods to calculate the relative sizes of the Sun and Moon and their relative distances from the Earth.

BİRİNCİ DERECEDEN EŞİTSİZLİKLER
FIRST DEGREE INEQUALITIES

Bu bölüm, 18 çözümlü örnek, 12 çözümlü test sorusu, 6 çözümlü YÖS sorusu, 32 test sorusu içermektedir.

This chapter includes 18 examples with solutions, 12 tests questions with solutions, 6 YÖS questions with solutions and 32 test questions.

BÖLÜM / CHAPTER 8 ..... 301 - 324

BİRİNCİ DERECEDEN EŞİTSİZLİKLER / FIRST DEGREE INEQUALITIES

- Birinci Dereceden Eşitsizlikler / First Degree Inequalities ..... 303 - 314
  - Reel Sayı Doğrusu Üzerinde Sınırlı Aralıklar / Bounded Intervals on the Real Numbers Line 303
  - Reel Sayı Doğrusu Üzerinde Sınırsız Aralıklar / Unbounded Intervals on the Real Numbers Line 304
  - Eşitsizliklerin Özelikleri / Properties of Inequalities ..... 305 - 310
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BÖLÜM
08
CHAPTER

BİRİNCİ DERECEDEN EŞİTSİZLİKLER
FIRST DEGREE INEQUALITIES

ÖNEMLİ / IMPORTANT

• <, >, ≤, ≥ sembolleri, **eşitsizlik sembolleri** olarak adlandırılır.

• The symbols <, >, ≤, ≥ are called **inequality symbols.**

BİRİNCİ DERECEDEN EŞİTSİZLİKLER
FIRST DEGREE INEQUALITIES

Tanım: $a \neq 0$, a ve b birer reel sayı olmak üzere,
$ax + b > 0$, $ax + b \geq 0$, $ax + b < 0$, $ax + b \leq 0$ biçimindeki ifadelere *birinci dereceden eşitsizlikler* denir. Eşitsizliği sağlayan sayıların oluşturduğu kümeye *çözüm kümesi* denir. Eşitsizliklerin çözüm kümeleri reel sayı aralıkları ile gösterilir.

Definition: Provided that $a \neq 0$, a and b are real numbers, the expressions in $ax + b > 0$, $ax + b \geq 0$, $ax + b < 0$, $ax + b \leq 0$ forms are called *first degree inequalities*. The set of all real numbers that make the inequality valid is the solution set of the inequality. The *solution set of* inequalities are shown by intervals.

Sembol / Symbol	Anlamı / Meaning
=	eşit / is equal to
≠	eşit değil / is not equal to
<	küçük / is less than
>	büyük / is greater than
≤	küçük veya eşit / is less than or equal to
≥	büyük veya eşit / is greater than or equal to

ÖNEMLİ / IMPORTANT

• (a,b) açık aralığı, koordinat sistemindeki x ekseni a, y ekseni b olan sıralı ikili ile karıştırılmamalıdır.

• The open interval (a,b), should not to be confused with a point in coordinate system with abscissa a and ordinate b.

REEL SAYI DOĞRUSU ÜZERİNDE SINIRLI ARALIKLAR
BOUNDED INTERVALS on the REAL NUMBERS LINE

$a, b \in R$, $a < b$ olmak üzere / if

Gösterim / Notation	Aralık türü / Interval Type	Eşitsizlik / Inequality	Şekilsel Gösterim / Representation
[a, b]	kapalı / closed	$a \leq x \leq b$	
(a, b)	açık / open	$a < x < b$	
[a, b)	yarı açık / half-open	$a \leq x < b$	
(a, b]	yarı açık / half-open	$a < x \leq b$	

REEL SAYI DOĞRUSU ÜZERİNDE SINIRSIZ ARALIKLAR
UNBOUNDED INTERVALS on the REAL NUMBERS LINE

a reel sayısını ele alalım. Reel sayı doğrusu üzerinde aşağıdaki aralıklar sınırsızdır.
Let a be a real number. The following intervals on the real number line are unbounded.

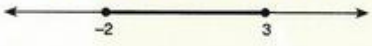
Gösterim Notation	Aralık türü Interval Type	Eşitsizlik Inequality	Şekilsel Gösterim Representation
(a, ∞)	açık / open	$a < x$	
$(-\infty, a)$	açık / open	$x < a$	
$[a, \infty)$	yarı açık / half-open	$a \le x$	
$(-\infty, a]$	yarı açık / half-open	$x \le a$	
$(-\infty, \infty)$	tüm reel eksen / entire real line		

ÖNEMLİ / IMPORTANT

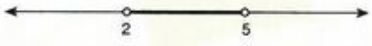
- Bir kapalı aralık, bitim noktalarının ikisini de; yarı açık aralık, bitim noktalarının sadece birisini içerir. Açık aralık, bitim noktalarının hiçbirini içermez.
- A closed interval contains both of its endpoints, a half open interval contains only one of its endpoints, and an open interval does not contain either of its endpoints.

Etkinlikler / Activities

a. $-2 \le x \le 3$ eşitsizliği, $x \ge -2$ ve $x \le 3$ anlamındadır. Bu eşitsizlik, aşağıda gösterildiği gibi -2 ve 3 dahil olmak üzere -2 ve 3 arasındaki tüm reel sayıları gösterir.
The inequality $-2 \le x \le 3$ means that $x \ge -2$ and $x \le 3$. This double inequality denotes all real numbers between -2 and 3, including -2 and 3, as shown below.



b. $2 < x < 5$ eşitsizliği, $x > 2$ ve $x < 5$ anlamındadır. Bu eşitsizlik, aşağıda gösterildiği gibi 2 ve 5 dahil olmamak üzere, 2 ve 5 arasındaki tüm reel sayıları gösterir.
The inequality $2 < x < 5$ means that $x > 2$ and $x < 5$. This double inequality denotes all real numbers between 2 and 5, not including 2 and 5, as shown below.



c. $2 \le x < 4$ eşitsizliği, $x \ge 2$ ve $x < 4$ anlamındadır. Bu eşitsizlik, aşağıda gösterildiği gibi 2 dahil ve 4 dahil olmamak üzere, 2 ve 4 arasındaki tüm reel sayıları gösterir.
The inequality $2 \le x < 4$ means that $x \ge 2$ and $x < 4$. This double inequality denotes all real numbers between 2 and 4, including 2 but not 4, as shown below.



d. $-3 < x \le 7$ eşitsizliği, $x > -3$ ve $x \le 7$ anlamındadır. Bu eşitsizlik, aşağıda gösterildiği gibi 7 dahil, -3 dahil olmamak üzere -3 ve 7 arasındaki tüm reel sayıları gösterir.
The inequality $-3 < x \le 7$ means that $x > -3$ and $x \le 7$. This double inequality denotes all real numbers between -3 and 7, including 7 but not -3, as shown below.



EŞİTSİZLİKLERİN ÖZELLİKLERİ
PROPERTIES of INEQUALITIES

ÖNEMLİ / IMPORTANT

• Toplama / Çıkarma özelliği, ≤ ve ≥ sembolleri için de her zaman geçerlidir.

• The Addition / Subtraction property, is also always true for an inequality that contains the symbol ≤ or ≥.

Özellik / Property

★ **Üç hal özelliği:** a ve b reel sayıları için a = b, a > b , a < b durumlarından birisi geçerlidir.

The Trichotomy property: For real numbers a and b, one and only one of the following is true: a = b, a > b, a < b

Özellik / Property

★ a, b, c ∈ R,

$a < b \Rightarrow a \mp c < b \mp c$

ÖNEMLİ / IMPORTANT

• Çarpma / Bölme özelliği, ≤ ve ≥ sembolleri için de her zaman geçerlidir.

• The multiplication / division property of inequalities is also always true for the symbols ≤ and ≥.

Özellik / Property

★ **I.** a, b ∈ R, c ∈ R^+

$a < b \Rightarrow a \cdot c < b \cdot c$

II. a, b ∈ R, c ∈ R^-

$a < b \Rightarrow a \cdot c > b \cdot c$

Özellik / Property

★ $\left.\begin{matrix} a < b \\ c < d \end{matrix}\right\} \Rightarrow a + c < b + d$

ÖNEMLİ / IMPORTANT

• Eşitsizliklerde taraf tarafa toplama yapılabilir, taraf tarafa çarpma ve bölme yapılamaz.

• In equalities adding respective sides can be done, but multiplying and dividing respective sides can not be done.

Özellik / Property

★ **Geçişme özelliği:** a < b ve b < c ⇒ a < c

The Transitive property: a < b and b < c ⇒ a < c

Özellik / Property

★ **I.** a, b ∈ R, a·b > 0

$a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$

II. a, b ∈ R, a·b < 0

$a < b \Rightarrow \frac{1}{a} < \frac{1}{b}$

ÖNEMLİ / IMPORTANT

• Eşitsizliklerde taraf tarafa toplama aynı türden (ikisi de küçük veya ikisi de büyük) eşitsizliklerde yapabilir.

• In equalities adding and subtracting respective sides can be done if both the inequalities are from same type of inequalities, (both greater or both less).

Özellik / Property

★ n pozitif bir tamsayı ve a < b ise $a^{2n+1} < b^{2n+1}$ dir.

If n is a positive integer and a < b then $a^{2n+1} < b^{2n+1}$.

Örnek / Example 1

$3x - 18 < 0 \Rightarrow$ ÇK(SS) = ?

A) $(-\infty, 4)$ B) $(-\infty, 6)$ C) $(4, \infty)$ D) $(6, \infty)$ E) $(4, 6)$

Çözüm / Solution

Yanıt / Answer **B**

$3x - 18 < 0$

$3x < 18$

$x < 6$

ÇK(SS) = $(-\infty, 6)$

Örnek / Example 2

$\frac{x}{2} - 3 \le \frac{3x-1}{5} \Rightarrow$ ÇK(SS) = ?

A) $[-28, +\infty)$ B) $(-28, +\infty)$ C) $(-28, -24)$

D) $[28, +\infty)$ E) $(-\infty, 28)$

Çözüm / Solution

Yanıt / Answer **A**

$\frac{x}{2} - 3 \le \frac{3x-1}{5}$

$10 \cdot \frac{x-6}{2} \le \frac{3x-1}{5} \cdot 10$

$5(x - 6) \le (3x - 1) \cdot 2$

$5x - 30 \le 6x - 2$

$2 - 30 \le 6x - 5x$

$-28 \le x$

ÇK(SS) = $[-28, +\infty)$

PEKİŞTİRME / REINFORCE

✓ $2x - 8 < 14 \Rightarrow$ ÇK(SS) = ?

..... [x < 11]

✓ $3x - 7 \le 11 \Rightarrow$ ÇK(SS) = ?

..... [x ≤ 6]

✓ $3(x - 1) \ge 9 \Rightarrow$ ÇK(SS) = ?

..... [x ≥ 4]

✓ $2(x + 1) > -4 \Rightarrow$ ÇK(SS) = ?

..... [x > −3]

✓ $3(x - 7) \ge -12 \Rightarrow$ ÇK(SS) = ?

..... [x ≥ 3]

Örnek / Example 3

$3 < \frac{5-7x}{6} \leq 5 \Rightarrow$ ÇK(SS) = ?

A) $\left[-\frac{25}{7}, -\frac{13}{7}\right)$ B) $\left(-\frac{25}{7}, -\frac{13}{7}\right)$ C) $\left(-\frac{25}{7}, -\frac{13}{7}\right]$

D) $\left(-\frac{13}{7}, \frac{25}{7}\right]$ E) $\left(-\frac{25}{7}, \frac{13}{7}\right]$

Çözüm / Solution Yanıt / Answer **A**

$3 < \frac{5-7x}{6} \leq 5$

$6 \cdot 3 < 5 - 7x \leq 6 \cdot 5$

$18 < 5 - 7x \leq 30$

$-5 + 18 < -5 + 5 - 7x \leq -5 + 30$

$13 < -7x \leq 25$

$\frac{13}{-7} > \frac{-7x}{-7} \geq \frac{25}{-7}$

$\frac{13}{-7} > x \geq -\frac{25}{7}$

$ÇK(SS) = \left[-\frac{25}{7}, -\frac{13}{7}\right)$

PEKİŞTİRME / REINFORCE

✓ $2 \leq 2x - 6 \leq 10 \Rightarrow$ ÇK(SS) = ?

..... **[4, 8]**

✓ $-4 \leq 3x + 5 < 14 \Rightarrow$ ÇK(SS) = ?

..... **[–3, 3)**

✓ $-3 < x - 7 \leq 9 \Rightarrow$ ÇK(SS) = ?

..... **(4, 16]**

✓ $-3 \leq 2x - 5 < 7 \Rightarrow$ ÇK(SS) = ?

..... **[1, 6)**

Örnek / Example 4

$x \in Z$,

$3x - 5 \leq \frac{2x+10}{4} \Rightarrow$ max(x) = ?

A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution Yanıt / Answer **C**

$3x - 5 \leq \frac{2x+10}{4}$

$4 \cdot (3x - 5) \leq 4 \cdot \frac{2x+10}{4}$

$12x - 20 \leq 2x + 10$

$12x - 2x \leq 10 + 20$

$10x \leq 30$

$x \leq 3$

$max(x) = 3$

Örnek / Example 5

$x \in Z$,

$\frac{x-8}{2} > \frac{x-4}{3} \Rightarrow \min(x) = ?$

A) 15 B) 17 C) 19 D) 21 E) 23

Çözüm / Solution

Yanıt / Answer **B**

$\frac{x-8}{2} > \frac{x-4}{3}$

$6 \cdot \frac{x-8}{2} > 6 \cdot \frac{x-4}{3}$

$3(x-8) > 2(x-4)$

$3x - 24 > 2x - 8$

$x > 16$

$x \in Z,\ x > 16 \Rightarrow \min(x) = 17$

Örnek / Example 6

$x \in Z$,

$-4 \le 3x - 7 < 5 \Rightarrow \sum x = ?$

A) 2 B) 4 C) 6 D) 8 E) 10

Çözüm / Solution

Yanıt / Answer **C**

$-4 \le 3x - 7 < 5$

$-4 + 7 \le 3x < 5 + 7$

$3 \le 3x < 12$

$\frac{3}{3} \le \frac{3x}{3} < \frac{12}{3}$

$1 \le x < 4$



$\sum x = 1 + 2 + 3 = 6$

PEKİŞTİRME / REINFORCE

$x \in Z$,

✓ $3x - 8 \le 2x - 7 \Rightarrow \max(x) = ?$

..... [1]

✓ $2x - 6 \ge 3x - 8 \Rightarrow \max(x) = ?$

..... [2]

✓ $3(x-2) \le 2(x-1) \Rightarrow \max(x) = ?$

..... [4]

✓ $\frac{x}{2} - \frac{x}{3} < 1 \Rightarrow \max(x) = ?$

..... [5]

✓ $\frac{x-1}{2} - \frac{x-1}{3} < -1 \Rightarrow \max(x) = ?$

..... [–6]

Örnek / Example 7

$(3x - 2y) \in Z$,

$-3 < 2x - 5 \leq 7$

$6 < 5y - 4 < 21 \Rightarrow$ **min(3x – 2y) + max(3x – 2y) = ?**

A) 3 B) 4 C) 5 D) 6 E) 7

Çözüm / Solution Yanıt / Answer **E**

$-3 < 2x - 5 \leq 7$

$2 < 2x \leq 12$

$1 < x \leq 6$

$3 < 3x \leq 18$

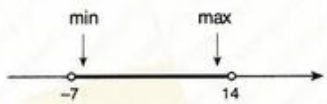
$6 < 5y - 4 < 21$

$10 < 5y < 25$

$2 < y < 5$

$-10 < -2y < -4$

$$\begin{array}{r} 3 < 3x \leq 18 \\ + \quad -10 < -2y < -4 \\ \hline -7 < 3x - 2y < 14 \end{array}$$



$\min(3x - 2y) = -6$,

$\max(3x - 2y) = 13 \Rightarrow \min(3x - 2y) + \max(3x - 2y) = -6 + 13 = 7$

ÖNEMLİ / IMPORTANT

$x,y \in Z$

- x ve y en büyük ise, x + y en büyüktür.
- x ve y en küçük ise, x + y en küçüktür.
- x en küçük, y en büyük ise, x – y en küçüktür. Çünkü bu işlem çıkarma işlemidir.
- x en büyük, y en küçük ise, x – y en büyüktür. Çünkü bu işlem çıkarma işlemidir.

$x,y \in Z$.

- The expression x + y is maximum if x and y are maximum.
- The expression x + y is minimum if x and y are minimum.
- The expression x – y is minimum if x is minimum, y is maximum. Because this operation is subtraction.
- The expression x – y is maximum if x is maximum, y is minimum. Because this operation is subtraction.

Örnek / Example 8

$x, y \in Z$,

$3 \leq x \leq 9$

$5 \leq y < 6 \Rightarrow$ **min(2x – y) + max(2x – y) = ?**

A) 11 B) 12 C) 13 D) 14 E) 15

Çözüm / Solution Yanıt / Answer **D**

$3 \leq x \leq 9 \Rightarrow \min(x) = 3$

$5 \leq y < 6 \Rightarrow \max(y) = 5$

$\min(2x - y) = 2 \cdot 3 - 5 = 6 - 5 = 1$

$3 \leq x \leq 9 \Rightarrow \max(x) = 9$

$5 \leq y < 6 \Rightarrow \min(y) = 5$

$\max(2x - y) = 2 \cdot 9 - 5 = 13$

$\Rightarrow \min(2x - y) + \max(2x - y) = 1 + 13 = 14$

Örnek / Example 9

$(x^2 - y) \in Z$,
$-3 \le x < 5$
$-3 \le y < 5$
$\Rightarrow \min(x^2 - y) = ?$

A) -4 B) -2 C) 0 D) 2 E) 4

Çözüm / Solution

Yanıt / Answer **A**

$-3 \le x < 5 \Rightarrow \quad 0 \le x^2 < 25$

$-3 \le y < 5 \Rightarrow \; + \; -5 < -y \le 3$

$-5 < x^2 - y < 28 \Rightarrow \min(x^2 - y) = -4$

Örnek / Example 10

$-5 \le x < 4$,
$3 < y \le 9 \Rightarrow ? < x \cdot y < ?$

A) $-45 < x \cdot y < 36$
B) $-45 \le x \cdot y < 36$
C) $-45 \le x \cdot y \le 36$
D) $-45 < x \cdot y \le 36$
E) $36 \le x \cdot y < 45$

Çözüm / Solution

Yanıt / Answer **B**

$-5 \cdot 3 = -15$
$-5 \cdot 9 = -45$
$4 \cdot 3 = 12$
$4 \cdot 9 = 36$
$\Rightarrow$ $\min(x \cdot y) = -45$, $\max(x \cdot y) = 36$

$\Rightarrow -45 \le x \cdot y < 36$

PEKİŞTİRME / REINFORCE

✓ $4 < x < 6$, $2 \le y < 5 \Rightarrow ? < x \cdot y < ?$
.....[$8 < x \cdot y < 30$]

✓ $-2 < x < -1$, $-6 < y < -4 \Rightarrow ? < x \cdot y < ?$
.....[$4 < x \cdot y < 12$]

✓ $-8 \le x < 5$, $-4 \le y \le 3 \Rightarrow ? < x \cdot y < ?$
.....[$-24 < x \cdot y < 32$]

✓ $-4 < x < 9$, $3 < y < 5 \Rightarrow ? < x \cdot y < ?$
.....[$-20 < x \cdot y < 45$]

EŞİTSİZLİK SİSTEMLERİ / SYSTEMS of INEQUALITIES

Eşitsizlik sistemleri çözülürken, eşitsizlikler ayrı ayrı çözülür, daha sonra bulunan çözüm kümelerinin kesişimi alınır.

To solve systems of ineqalities, solve inequalities one by one then take the intersection of solutions.

PEKİŞTİRME / REINFORCE

✓ $3x - 2 \geq 4,\ x + 2 > 2 \Rightarrow$ ÇK(SS) = ?

.....[[2, ∞)]

✓ $3x - 4 \leq 7,\ x + 1 \leq 2 \Rightarrow$ ÇK(SS) = ?

.....[(−∞, 1]]

✓ $3(x - 1) + 4 < -5,$

$2x + 4 \geq -6 \Rightarrow$ ÇK(SS) = ?

.....[[−5, −2)]

✓ $2(x + 1) - 3 \leq -7,$

$3x - 2 \leq 2x - 7 \Rightarrow$ ÇK(SS) = ?

.....[(−∞, −5]]

Örnek / Example 11

$x, y \in Z$

$$\left.\begin{matrix} -7 \leq x < 6 \\ -4 < y < 2 \end{matrix}\right\} \Rightarrow \max(x \cdot y) + \min(x \cdot y) = ?$$

A) 5 B) 6 C) 7 D) 8 E) 9

Çözüm / Solution

Yanıt / Answer **B**

$\Rightarrow \max(x \cdot y) = (-7) \cdot (-3) = 21$

$\Rightarrow \min(x \cdot y) = 5 \cdot (-3) = -15$

$\max(x \cdot y) + \min(x \cdot y) = 6$

Örnek / Example 12

$a, b \in Z$

$A = x^2 - 6x + 5$

$-4 < x < 7 \Rightarrow$ **? < A < ?**

A) $-4 \leq A < 45$

B) $-4 \leq A < 49$

C) $0 \leq A < 45$

D) $0 \leq A < 49$

E) $-4 \leq A \leq 45$

Çözüm / Solution

Yanıt / Answer **A**

$A = x^2 - 6x + 5 = x^2 - 6x + 9 - 4$

$= (x - 3)^2 - 4$

$-4 < x < 7 \Rightarrow -7 < x - 3 < 4$

$0 \leq (x - 3)^2 < 49$

$-4 \leq (x - 3)^2 - 4 < 45$

$-4 \leq A < 45$

Örnek / Example 13

$\left.\begin{array}{l} 3x-7 \geq 2x+1 \\ 5x+3 < 3x-7 \end{array}\right\} \Rightarrow$ ÇK(SS) = ?

A) (–5, 8) B) (–5, 8] C) [–5, 8) D) R E) ∅

Çözüm / Solution

Yanıt / Answer **E**

$3x - 7 \geq 2x + 1$ $\qquad$ $5x + 3 < 3x - 7$

$3x - 2x \geq 7 + 1$ $\qquad$ $5x - 3x < -3 - 7$

$x \geq 8$ $\qquad$ $2x < -10$

$x < -5$



Eşitsizliklerin ikisini de sağlayan ortak sayılar olmadığından çözüm kümesi boş kümedir.

Since there is no intersection satisfying both inequalities the solution set is empty.

Örnek / Example 14

$x \in Z$,

$\left.\begin{array}{l} 2x-2 < 6 \\ x+3 > 0 \end{array}\right\} \Rightarrow \sum x = ?$

A) 1 B) 2 C) 3 D) 4 E) 6

Çözüm / Solution

Yanıt / Answer **C**

$2x - 2 < 6$ $\qquad$ $x + 3 > 0$

$2x < 8$ $\qquad$ $x > -3$

$x < 4$

$\Rightarrow \sum x = (-2) + (-1) + 0 + 1 + 2 + 3 = 3$

Örnek / Example 15

$$\left.\begin{array}{l}\frac{x-12}{4} \le 3\\ \frac{5-x}{6} \le 2\end{array}\right\} \Rightarrow \; ? \le ? \le ?$$

A) $-3 \le x \le 7$

B) $-5 \le x \le 12$

C) $-6 \le x \le 18$

D) $-7 \le x \le 24$

E) $-9 \le x < 27$

Çözüm / Solution Yanıt / Answer **D**

$\frac{x-12}{4} \le 3$ $\qquad$ $\frac{5-x}{6} \le 2$

$x - 12 \le 12$ $\qquad$ $5 - x \le 12$

$x \le 24$ $\qquad$ $-7 \le x$

$-7 \le x \le 24$

Örnek / Example 16

$a, b \in Z$

$$\left.\begin{array}{l}20 < a+b < 35\\ \frac{a+b}{b} = 5\end{array}\right\} \Rightarrow \; \max(a-b) = ?$$

A) 17 B) 18 C) 19 D) 20 E) 21

Çözüm / Solution Yanıt / Answer **B**

$\frac{a+b}{b} = 5$

$a + b = 5b$

$20 < 5b < 35$

$4 < b < 7$

$\frac{a+6}{6} = 5$

$a = 24$

$a - b = 18$

Örnek / Example 17

$x \in Z^+,\ y, z \in R^+$

$x < y < z$

$\frac{3}{x} + \frac{4}{y} + \frac{11}{z} = \frac{1}{4} \Rightarrow$ **max(x) = ?**

A) 72 B) 71 C) 70 D) 69 E) 68

Çözüm / Solution

Yanıt / Answer **B**

$x = y = z \Rightarrow \frac{3}{x} + \frac{4}{x} + \frac{11}{x} = \frac{1}{4}$

$\frac{18}{x} = \frac{1}{4}$

$x = 72$

$x < y < z \Rightarrow \max(x) = 71$

Örnek / Example 18

$a, b \in R,\ c \in Z$

$a < b < c$

$3a + 4b + 5c = 156$

$\Rightarrow$ **min(c) = ?**

A) 16 B) 15 C) 14 D) 13 E) 12

Çözüm / Solution

Yanıt / Answer **C**

$a = b = c \Rightarrow 3c + 4c + 5c = 156$

$12c = 156$

$c = 13$

$a < b < c \Rightarrow \min(c) = 14$

1. $2x-4<6 \Rightarrow$ **ÇK(SS) = ?**

A) $x>10$ B) $x>6$ C) $x>5$

D) $x<5$ E) $x<6$

2. $5-2x \le x+2 \Rightarrow x \ge ?$

A) $x \ge -3$ B) $x \ge -2$ C) $x \ge -1$

D) $x \ge 0$ E) $x \ge 1$

3. $3x-[(2-x)+2x]+1<x-5 \Rightarrow x<?$

A) $x<-6$ B) $x<-5$ C) $x<-4$

D) $x<-3$ E) $x<-2$

4. $\frac{2x-3}{5}<\frac{x-2}{7}+\frac{1}{3} \Rightarrow x<?$

A) $x<\frac{27}{68}$ B) $x<\frac{68}{27}$ C) $x<\frac{27}{50}$

D) $x<\frac{50}{27}$ E) $x<\frac{27}{40}$

5. $\frac{x-1}{4}-3 \le \frac{2x-1}{3}+x \Rightarrow ? \le x$

A) $-\frac{35}{17} \le x$ B) $-2<x$ C) $\frac{33}{17} \le x$

D) $-\frac{32}{17} \le x$ E) $-\frac{31}{17} \le x$

6. $4\cdot(x+1)-2>-\frac{1}{2}(x-1) \Rightarrow x>?$

A) $x<0$ B) $x>\frac{1}{2}$ C) $x>-\frac{1}{3}$

D) $x<\frac{1}{3}$ E) $x<-1$

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7. $5x + 2 < 3 \cdot (x - 1) \Rightarrow x < ?$

A) $x < -\frac{5}{2}$ B) $x < -3$ C) $x < -\frac{7}{2}$

D) $x < -4$ E) $x < -\frac{9}{2}$

8. $\frac{-2}{1-x} > 0 \Rightarrow$ ÇK(SS) = ?

A) $x < 0$ B) $0 < x < 1$ C) $x < -1$

D) $-1 < x < 0$ E) $1 < x$

9. $x + 2y - 12 = 0$, $2 < y < 6 \Rightarrow ? < x < ?$

A) $0 < x < 6$

B) $0 < x < 8$

C) $2 < x < 6$

D) $2 < x < 8$

E) $8 < x < 10$

10. $4(x + 1) - 2 < x - 1 \Rightarrow$ ÇK(SS) = ?

A) $x < 0$ B) $x > \frac{1}{2}$ C) $x > -\frac{1}{3}$

D) $x < \frac{1}{3}$ E) $x < -1$

11. $2(x - 1) \leq -1 + 3(x + 2) \Rightarrow$ ÇK (SS) = ?

A) $x \geq 3$ B) $x \geq 1$ C) $x \leq -1$

D) $x \geq -3$ E) $x \geq -7$

12. $\frac{3}{4}(x+8) \geq -\frac{1}{2}(x-4) + x \Rightarrow$ ÇK(SS) = ?

A) $x \geq -2$

B) $x > -\frac{8}{5}$

C) $x > -1$

D) $x > \frac{1}{2}$

E) $x \geq -16$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
D	E	C	B	A	C	A	E	B	E	E	E

1. $2x-4<6$

$2x<6+4$

$2x<10$

$\frac{2x}{2}<\frac{10}{2}$

$x<5$

Yanıt / The Answer is **D**

2. $5-2x\le x+2$

$5-2\le x+2x$

$3\le 3\cdot x$

$\frac{3}{3}\le\frac{3\cdot x}{3}$

$1\le x \Rightarrow x\ge 1$

Yanıt / The Answer is **E**

3. $3x-[(2-x)+2x]+1<x-5$

$3x-[2-x+2x]+1<x-5$

$3x-2+x-2x+1<x-5$

$2x-1<x-5$

$2x-x<-5+1$

$x<-4$

Yanıt / The Answer is **C**

4. $\frac{2x-3}{5}<\frac{x-2}{7}+\frac{1}{3} \Rightarrow \frac{42x-63}{105}<\frac{15x-30+35}{105}$

(21) 15 35

$42x-63<15x+5$

$27x<68$

$x<\frac{68}{27}$

Yanıt / The Answer is **B**

5. $\frac{x-1}{4}-\frac{3}{12}\le\frac{2x-1}{3}+\frac{x}{12} \Rightarrow \frac{3x-3-36}{12}\le\frac{8x-4+12x}{12}$

3 12 4 12

$3x-39\le 20x-4$

$-39+4\le 20x-3x$

$-35\le 17x$

$\frac{-35}{17}\le\frac{17x}{17}$

$\frac{-35}{17}\le x$

Yanıt / The Answer is **A**

6. $4\cdot(x+1)-2>-\frac{x-1}{2} \Rightarrow 4x+4-2>\frac{1-x}{2}$

$\frac{4x+2}{1}>\frac{1-x}{2}$

$\frac{8x+4}{2}>\frac{1-x}{2}$

$8x+4>1-x$

$8x+x>1-4$

$\frac{9x}{9}>-\frac{3}{9}$

$x>-\frac{1}{3}$

Yanıt / The Answer is **C**

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7. $5x + 2 < 3 \cdot (x - 1)$

$5x + 2 < 3x - 3$

$5x - 3x < -3 - 2$

$2x < -5$

$\frac{2x}{2} < -\frac{5}{2}$

$x < -\frac{5}{2}$

Yanıt / The Answer is A

8. $-\frac{2}{1-x} > 0$

$1 - x < 0$

$1 < x$

Yanıt / The Answer is E

9. $x + 2y - 12 = 0$

$2y = 12 - x$

$\frac{2y}{2} = \frac{12-x}{2}$

$y = \frac{12-x}{2}$

$2 < y < 6 \Rightarrow 2 < \frac{12-x}{2} < 6$

$4 < 12 - x < 12$

$-8 < -x < 0$

$8 > x > 0$

Yanıt / The Answer is B

10. $4(x + 1) - 2 < x - 1$

$4x + 4 - 2 < x - 1$

$4x - x < -1 - 2$

$3x < -3$

$x < -1$

Yanıt / The Answer is E

11. $2(x - 1) \leq -1 + 3(x + 2)$

$2x - 2 \leq -1 + 3x + 6$

$-2 - 5 \leq 3x - 2x$

$-7 \leq x$

Yanıt / The Answer is E

12. $\frac{3}{4}(x+8) \geq \frac{-1}{2}(x-4) + x$

$\frac{3x}{4} + \frac{24}{4} \geq \frac{-x}{2} + \frac{4}{2} + x$

$\underset{(1)}{\frac{3x+24}{4}} \geq \underset{(2)}{\frac{-x+4+2x}{2}}$

$3x + 24 \geq 2x + 8$

$x \geq -16$

Yanıt / The Answer is E

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1. $\left.\begin{matrix} -3 \le x \le 4 \\ -4 \le y \le 2 \end{matrix}\right\} \Rightarrow \min(x^2 - 4y) = ?$

A) -16 B) -8 C) 0 D) 4 E) 8

[HARRAN ÜNİVERSİTESİ – YÖS 2020]

2. $\left.\begin{matrix} a,c \in Z \\ b \in z^+ \\ c < 0 \\ a < 2c \\ a+b+c = -19 \end{matrix}\right\} \Rightarrow \min(b) = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2017]

3. $k, p, m \in Z^+$

$\left.\begin{matrix} 13 \le k < 21 \\ 9 < p \le 19 \\ 2 < m \le 6 \end{matrix}\right\} \Rightarrow \max\left(\frac{k-p}{m}\right) = ?$

A) $\frac{10}{3}$ B) 2 C) $\frac{12}{5}$ D) $\frac{1}{5}$ E) $\frac{5}{6}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

4. $a,c \in Z,\ b \in Z^+$

$\left.\begin{matrix} c < 0 \\ a < 2c \\ a+b+c = -15 \end{matrix}\right\} \Rightarrow \min(b) = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

5. $\frac{2}{5} < x < \frac{3}{4} \Rightarrow x = ?$

A) $\frac{3}{10}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{4}{5}$ E) $\frac{5}{6}$

[İSTANBUL ÜNİVERSİTESİ – YÖS 2013]

6. $a < b < 0$

$c = \frac{3a-b}{a}$

? < c < ?

A) $1 < c < 2$ B) $2 < c < 3$ C) $3 < c < 4$ D) $4 < c < 5$ E) $5 < c < 6$

[YÖS 2008]

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YANITLAR / ANSWERS

1	2	3	4	5	6
B	A	A	A	C	B

1. $-3 \le x \le 4 \Rightarrow 0 \le x^2 \le 16$

$-4 \le y \le 2 \Rightarrow -8 \le -4y \le 16$

$-8 \le x^2 - 4y \le 32$

$\min(x^2 - 4y) = -8$

Yanıt / The Answer is B

2. $\underset{-18}{a} + \underset{1}{b} + \underset{-2}{c} = -19$

Yanıt / The Answer is A

3. $k, p, m \in Z^+$

$13 \le k < 21 \quad k = 20$

$9 < p \le 19 \quad p = 10$

$2 < m \le 6 \quad m = 3$

$\max\left(\frac{k-p}{m}\right) = \frac{20-10}{3} = \frac{10}{3}$

Yanıt / The Answer is A

4. $a, c \in Z, \ b \in Z^+$

$c < 0 \quad c = -5$

$a < 2c \quad a < -10 \quad a = -11$

$a + b + c = -15$

$\downarrow \quad \downarrow$

$-11 \quad -5$

$b = 1$

Yanıt / The Answer is A

5. $\underset{(4)}{\frac{2}{5}} < x < \underset{(5)}{\frac{3}{4}} \Rightarrow \frac{8}{20} < x < \frac{15}{20}$

$x = \frac{10}{20} = \frac{1}{2}$

Yanıt / The Answer is C

6. $c = \frac{3a-b}{a} = \frac{3a}{a} - \frac{b}{a} = 3 - \frac{b}{a}$

$a < b < 0 \Rightarrow 0 < \frac{b}{a} < 1$

$-1 < -\frac{b}{a} < 0$

$-1+3 < 3-\frac{b}{a} < 0+3$

$2 < c < 3$

Yanıt / The Answer is B

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1. $x \in Z$,

$2(x-1) - 3(x-2) > 0$

$\Rightarrow$ **max(x) = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

2. $2(x-1) + 3(2x-2) - 5(x-1) > 0$

$\Rightarrow$ **ÇK(SS) = ?**

A) $[1, \infty)$ B) $(1, \infty)$ C) $(-1, \infty)$

D) $(-\infty, 1]$ E) $(-\infty, 1)$

3. $2(x-2) + 3x > 5(x+1)$

$\Rightarrow$ **ÇK(SS) = ?**

A) $[2, \infty)$ B) $(\infty, 2]$ C) $(2, \infty)$

D) R E) $\varnothing$

4. $3(x-2) + x - 1 > 2(2x-2) \Rightarrow$ **ÇK(SS) = ?**

A) $\varnothing$ B) R C) $(1, \infty)$

D) $[2, \infty)$ E) $(-\infty, 1)$

5. $\frac{x+3}{2} - \frac{x-2}{3} \leq 5 \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, 17]$ B) $(-\infty, 17)$ C) $(2, 17)$

D) $[17, \infty)$ E) $(17, \infty)$

6. $\frac{2x}{3} - \frac{3x}{2} \leq -25$, $x \in Z \Rightarrow$ **min(x) = ?**

A) 21 B) 22 C) 24

D) 30 E) 32

7. $x < 0 < y$ ise, **aşağıdakilerden hangisi kesinlikle doğrudur?**

If $x < 0 < y$, which of the followings is definitely true?

A) $x + y < 0$

B) $x + y > 0$

C) $x \cdot y = 0$

D) $x - y > 0$

E) $x \cdot y < 0$

8. $x \in Z$,

$-3 \leq x < 5 \Rightarrow$ **min(2x + 3) + max(2x + 3) = ?**

A) 2 B) 4 C) 6

D) 8 E) 9

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9. $(2x-y) \in Z$,
$3 < x \leq 7$,
$-1 < y \leq 3 \Rightarrow \min(2x-y) = ?$

A) 1 B) 2 C) 3
D) 4 E) 5

10. $-4 < x < 5,\ 1 < y < 2 \Rightarrow ? < x+y < ?$

A) $-5 < x+y < 7$
B) $-3 < x+y < 3$
C) $-3 < x+y < 7$
D) $-6 < x+y < 6$
E) $-6 < x+y < 4$

11. $x \in Z$,

$$\left.\begin{array}{l} -3 \leq 2x-1 \leq 3 \\ -5 < x+1 < 4 \end{array}\right\} \Rightarrow \min(x)+\max(x) = ?$$

A) -1 B) 0 C) 1
D) 2 E) 3

12. $x \in Z$,

$$\left.\begin{array}{l} -3 \leq -2x+5 \leq 7 \\ 5 < 3x-4 \leq 14 \end{array}\right\} \Rightarrow x = ?$$

A) 1 B) 2 C) 3
D) 4 E) 5

13. $a,b,c \in R$

$$\left.\begin{array}{l} a \cdot b < 0 \\ a > b \\ \frac{c}{b} = 0 \end{array}\right\} \Rightarrow ? > ? > ?$$

A) $b < a < c$ B) $b < c < a$ C) $a << b < c$
D) $c < b < a$ E) $a < c < b$

14. $a \in N$

$$\left.\begin{array}{l} 36 < \frac{3a}{4} \\ \frac{4a}{5} < 40 \end{array}\right\} \Rightarrow a = ?$$

A) 43 B) 45 C) 48
D) 49 E) 50

15. $a,b \in Z$

$$\left.\begin{array}{l} -2 < a < 4 \\ 2 < b < 5 \end{array}\right\} \Rightarrow \min\left(\frac{a-b-1}{a-b}\right) = ?$$

A) 1 B) $\frac{3}{2}$ C) $\frac{6}{5}$
D) $\frac{5}{2}$ E) 3

16. $x,y \in R$

$$\left.\begin{array}{l} 1 < x < 4 \\ 2 < y < 4 \\ a < \frac{2x+y}{4} < b \end{array}\right\} \Rightarrow (a,b) = ?$$

A) (1,3) B) $\left(\frac{1}{3},1\right)$ C) $\left(\frac{1}{2},1\right)$
D) $\left(1,\frac{3}{2}\right)$ E) $\left(\frac{1}{2},\frac{3}{2}\right)$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	B	E	A	A	D	E	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	C	C	D	B	D	C	A

1. $5 < \frac{x}{2} - 3 < 6,\ x \in Z \Rightarrow \min(x) = ?$

A) 11 B) 13 C) 15

D) 17 E) 19

2. $3 \le \frac{x-2}{5} \le 7,\ x \in Z \Rightarrow \max(x) = ?$

A) 17 B) 20 C) 23

D) 35 E) 37

3. $2 \le -x + 5 \le 4,\ x \in Z \Rightarrow \sum x = ?$

A) 4 B) 5 C) 6

D) 7 E) 8

4. $\frac{3x}{2} \le x - 2 \le 6,\ x \in Z \Rightarrow \max(x) = ?$

A) −1 B) −2 C) −3

D) −4 E) −5

5. $2x + 5 \le 3x - 2 \le 5x + 6,\ x \in Z \Rightarrow \min(x) = ?$

A) 1 B) 3 C) 4

D) 5 E) 7

6. $-5 \le 3y - 5 < 7,$

$y = 5x - 5 \Rightarrow ? < x < ?$

A) $\left(1, \frac{9}{5}\right)$ B) $\left[1, \frac{9}{5}\right)$ C) $\left(-1, \frac{9}{5}\right)$

D) $\left[-1, \frac{9}{5}\right)$ E) $\left[\frac{9}{5}, 2\right)$

7. $x^2 \in Z,$

$25 < 3x - 8 \le 49 \Rightarrow \min(x^2) = ?$

A) 100

B) 121

C) 122

D) 144

E) 152

8. $-4 \le x < 5,$

$3 < y < 9,\ (x \cdot y) \in Z,$

$\Rightarrow \min(x \cdot y) + \max(x \cdot y) = ?$

A) 5 B) 7 C) 9

D) 11 E) 13

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9. $2x - 5 < 3x + 7$

$3x - 2 \leq 5x + 6$

$\Rightarrow$ **ÇK(SS) = ?**

A) $(-12, \infty)$ B) $[-12, \infty)$ C) $[-4, \infty)$

D) $(-4, \infty)$ E) $[-\infty, 4)$

10. $x^2 \in Z$,

$-7 \leq x < 12 \Rightarrow$ **max($x^2 - 3$) = ?**

A) 0

B) 48

C) 49

D) 50

E) 140

11. $(x^2 - 3) \in Z$,

$-7 < x < -4 \Rightarrow$ **min($x^2 - 3$) = ?**

A) 10 B) 11 C) 13

D) 14 E) 45

12. $2x + 1 < 3x + 11 < x - 5$, $x \in Z \Rightarrow \sum x = ?$

A) -9 B) -12 C) -16

D) -18 E) -20

13. $\left.\begin{array}{l} x^4 \cdot y \cdot z^2 < 0 \\ x^2 \cdot y^3 \cdot z > 0 \\ x \cdot y \cdot z^4 < 0 \end{array}\right\} \Rightarrow$ **x,y,z = ?,?,?**

A) +,+,+ B) +,−,− C) −,+,−

D) −,−,− E) −,−,+

14. $\left.\begin{array}{l} x \cdot y^3 \cdot z^5 < 0 \\ x^4 \cdot y > 0 \\ y + z < 0 \end{array}\right\} \Rightarrow$ **x,y,z = ?,?,?**

A) −,−,+ B) −,+,− C) +,+,+

D) +,+,− E) −,−,−

15. $x \in Z$

$1 < \frac{x}{2} - \frac{x+1}{3} < 4 \Rightarrow$ **max(x) = ?**

A) 24 B) 25 C) 26

D) 27 E) 28

16. $x \in R$, $\frac{3x+1}{x} \in Z$

$\frac{1}{9} < x < \frac{1}{5} \Rightarrow \sum x = ?$

A) 20 B) 25 C) 30

D) 35 E) 40

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	E	C	D	E	B	C	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	E	D	A	B	D	B	C

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BÖLÜM CHAPTER 9

TARİHSEL NOT / HISTORICAL NOTE

Banach Stefan [1892 - 1945]

Leh matematikçi, Fonsiyonel analiz konusuna katkıları olan ilk kişilerdendir.

Polish mathematician who was a major contributor to the subject known as functional analysis.

MUTLAK DEĞER
ABSOLUTE VALUE

Bu bölüm, 34 çözümlü örnek, 12 çözümlü test sorusu, 7 çözümlü YÖS sorusu, 64 test sorusu içermektedir.

This chapter includes 34 examples with solutions, 12 tests questions with solutions, 7 YÖS questions with solutions and 64 test questions.

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MUTLAK DEĞER / ABSOLUTE VALUE

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BÖLÜM
09
CHAPTER

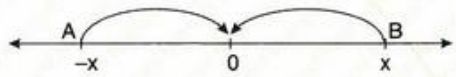
MUTLAK DEĞER
ABSOLUTE VALUE

MUTLAK DEĞER / ABSOLUTE VALUE

Tanım: Sayı doğrusu üzerinde, x sayısının sıfıra olan uzaklığına, x sayısının *mutlak değeri* denir ve |x| ile gösterilir.

Definition: *The absolute value* of a number is its distance from zero on the number line. The absolute value of the number x is denoted by |x|.

$$|OB| = |x|, \quad |OA| = |-x|$$



Buna göre,

$$|x| = \begin{cases} x, & x > 0 \text{ ise} \\ 0, & x = 0 \text{ ise} \\ -x, & x < 0 \text{ ise} \end{cases}$$

olarak tanımlanır.

According to that, |x| is defined.

Örnek / Example 1

$a > 5$, $|a - 5| = 3a - 21 \Rightarrow a = ?$

A) 6 B) 7 C) 8 D) 9 E) 10

Çözüm / Solution

Yanıt / Answer **C**

$a > 5 \Rightarrow a - 5 > 0$

$|a - 5| = a - 5$

$a - 5 = 3a - 21$

$21 - 5 = 3a - a$

$16 = 2a$

$a = 8$

Örnek / Example 2

$a < 7$,

$|a - 7| = 5a - 47 \Rightarrow$ ÇK(SS) = ?

A) {2} B) {3} C) {4} D) {5} E) ∅

Çözüm / Solution

Yanıt / Answer **E**

$a < 7 \Rightarrow a - 7 < 0$

$|a - 7| = 7 - a$

$7 - a = 5a - 47$

$7 + 47 = 5a + a$

$54 = 6a$

$a = 9$

$a = 9 > 7 \Rightarrow$ ÇK(SS) = Ø

Örnek / Example 3

$|-4| + |3 - \sqrt{2}| + |\sqrt{2} - 3| + |1 - \sqrt{2}| + |3 + \sqrt{2}| = ?$

A) 12 B) 10 C) $10 + 2\sqrt{2}$

D) $8 - 2\sqrt{2}$ E) $4\sqrt{2}$

Çözüm / Solution

Yanıt / Answer **A**

$-4 < 0 \Rightarrow |-4| = -(-4) = 4$

$\sqrt{9} > \sqrt{2} \Rightarrow 3 - \sqrt{2} > 0 \Rightarrow |3 - \sqrt{2}| = 3 - \sqrt{2}$

$\sqrt{2} < \sqrt{9} \Rightarrow \sqrt{2} - 3 < 0 \Rightarrow |\sqrt{2} - 3| = -(\sqrt{2} - 3) = 3 - \sqrt{2}$

$\sqrt{1} < \sqrt{2} \Rightarrow 1 - \sqrt{2} < 0 \Rightarrow |1 - \sqrt{2}| = -(1 - \sqrt{2}) = \sqrt{2} - 1$

$3 + \sqrt{2} > 0 \Rightarrow |3 + \sqrt{2}| = 3 + \sqrt{2} \Rightarrow |-4| + |3 - \sqrt{2}| + |\sqrt{2} - 3| + |1 - \sqrt{2}| + |3 + \sqrt{2}|$

$= 4 + 3 - \sqrt{2} + 3 - \sqrt{2} + \sqrt{2} - 1 + 3 + \sqrt{2}$

$= 4 + 3 + 3 + 3 - 1 = 12$

Örnek / Example 4

$a < 0 \Rightarrow |-a| + |a| - |a - 3| + |1 - a| = ?$

A) $-2a$ B) $-2a - 2$ C) $2a + 1$

D) $2a + 2$ E) $2a$

Çözüm / Solution

Yanıt / Answer **B**

$a < 0 \Rightarrow -a > 0 \Rightarrow |-a| = -a$

$a - 3 < 0 \Rightarrow |a - 3| = 3 - a$

$1 - a > 0 \Rightarrow |1 - a| = 1 - a$

$|-a| + |a| - |a - 3| + |1 - a| = (-a) + (-a) - (3 - a) + (1 - a)$

$= -a - a - 3 + a + 1 - a$

$= -2a - 2$

PEKİŞTİRME / REINFORCE

$x > 0$,

✓ $|x| + |x + 2| + |-x - 5| = ?$ ..... [3x + 7]

✓ $|x + 2| = x + 3 \Rightarrow$ ÇK(SS) = ? ..... [Ø]

✓ $|2x + 4| = x - 5 \Rightarrow$ ÇK(SS) = ? ..... [Ø]

✓ $|3x + 9| = x + 1 \Rightarrow$ ÇK(SS) = ? ..... [Ø]

PEKİŞTİRME / REINFORCE

✓ $|-2| + |-5| + |3| = ?$ ..... [10]

✓ $|1 - 2| + |4 - 7| + |8 - 3| = ?$ ..... [9]

✓ $|3 - 7| + |4| - |24| = ?$ ..... [−16]

✓ $|4 - \sqrt{6}| + |\sqrt{6} - 2| = ?$ ..... [2]

✓ $|2 - 4\sqrt{2}| + |2\sqrt{2} - 3| = ?$ ..... [$1 + 2\sqrt{2}$]

PEKİŞTİRME / REINFORCE

$x < 0 < y$,

✓ $|x| + |y - x| = ?$

..... [$y - 2x$]

✓ $|x| + |2y - 2x| = ?$

..... [$2y - 3x$]

✓ $|-3x| + |x - y| = ?$

..... [$y - 4x$]

✓ $|-2y| + |3x| = ?$

..... [$2y - 3x$]

Örnek / Example 5

$b - 3c < 2a$ ve / and $|b - 3c - 2a| = 3c - 4a \Rightarrow \frac{4a+3b}{b-5a} = ?$

A) 16 B) 18 C) 20 D) 22 E) 24

Çözüm / Solution Yanıt / Answer **D**

$b - 3c < 2a \Rightarrow b - 3c - 2a < 0 \Rightarrow |b - 3c - 2a| = -b + 3c + 2a$

$|b - 3c - 2a| = 3c - 4a$

$2a - (b - 3c) = 3c - 4a$

$2a - b + 3c = 3c - 4a$

$6a = b$

$\frac{4a+3b}{b-5a} = \frac{4a+3 \cdot 6a}{6a-5a} = \frac{22a}{a} = 22$

Örnek / Example 6

$3 < x < 8 \Rightarrow |2 - x| + |8 - x| = ?$

A) 2 B) 4 C) 6 D) 8 E) 10

Çözüm / Solution Yanıt / Answer **C**

$3 < x \Rightarrow |2 - x| = -(2 - x)$

$x < 8 \Rightarrow |8 - x| = (8 - x)$

$-(2 - x) + (8 - x) = -2 + x + 8 - x = 6$

Örnek / Example 7

$4 < x < 8 \Rightarrow |3 - x| - |x - 4| - |x - 8| = ?$

A) $x + 7$ B) $x - 7$ C) $3x - 7$

D) $3x + 7$ E) $x + 15$

PEKİŞTİRME / REINFORCE

✓ $1 < x < 5 \Rightarrow |x - 5| = ?$

..... [$5 - x$]

✓ $-1 < x < 0 \Rightarrow |1 - x| = ?$

..... [$1 - x$]

✓ $-2 < x < -1 \Rightarrow |x + 1| + |x + 2| = ?$

..... [1]

Çözüm / Solution Yanıt / Answer **B**

$4 < x \Rightarrow |3 - x| = x - 3$

$4 < x \Rightarrow |x - 4| = x - 4$

$x < 8 \Rightarrow |x - 8| = 8 - x$

$\Rightarrow |3 - x| - |x - 4| - |x - 8| = x - 3 - (x - 4) - (8 - x)$

$= x - 3 - x + 4 - 8 + x$

$= x - 7$

TEMEL ÖZELLİKLER / BASIC PROPERTIES

Özellik / Property

★ **Herhangi bir reel sayının mutlak değeri ya pozitiftir ya da sıfırdır.**
The absolute value of any real number is either positive or zero.

★ $a, b \in R^+$, $a > b$,
$|a - b| = |b - a| = a - b$

★ $|a \cdot b| = |a| \cdot |b|$

★ $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $b \neq 0$

★ $|a + b| \leq |a| + |b|$

★ $A = |f(x)| \Rightarrow \min(A) = 0$

★ $a, b, c \in R$
$|a| + |b| + |c| = 0$
$\Rightarrow a = 0,\ b = 0,\ c = 0$

★ $a \in R^-$,
n tek bir doğal sayı ise $|a^n| \neq a^n$, n çift bir doğal sayı ise $|a^n| = a^n$ dir.
if n is odd number, then $|a^n| \neq a^n$, if n is even number, then $|a^n| = a^n$.

Etkinlikler / Activities

a. $|(x-3)^3| \neq (x-3)^3$

b. $|(x-2)^2| = (x-2)^2$

c. $|(-7)^5| \neq (-7)^5$

d. $|3 - 14| = 14 - 3 = 11$

e. $|12 - 21| = 21 - 12 = 9$

f. $|17 - 11| = 17 - 11 = 6$

g. $|x - 7| = x - 7 \Rightarrow x - 7 \geq 0 \Rightarrow x \geq 7$

h. $|x - 3| = 3 - x \Rightarrow x - 3 \leq 0 \Rightarrow x \leq 3$

PEKİŞTİRME / REINFORCE

✓ $|x-2|=0 \Rightarrow x=?$

..... [2]

✓ $|3x-7|=0 \Rightarrow x=?$

..... [7/3]

✓ $|x^2-4|=0 \Rightarrow \prod x=?$

..... [–4]

✓ $|x^2-9|=0 \Rightarrow \sum x=?$

..... [0]

Örnek / Example 8

$A=|x-4|$, $\min(A) \Rightarrow x=?$

A) –2 B) 0 C) 3 D) 4 E) 5

Çözüm / Solution

Yanıt / Answer **D**

$|x-4|=0 \Rightarrow x-4=0$

$x=4$

Örnek / Example 9

$A=|a-4b|, \min(A) \Rightarrow \frac{2a+b}{a-3b}=?$

A) 3 B) 7 C) 8 D) 9 E) 10

Çözüm / Solution

Yanıt / Answer **D**

$|a-4b|$ ifadesinin en küçük değeri 0 olduğundan,

Since the minimum value of $|a-4b|$ is 0,

$|a-4b|=0$

$a-4b=0$

$a=4b$

$\frac{2a+b}{a-3b}=\frac{2\cdot 4b+b}{4b-3b}=\frac{9b}{b}=9$

Örnek / Example 10

$x \neq y \neq 0$,

$A=|3x+y|$, $\min(A) \Rightarrow \frac{3x-2y}{y+2x}=?$

A) – 10 B) –9 C) –8 D) –7 E) –6

Çözüm / Solution

Yanıt / Answer **B**

$|3x+y|=0 \Rightarrow 3x+y=0$

$y=-3x$

$\frac{3x-2y}{y+2x}=\frac{3x-2(-3x)}{-3x+2x}$

$=\frac{3x+6x}{-x}$

$=\frac{9x}{-x}$

$=-9$

Örnek / Example 11

$|a-2|+|b-3|+|c-4|=0 \Rightarrow$ **$2a+3b+4c=?$**

A) 27 B) 29 C) 32 D) 35 E) 38

Çözüm / Solution

Yanıt / Answer **B**

$|a-2|+|b-3|+|c-4|=0$

$a-2=0 \Rightarrow a=2$

$b-3=0 \Rightarrow b=3$

$c-4=0 \Rightarrow c=4$

$2a+3b+4c=2\cdot 2+3\cdot 3+4\cdot 4$

$=4+9+16$

$=29$

Örnek / Example 12

$A=|x-5|+|x-7| \Rightarrow$ **$\min(A)=?$**

A) 0 B) 1 C) 2 D) 3 E) 4

Çözüm / Solution

Yanıt / Answer **C**

$x-5=0 \Rightarrow x=5$ $\quad A=|5-5|+|5-7|$ $\quad =0+2=2$

$x-7=0 \Rightarrow x=7$ $\quad A=|7-5|+|7-7|$ $\quad =2+0=2$

O halde A'nın en küçük değeri 2 dir.

Therefore, the minimum value of A is 2

Örnek / Example 13

$a\cdot(|x+3|+|x+9|)=24 \Rightarrow$ **$\max(a)=?$**

A) 1 B) 2 C) 4 D) 7 E) 10

Çözüm / Solution

Yanıt / Answer **C**

$(|x+3|+|x+9|)$ ifadesi minimum iken a maximum olur.

a is maximum when $(|x+3|+|x+9|)$ is minimum.

$x+3=0 \Rightarrow x=-3$

$a(|-3+3|+|-3+9|)=24$

$a\cdot(0+6)=24$

$6a=24$

$a=4$

PEKİŞTİRME / REINFORCE

✓ $|2x-4|+|y-3|=0 \Rightarrow$ **$x+y=?$**

..... [5]

✓ $|x-2|+|y-4|=0 \Rightarrow$ **$x\cdot y=?$**

..... [8]

✓ $|x-2|+|y-4|=0 \Rightarrow$ **$x+y=?$**

..... [6]

✓ $|3a-6|+|2b-8|=0 \Rightarrow$ **$a\cdot b=?$**

..... [8]

PEKİŞTİRME / REINFORCE

✓ $x < 0$

$\Rightarrow \sqrt{x^2-5x+8}+\sqrt{x^2-2x+1} = ?$

.....[3 – x]

Özellik / Property

★ $\forall x \in R, \sqrt{(f(x))^2} = |f(x)|$

Örnek / Example 14

$x < 0 \Rightarrow \frac{\sqrt{x^2}}{x}+3 = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution Yanıt / Answer **B**

$x < 0 \Rightarrow |x| = -x$

$$\frac{\sqrt{x^2}}{x}+3 = \frac{|x|}{x}+3$$

$$= \frac{-x}{x}+3$$
$$= -1+3$$
$$= 2$$

MUTLAK DEĞERLİ DENKLEMLER
EQUATIONS CONTAINING ABSOLUTE VALUE

▼ $a \in R^+$, $|f(x)| = a \Rightarrow f(x) = a$, $f(x) = -a$

▼ $|f(x)| = g(x)$

I. $f(x) = g(x)$, II. $f(x) = -g(x)$

▼ $|f(x)| = |g(x)| \Rightarrow f(x) = g(x)$ v $f(x) = -g(x)$

Etkinlikler / Activities

a. $|x| = 3 \Rightarrow x_1 = 3$, $x_2 = -3$

b. $|2x-1| = 5 \Rightarrow 2x_1 - 1 = 5$, $2x_2 - 1 = -5$

$2x_1 = 6$ $2x_2 = -4$

$x_1 = 3$ $x_2 = -2$

c. $|3x-2| = 7 \Rightarrow 3x_1 - 2 = 7$, $3x_2 - 2 = -7$

$3x_1 = 9$ $3x_2 = -5$

$x_1 = 3$ $x_2 = -\frac{5}{3}$

Örnek / Example 15

$|7 - 4x| = -3 \Rightarrow$ ÇK(SS) = ?

A) {–2} B) {1} C) ∅ D) {–1} E) {2}

Çözüm / Solution

Yanıt / Answer **C**

$|7 - 4x| \geq 0 \Rightarrow |7 - 4x| \neq -3$

ÇK(SS) = ∅

PEKİŞTİRME / REINFORCE

✓ $|x - 1| = 3 \Rightarrow$ ÇK(SS) = ?

.....[{4, –2}]

✓ $|x + 1| = -1 \Rightarrow$ ÇK(SS) = ?

.....[∅]

✓ $|3x - 1| = 5, x \in Z \Rightarrow \sum x$ = ?

.....[2]

✓ $|x - 2| = 1 \Rightarrow$ ÇK(SS) = ?

.....[{3, 1}]

Örnek / Example 16

$|x - 1| + |2x - 2| + |3 - 3x| = 18 \Rightarrow$ ÇK(SS) = ?

A) {–1, 3} B) {0, 4} C) {–4, 2}

D) {–3, 4} E) {–2, 4}

Çözüm / Solution

Yanıt / Answer **E**

$|x - 1| + |2x - 2| + |3 - 3x| = 18$

$|x - 1| + |2|\cdot|x - 1| + |-3|\cdot|x - 1| = 18$

$|x - 1| + 2\cdot|x - 1| + 3\cdot|x - 1| = 18$

$6\cdot|x - 1| = 18$

$|x - 1| = 3$

$x - 1 = 3$ $x - 1 = -3$

$x = 4$ $x = -2$

$\Rightarrow$ ÇK(SS) = {–2, 4}

PEKİŞTİRME / REINFORCE

✓ $|2 - |x|| = 5 \Rightarrow$ ÇK(SS) = ?

.....[{ −7, 7}]

✓ $|3 + |x|| = 7 \Rightarrow$ ÇK(SS) = ?

.....[{ −4, 4}]

✓ $|3 - |3x|| = 9 \Rightarrow$ ÇK(SS) = ?

.....[{ −4, 4}]

✓ $x > 0, |-4 - |x|| = 5 \Rightarrow$ ÇK(SS) = ?

.....[{ 1 }]

Örnek / Example 17

$|4 - |2x|| = 6 \Rightarrow$ ÇK(SS) = ?

A) {−5, −1, 1, 5} B) {−1, 1} C) {−5, 5}

D) ∅ E) {−1, 5}

Çözüm / Solution

Yanıt / Answer **C**

$|4 - |2x|| = 6$

$4 - |2x| = 6$ $\quad$ $4 - |2x| = -6$

$|2x| \neq -2$ $\quad$ $|2x| = 4 + 6 = 10$

$2x = 10$ $\quad$ $2x = -10$

$x = 5$ $\quad$ $x = -5$

$\Rightarrow$ ÇK(SS) = {−5, 5}

Örnek / Example 18

$||4x - 2| - 4| = 2 \Rightarrow \prod x = ?$

A) −2 B) −6 C) 0 D) 10 E) 12

Çözüm / Solution

Yanıt / Answer **C**

$||4x - 2| - 4| = 2$

$|4x - 2| - 4 = 2$ $\quad$ $|4x - 2| - 4 = -2$

$|4x - 2| = 6$ $\quad$ $|4x - 2| = 2$

$4x - 2 = 6$ $\quad$ $4x - 2 = -6$ $\quad$ $4x - 2 = 2$ $\quad$ $4x - 2 = -2$

$4x = 8$ $\quad$ $4x = -4$ $\quad$ $4x = 4$ $\quad$ $4x = 0$

$x = 2$ $\quad$ $x = -1$ $\quad$ $x = 1$ $\quad$ $x = 0$

$\Rightarrow \prod x = 2 \cdot (-1) \cdot (1) \cdot 0 = 0$

Örnek / Example 19

$|3x - 4| = |2x - 6| \Rightarrow$ ÇK(SS) = ?

A) $\{-2, 2\}$ B) $\{-1, 1\}$ C) $\{-1, 3\}$

D) $\{-3, 5\}$ E) $\varnothing$

Çözüm / Solution

Yanıt / Answer **A**

$$|3x - 4| = |2x - 6|$$

$3x - 4 = 2x - 6$ $\qquad$ $3x - 4 = -2x + 6$

$3x - 2x = -6 + 4$ $\qquad$ $3x + 2x = 6 + 4$

$x = -2$ $\qquad$ $x = 2$

$\Rightarrow$ ÇK(SS) = $\{-2, 2\}$

Örnek / Example 20

$|2x + 1| = |3x - 2| \Rightarrow$ ÇK(SS) = ?

A) $\{-3, 2\}$ B) $\{-3, \frac{1}{5}\}$ C) $\{0, \frac{1}{5}\}$

D) $\{\frac{1}{5}, 3\}$ E) $\{\frac{2}{5}, 3\}$

Çözüm / Solution

Yanıt / Answer **D**

$$|2x + 1| = |3x - 2|$$

$2x + 1 = 3x - 2$ $\qquad$ $2x + 1 = -3x + 2$

$3 = x$ $\qquad$ $5x = 1$

$x = \frac{1}{5}$

PEKİŞTİRME / REINFORCE

✓ $|x - 1| = |x + 1| \Rightarrow$ ÇK(SS) = ?

.....[{0}]

✓ $|x - 3| = |2x - 4| \Rightarrow$ ÇK(SS) = ?

.....[{1, 7/3}]

✓ $|2x - 1| = |3x + 4| \Rightarrow$ ÇK(SS) = ?

.....[{−5, −3/5}]

✓ $|x + 1| = |2x - 1| \Rightarrow$ ÇK(SS) = ?

.....[{0, 2}]

ÖNEMLİ / IMPORTANT

- $|f(x)| = g(x)$ denkleminin kökleri, mutlak değerli ifadeler sıfırdan küçük olmayacağı için denklemde yerine yazılarak denenmelidir.
- Roots of the equation $|f(x)| = g(x)$ should be tested by replacing in the equation as the absolute value expressions can not be less than zero.

Örnek / Example 21

$|2x - 5| = 3x - 1 \Rightarrow$ ÇK(SS) = ?

A) $\left\{-5, \frac{5}{2}\right\}$ B) $\left\{-6, \frac{5}{2}\right\}$ C) $\{-6\}$ D) $\left\{-\frac{5}{2}\right\}$ E) $\left\{\frac{6}{5}\right\}$

Çözüm / Solution

Yanıt / Answer **E**

$$|2x - 5| = 3x - 1$$

$$2x - 5 = 3x - 1 \qquad 2x - 5 = 1 - 3x$$

$$-4 = x \qquad 5x = 6$$

$$x = -4 \Rightarrow |2x - 5| \neq -13 \qquad x = \frac{6}{5}$$

$$\Rightarrow \text{ÇK(SS)} = \emptyset$$

$$\text{ÇK(SS)} = \left\{\frac{6}{5}\right\}$$

Örnek / Example 22

$|2x - 5| = x - 12 \Rightarrow$ ÇK(SS) = ?

A) $\{-7\}$ B) $\left\{\frac{17}{3}\right\}$ C) $\left\{-7, \frac{17}{3}\right\}$ D) $\emptyset$ E) $\left\{-7, \frac{17}{6}\right\}$

Çözüm / Solution

Yanıt / Answer **D**

$$|2x - 5| = 3x - 1$$

$$2x - 5 = x - 12 \qquad 2x - 5 = 12 - x$$

$$x = -7 \qquad 3x = 17$$

$$x = -7 \Rightarrow |2x - 5| < 0 \qquad x = \frac{17}{3}$$

$$x = \frac{17}{3} \Rightarrow |2x - 5| < 0$$

$$\Rightarrow \text{ÇK(SS)} = \emptyset$$

Örnek / Example 23

$|x + 3| + |2x - 4| = 8 \Rightarrow$ ÇK(SS) = ?

A) {1, 3} B) {3, 4} C) {−1, 3} D) {−3, 1} E) ∅

Çözüm / Solution

Yanıt / Answer **C**

$x + 3 = 0 \Rightarrow x = -3$

$2x - 4 = 0 \Rightarrow x = 2$

$|x + 3|$ değeri, $x \geq -3$ için pozitif, $x < -3$ için negatiftir.

$|2x - 4|$ değeri $x \geq 2$ için pozitif, $x < 2$ için negatiftir.

The value $|x + 3|$ is positive when $x \geq -3$, and it is negative when $x < -3$.

The value $|2x - 4|$ is positive when $x \geq 2$ and it is negative when $x < 2$.

x	$-\infty$		−3		2		$+\infty$
x + 3	−		0	+		+	
2x − 4	−			−	0	+	
	III. bölge Quadrant III		II. bölge Quadrant II		I. bölge Quadrant I		

I. Bölge / Interval I

$2 \leq x < \infty$

$x + 3 + 2x - 4 = 8$

$3x - 1 = 8$

$3x = 9$

$x = 3$

$3 \in [2, \infty)$ olduğundan $x = 3$ denklemin köküdür. / Since $3 \in [2, \infty)$, $x = 3$ is root of equation.

II. Bölge / Interval II

$-3 \leq x < 2$

$x + 3 - 2x + 4 = 8$

$-x + 7 = 8$

$-x = 1$

$x = -1$

$-1 \in [-3, 2)$ olduğundan $x = -1$ denklemin köküdür./Since $-1 \in [-3, 2)$, $x = -1$ is root of equation.

III. Bölge / Interval III

$-\infty < x < -3$

$-x - 3 - 2x + 4 = 8$

$-3x + 1 = 8$

$-3x = 3$

$x = -\frac{7}{3}$

$-\frac{7}{3} \notin (-\infty, -3)$ olduğundan $x = -\frac{7}{3}$ denklemin kökü değildir.

Since $-\frac{7}{3} \notin (-\infty, -3)$, $x = -\frac{7}{3}$ is not root of equation.

O halde, denklemin çözüm kümesi, {−1, 3} tür.

Therefore, the solution set of equation is {−1, 3}.

PEKİŞTİRME / REINFORCE

$|x + 3| + |x - 4| = 7 \Rightarrow$ ÇK(SS) = ?

..... [–3,4]

Örnek / Example 24

$|x + 3| + |x - 2| = 5 \Rightarrow$ ÇK(SS) = ?

A) $[-3, 2]$ B) $[-3, 2)$ C) $(-3, 2]$

D) $(-3, 2)$ E) $\varnothing$

Çözüm / Solution Yanıt / Answer **A**

x	$-\infty$	–3	2	$+\infty$
x + 3	–	0	+	+
x – 2	–	–	0	+

i. $2 \le x < \infty$

$x + 3 + x - 2 = 5$

$2x + 1 = 5$

$2x = 4$

$x = 2$

$2 \in [2, \infty)$ olduğundan $x = 2$ denklemin köküdür.

Since $2 \in [2, \infty)$, $x = 2$ is root of the equation.

ii. $-3 \le x < 2$

$x + 3 - x + 2 = 5$

$5 = 5$

olduğundan $-3 \le x < 2$ aralığındaki tüm reel sayılar, bu denklemi sağlayacağından denklemin kökleri $[-3, 2)$ aralığındadır.

The all real numbers satisfy equation in interval $-3 \le x < 2$ and all are roots of equation. So roots are in interval $[-3, 2)$.

iii. $-\infty < x < -3$

$-x - 3 - x + 2 = 5$

$-2x - 1 = 5$

$-2x = 6$

$x = -3$

$-3 \notin (-\infty, -3)$ olduğundan denklemin kökü değildir.

Since $-3 \notin (-\infty, -3)$, it is not root of equation.

O halde denklemin çözüm kümesi, $[-3, 2) \cup \{2\} = [-3, 2]$ dir.

Therefore, the solution set is $[-3, 2) \cup \{2\} = [-3, 2]$.

MUTLAK DEĞERLİ EŞİTSİZLİKLER
INEQUALITIES CONTAINING ABSOLUTE VALUE

▼ $a \in R^+$,

$|f(x)| < a \Rightarrow -a < f(x) < a$

▼ $a \in R^+$, $|f(x)| > a \Leftrightarrow f(x) > a$, $f(x) < -a$

▼ $a, b \in R^+$, $a < b$,

$a < |f(x)| < b$

$a < f(x) < b$ $\qquad -b < f(x) < -a$

▼ $a, b \in R - \{0\}$, $|f(x)| < |g(x)| \Rightarrow f^2(x) < g^2(x)$

Etkinlikler / Activities

a. $|x| < 5 \Leftrightarrow -5 < x < 5$

b. $|x| < \frac{3}{4} \Leftrightarrow -\frac{3}{4} < x < \frac{3}{4}$

c. $|x| < 0 \rightarrow$ çözüm yok / there is not-solution

d. $|x| < -3 \rightarrow$ çözüm yok / there is not-solution

e. $|x| > 3 \Rightarrow x > 3$, $x < -3$

f. $|x| > -5 \Rightarrow$ ÇK(SS) = R

g. $-4 < |x| < 5 \Rightarrow |x| < 5 \Rightarrow -5 < x < 5$

Örnek / Example 25

$x \in Z$

$|2x - 5| < 7 \Rightarrow \sum x = ?$

A) 14 B) 15 C) 16 D) 17 E) 18

Çözüm / Solution

Yanıt / Answer **B**

$|2x - 5| < 7$

$-7 < 2x - 5 < 7$

$-2 < 2x < 12$

$-1 < x < 6$

$\sum x = 0 + 1 + 2 + 3 + 4 + 5 = 15$

PEKİŞTİRME / REINFORCE

✓ $|x| < -3 \Rightarrow ? < x < ?$

.....[Ø]

✓ $|x| < -4 \Rightarrow ? < x < ?$

.....[Ø]

✓ $|x| > 7 \Rightarrow ? < x < ?$

..... [x > 7, x < −7]

✓ $|x| > -9 \Rightarrow ? < x < ?$

.....[R]

Örnek / Example 26

$|x| < 10$ ve $x - 2y = 4 \Rightarrow ? < y < ?$

A) $-7 < y < -3$ B) $-7 < y < -1$ C) $-7 < y < 3$

D) $-6 < y < 2$ E) $-6 < y < 3$

Çözüm / Solution

Yanıt / Answer **C**

$x - 2y = 4 \Rightarrow x = 2y + 4$

$|x| < 10 \Rightarrow -10 < x < 10$

$-10 < 2y + 4 < 10$

$-14 < 2y < 6$

$-7 < y < 3$

Örnek / Example 27

$|x| < 6$ ve $y = \frac{x}{2} + 4$, $y \in Z \Rightarrow \sum y = ?$

A) 17 B) 18 C) 19 D) 20 E) 21

Çözüm / Solution

Yanıt / Answer **D**

$y = \frac{x}{2} + 4 \Rightarrow y - 4 = \frac{x}{2}$

$\Rightarrow x = 2y - 8$

$|x| < 6 \Leftrightarrow -6 < x < 6$

$-6 < 2y - 8 < 6$

$2 < 2y < 14$

$1 < y < 7$

$2 \quad 3 \quad 4 \quad 5 \quad 6$

$\sum y = 2 + 3 + 4 + 5 + 6 = 20$

PEKİŞTİRME / REINFORCE

$x \in Z$

✓ $|x - 1| < 2 \Rightarrow \sum x = ?$

..... [3]

✓ $|x + 3| \leq 2 \Rightarrow \sum x = ?$

..... [−15]

✓ $|2x - 4| < 6 \Rightarrow \prod x = ?$

..... [0]

Örnek / Example 28

$\left|\frac{2x-3}{5}\right| < 3 \Rightarrow \; ? < x < ?$

A) $-6 < x < 9$ B) $-6 < x < 8$ C) $-7 < x < 9$

D) $-8 < x < 10$ E) $-8 < x < 11$

Çözüm / Solution

Yanıt / Answer **A**

$$\left|\frac{2x-3}{5}\right| < 3 \Rightarrow -3 < \frac{2x-3}{5} < 3$$
$$-15 < 2x-3 < 15$$
$$-12 < 2x < 18$$
$$-6 < x < 9$$

Örnek / Example 29

$x \in Z$,

$\frac{|2x-1|-9}{|x-3|} < 0 \Rightarrow \sum x = ?$

A) 0 B) 1 C) 2 D) 3 E) 4

Çözüm / Solution

Yanıt / Answer **B**

$|x-3| > 0$ ve $x-3 \neq 0, x \neq 3$

$|2x-1|-9 < 0 \Rightarrow |2x-1| < 9$

$-9 < 2x-1 < 9$

$-8 < 2x < 10$

$-4 < x < 5$



$x = -3-2-1+0+1+2+4 = 1$

PEKİŞTİRME / REINFORCE

✓ $|x-2| > 0 \Rightarrow$ ÇK(SS) = ?

..... [R – {2}]

✓ $|x+3| \geq 0 \Rightarrow$ ÇK(SS) = ?

..... [R]

✓ $|x-3| < 0 \Rightarrow$ ÇK(SS) = ?

..... [Ø]

✓ $|x+4| \leq 0 \Rightarrow$ ÇK(SS) = ?

..... [{–4}]

Örnek / Example 30

$x \in Z$,

$|x - 3| \geq 1 \Rightarrow x = ?$

A) –3 B) –2 C) 0 D) 2 E) 3

Çözüm / Solution

Yanıt / Answer **A**

$|x - 3| \geq 1$

$x - 3 \geq 1$ $\qquad$ $x - 3 \leq -1$

$x \geq 4$ $\qquad$ $x \leq 2$

$\{\ldots -5 -4 -3 -2 -1 +0 +1 +2 +4 +5 +6 \ldots\}$

$\sum x = -3$

Örnek / Example 31

$4 \leq |x + 2| < 7,\ x \in Z \Rightarrow \sum x = ?$

A) –21 B) –18 C) –12 D) –9 E) –6

Çözüm / Solution

Yanıt / Answer **C**

$4 \leq |x + 2| < 7$

$4 \leq -x - 2 < 7$ $\qquad$ $4 \leq x + 2 < 7$

$-9 < x \leq -6$ $\qquad$ $2 \leq x < 5$

$x = -6, -7, -8$ $\qquad$ $x = 2, 3, 4$

$\sum x = -6 - 7 - 8 + 2 + 3 + 4 = -12$

Örnek / Example 32

$x \in Z^-$

$-\frac{3}{2} \leq \left|\frac{x-2}{4}\right| \leq \frac{5}{4} \Rightarrow \sum x = ?$

A) –12 B) –10 C) –8 D) –6 E) –4

Çözüm / Solution

Yanıt / Answer **D**

$-\frac{3}{2} \leq \left|\frac{x-2}{4}\right| \leq \frac{5}{4}$

$-\frac{6}{4} \leq \frac{|x-2|}{4} \leq \frac{5}{4}$

$-6 \leq |x-2| \leq 5$

$|x-2| \leq 5$

$-5 \leq x - 2 \leq 5$

$-3 \leq x \leq 7,\ x \in Z^-$

$\Rightarrow \sum x = -3 - 2 - 1 = -6$

Örnek / Example 33

$|x-2| < |x+3| \Rightarrow$ ÇK(SS) = ?

A) $(-\infty, -1)$ B) $(-1,1)$ C) $\left(-\frac{1}{2},1\right)$ D) $\left(-\frac{1}{2},\infty\right)$ E) $(0, \infty)$

Çözüm / Solution

Yanıt / Answer **D**

$|x-2| < |x+3|$

$(x-2)^2 < (x+3)^2$

$x^2 - 4x + 4 < x^2 + 6x + 9$

$-5 < 10x$

$-\frac{1}{2} < x$

$ÇK(SS) = \left(-\frac{1}{2},\infty\right)$

Örnek / Example 34

$|x-1| \leq |x+2| \Rightarrow$ ÇK(SS) = ?

A) $(-\infty, -\frac{1}{2}]$ B) $(-\infty, \frac{1}{2}]$ C) $[-\frac{1}{2}, \infty)$ D) $[0, \infty)$ E) $[\frac{1}{2}, \infty)$

Çözüm / Solution

Yanıt / Answer **C**

$|x-1| \leq |x+2|$

$(x-1)^2 \leq (x+2)^2$

$x^2 - 2x + 1 \leq x^2 + 4x + 4$

$-3 \leq 6x$

$-\frac{1}{2} \leq x \Rightarrow ÇK(SS) = [-\frac{1}{2}, \infty)$

1. $a < b < 0 \Rightarrow |a+b| + |b-a| = ?$

A) $2b$ B) $-b$ C) $-a$

D) $-2a$ E) $2a$

2. $a < 0 < b \Rightarrow \sqrt{a^2-2ab+b^2} + |b-a| = ?$

A) $2b$ B) $2a-b$ C) $2b-2a$

D) $2a-2b$ E) $a-b$

3. $1 < x < 3 \Rightarrow \sqrt{x^2-6x+9} - \sqrt{x^2-2x+1} = ?$

A) $2x-3$ B) $2x-4$ C) $2x-5$

D) $-2x+4$ E) $-2x+5$

4. $a < b < 0 < c \Rightarrow$

$\sqrt{a^2} + \sqrt{(a-b)^2} - \sqrt{(c-b)^2} + \sqrt{(c-a)^2} = ?$

A) $2b-3a$ B) $2b-a$ C) $3b-a$

D) $3b-2a$ E) $2b-2a$

5. $|4-3x| = 2 \Rightarrow$ ÇK(SS) = ?

A) $\left\{\frac{2}{3}, 2\right\}$ B) $\left\{\frac{1}{3}, 2\right\}$ C) $\left\{\frac{2}{3}, 1\right\}$

D) $\{1, 2\}$ E) $\left\{\frac{1}{3}, 1\right\}$

6. $|3x-12| = 12-3x \Rightarrow x \leq ?$

A) $x \leq 3$ B) $x \leq 4$ C) $x \leq 5$

D) $x \leq 6$ E) $x \leq 7$

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7. $|5-x|+x=7 \Rightarrow$ **ÇK (SS) = ?**

A) { 4 } B) { 5 } C) { 6 }

D) { 7 } E) { 8 }

8. $x \in Z$,

$3 < |2x-1| < 7 \Rightarrow \sum x = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

9. $|x^2-3x-4| = |x-4| \Rightarrow \sum x = ?$

A) 0 B) 1 C) 2

D) 3 E) 4

10. $a < -2 \Rightarrow \sqrt{a^2-7a+18+\sqrt{a^2+4a+4}} = ?$

A) 4 – a B) a – 4 C) 2 – a

D) 2 + a E) a + 4

11. $|4-2x| \leq 8 \Rightarrow$ **ÇK (SS) = ?**

A) (3 , 5) B) $(-\infty , 6)$ C) $[-2, +\infty)$

D) (– 2 , 6) E) [– 2, 6]

12. $|2x-1| < 5 \Rightarrow$ **? < x < ?**

A) $-3 < x < 5$

B) $-5 < x < 0$

C) $-5 < x < 1$

D) $-3 < x < 2$

E) $-2 < x < 3$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
D	C	D	A	A	B	C	A	C	A	E	E

1. $a < 0, b < 0 \Rightarrow a + b < 0$

$a < b \Rightarrow 0 < b - a$

$|a+b| + |b-a| = -a - b + b - a$

$= -2a$

Yanıt / The Answer is D

2. $a < b \Rightarrow a - b < 0$

$\Rightarrow b - a > 0$

$\sqrt{a^2 - 2ab + b^2} + |b-a| = \sqrt{(a-b)^2} + |b-a|$

$= |a-b| + |b-a|$

$= -a + b + b - a$

$= 2b - 2a$

Yanıt / The Answer is C

3. $1 < x \Rightarrow 0 < x - 1$

$x < 3 \Rightarrow x - 3 < 0$

$\sqrt{x^2 - 6x + 9} - \sqrt{x^2 - 2x + 1} = \sqrt{(x-3)^2} - \sqrt{(x-1)^2}$

$= |x-3| - |x-1|$

$= -x + 3 - (x-1)$

$= -2x + 4$

Yanıt / The Answer is D

4. $a < b \Rightarrow a - b < 0$

$b < c \Rightarrow 0 < c - b$

$a < c \Rightarrow 0 < c - a$

$\sqrt{a^2} + \sqrt{(a-b)^2} - \sqrt{(c-b)^2} + \sqrt{(c-a)^2}$

$= |a| + |a-b| - |c-b| + |c-a|$

$= -a + (-a + b) - (c - b) + (c - a)$

$= -a - a + b - c + b + c - a$

$= 2b - 3a$

Yanıt / The Answer is A

5. $|4 - 3x| = 2$

$4 - 3x = 2$ ve $4 - 3x = -2$

$4 - 2 = 3x$ ve $3x = 4 + 2$

$2 = 3 \cdot x$ ve $3x = 6$

$\frac{2}{3} = x$ ve $x = 2$

$\Rightarrow$ ÇK(SS) $= \left\{\frac{2}{3}, 2\right\}$

Yanıt / The Answer is A

6. $|3x - 12| = 12 - 3x \Rightarrow 3x - 12 \leq 0$

$3x \leq 12$

$\frac{3x}{3} \leq \frac{12}{3}$

$x \leq 4$

Yanıt / The Answer is B

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7. $|5 - x| + x = 7$

$5 - x + x = 7$ veya / or $-5 + x + x = 7$

$5 \neq 7$ $2x = 7 + 5$

$x = 6$

Yanıt / The Answer is C

8. $3 < |2x - 1| < 7$

$3 < 2x - 1 < 7$ $-7 < 2x - 1 < -3$

$4 < 2x < 8$ $-6 < 2x < -2$

$2 < x < 4$ $-3 < x < -1$

$x = 3$ $x = -2$

$\Rightarrow \sum x = 3 - 2 = 1$

Yanıt / The Answer is A

9. $|x^2 - 3x - 4| = |x - 4|$

$|x - 4| \cdot |x + 1| = |x - 4|$

$|x - 4| = 0$ $|x + 1| = 1$

$x = 4$

$x + 1 = 1$ $x + 1 = -1$

$x = 0$ $x = -2$

$\Rightarrow \sum x = 4 + 0 + (-2) = 2$

Yanıt / The Answer is C

10. $a < -2 \Rightarrow a + 2 < 0$

$$\sqrt{a^2 - 7a + 18 + \sqrt{a^2 + 4a + 4}} = \sqrt{a^2 - 7a + 18 + \sqrt{(a+2)^2}}$$
$$= \sqrt{a^2 - 7a + 18 - a - 2}$$
$$= \sqrt{a^2 - 8a + 16}$$
$$= \sqrt{(a-4)^2}$$
$$= |a - 4|$$
$$= 4 - a$$

Yanıt / The Answer is A

11. $|4 - 2x| \leq 8 \Rightarrow -8 \leq 4 - 2x \leq 8$

$-12 \leq -2x \leq 4$

$6 \geq x \geq -2$

$\Rightarrow$ ÇK(SS) = $[-2, 6]$

Yanıt / The Answer is E

12. $|2x - 1| < 5 \Rightarrow -5 < 2x - 1 < 5$

$-4 < 2x < 6$

$-2 < x < 3$

Yanıt / The Answer is E

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1. $a, b, c \in Z^+$

$\frac{1}{a} < \frac{1}{b} < \frac{1}{c}$

$|2a - b| + |2b - c| - |2a - c| = ?$

A) $2c - b$ B) $-b$ C) c D) $b + 2c$ E) b

[KARABÜK ÜNİVERSİTESİ – YÖS 2020]

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2. $a, b \in Z$,

$|a - b - 1| + |3a + 2b - 15| = 1$ **ise** $a + b = ?$

A) 6 B) 7 C) 8 D) 9 E) 10

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2020]

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3. $x < 0 < y < z$

$|x - z| + |z - y| - |x - y| + |y + z| = ?$

A) $x - y - z$ B) $2x + y$ C) $-y + 3z$ D) $2x$ E) $2x + 2y - 2z$

[KARABÜK ÜNİVERSİTESİ – YÖS 2019]

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4. $y < 0 < x \Rightarrow |y| - |x - y| + |x| = ?$

A) x B) y C) $-x$ D) $-y$ E) 0

[BALIKESİR ÜNİVERSİTESİ – YÖS 2018]

5. $3 < x < 5 \Rightarrow \frac{|(x-2)(x-5)| + x - 5|}{x^2 - 4x + 3} = ?$

A) $\frac{3-x}{x-5}$ B) $\frac{3-x}{x-1}$ C) $\frac{5-x}{x-1}$ D) $\frac{x-3}{x-1}$ E) $\frac{x-5}{x-3}$

[İNÖNÜ ÜNİVERSİTESİ – YÖS 2018]

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6. $x, y \in N^+$,

$|x| + |y| = 2|x|$

$2x + y = 12 \Rightarrow$ **min(y) = ?**

A) -4 B) -10 C) -12 D) -24 E) -50

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2017]

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7. $a < 0 < b$

$\Rightarrow \frac{a \cdot |b - a|}{|a| + b|} = ?$

A) $-a$ B) a C) b D) $-b$ E) 1

[KARADENİZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

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8. $a = 1/36$,

$b = 1/4$,

$c = 1/3$

$\Rightarrow |a - |b - c|| = ?$

A) $-\frac{1}{18}$ B) $\frac{1}{18}$ C) $-\frac{1}{9}$ D) $\frac{1}{9}$ E) $\frac{1}{36}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2015]

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	A	C	E	C	C	B	B

1. $\frac{1}{a} < \frac{1}{b} < \frac{1}{c} \Rightarrow a > b > c$

$|2a - b| + |2b - c| - |2a - c|$

$= 2a - b + 2b - c - (2a - c)$

$= b$

Yanıt / The Answer is E

---

2. $a, b \in Z$

$|a - b - 1| + |3a + 2b - 15| = 0$

$a, b \in Z \Rightarrow (a - b - 1) \in Z$

$(3a + 2b - 15) \in Z$

$a - b - 1 = -1 \Rightarrow a - b = 0$

$3a + 2b - 15 = 0 \Rightarrow 3a + 2b = 15$

$\Rightarrow a = 3, b = 3$

$\Rightarrow a + b = 6$

Yanıt / The Answer is A

---

3. $\underbrace{|x-z|}_{-} + \underbrace{|z-y|}_{+} - \underbrace{|x-y|}_{-} + \underbrace{|y+z|}_{+}$

$= -x + z + z - y + x - y + y + z$

$= -y + 3z$

Yanıt / The Answer is C

---

4. $3 < x < 5$

$y < 0 < x \Rightarrow |y| - |x - y| + |x|$

$= -y - x + y + x = 0$

Yanıt / The Answer is E

---

5. $3 < x < 5$

$$\frac{|\,|(x-2)(x-5)| + x - 5\,|}{x^2 - 4x + 3} = \frac{|(x-2)(-x+5) + x - 5}{(x-3)(x-1)}$$

$$= \frac{|-x^2 + 8x - 15|}{(x-3)(x-1)}$$

$$= \frac{-(x-3)(x-5)}{(x-3)(x-1)}$$

$$= \frac{-x+5}{x-1}$$

Yanıt / The Answer is C

---

6. $x, y \in Z,$

$|x| + |y| = 2|x|$

$|y| = 2 \cdot |x| - |x|$

$|y| = |x|$

$y = -x$

$2x + y = 12$

$2x - x = 12$

$x = 12$

$y = -12$

Yanıt / The Answer is C

---

7. $b - a > 0 \Rightarrow \frac{a \cdot |b-a|}{|a|+b} = \frac{a \cdot (b-a)}{b-a} = a$

Yanıt / The Answer is B

---

8. $|a + b - c| = \left|\underset{(1)}{\frac{1}{36}} + \underset{(9)}{\frac{1}{4}} - \underset{(12)}{\frac{1}{3}}\right|$

$$= \left|\frac{1 + 9 - 12}{36}\right|$$

$$= \left|-\frac{1}{18}\right|$$

$$= \frac{1}{18}$$

Yanıt / The Answer is B

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $|4-\sqrt{8}|+|2-\sqrt{8}|=?$

A) 0 B) 1 C) 2 D) $\sqrt{2}$ E) $2\sqrt{2}$

2. $x<-1 \Rightarrow |1-x|+|x|-3|+2x=?$

A) –2 B) –1 C) 0 D) 1 E) 2

3. $x<0<y \Rightarrow |x|-|y|-|x-y|-|y-x|=?$

A) $-3x+y$ B) 0 C) $x+3y$ D) $x-3y$ E) $x+y$

4. $x<3 \Rightarrow |x-3|+5x-4=?$

A) $6x-1$ B) $6x-7$ C) $4x+2$ D) $2x+1$ E) $4x-1$

5. $x>0$,$y<0 \Rightarrow \sqrt{x^2}+\sqrt{y^2}=?$

A) 2x B) x – y C) x + y D) 2y E) 0

6. $a<-2 \Rightarrow \sqrt{a^2+4a+4}=?$

A) 4 – a B) – 2 – a C) 2 + a D) a + 4 E) a – 4

7. $1<a<2 \Rightarrow \sqrt{a^2-4a+4}-\sqrt{a^2-2a+1}=?$

A) 5 – a B) 3 – a C) 2 (a + 2) D) 4 – a E) 3 – 2a

8. $a<0$,$0<b \Rightarrow \sqrt{9a^2}-\sqrt{4b^2}-|a-b|=?$

A) –(a + b) B) –(2a + 3b) C) 3a – b D) 2(a – b) E) – 2(2a – b)

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $a<0<b \Rightarrow$

$|a-b|-|b-2a|-|a|=?$

A) b B) $-b$ C) $2b-a$

D) $2a$ E) $-a$

10. $a<0 \Rightarrow \dfrac{|-5a|+|a|+|-4a|}{|a|}=?$

A) $10a$ B) $-12a$ C) 15

D) -10 E) 10

11. $a<\dfrac{1}{2} \Rightarrow |1-2a|-|-2a+6|=?$

A) -10 B) -9 C) -5

D) 8 E) 12

12. $x<y<0<z \Rightarrow$

$\sqrt{x^2-2xy+y^2}-\sqrt{x^2-2xz+z^2}+\sqrt{y^2-2yz+z^2}=?$

A) 0 B) $3x$ C) y

D) $x+y+z$ E) z

13. $a<b<c$

$|a-b|-|b-c|+|c-a|=?$

A) $2b-2a$ B) $b-a$ C) $a-b$

D) $2a-2b$ E) $2b$

14. $a<0$

$|5a+|2a+|-a|||=?$

A) $-8a$ B) $-6a$ C) $-4a$

D) $4a$ E) $6a$

15. $a<b<0$

$\sqrt{(a-1)^2}+\sqrt[3]{(b-1)^3}-\sqrt{(a-b)^2}=?$

A) $-a$ B) $-b$ C) 0

D) a E) b

16. $y<0<x$

$\Rightarrow \dfrac{3|y-x|}{|5x|+5|y|}=?$

A) $\dfrac{-3}{10}$ B) $\dfrac{-3}{5}$ C) $\dfrac{-1}{2}$

D) $\dfrac{3}{5}$ E) $\dfrac{3}{10}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	A	D	E	B	B	E	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	E	C	A	A	C	C	D

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $3\cdot|-3+4-(-2)|-4\cdot|-3|=?$

A) -3 B) -2 C) -1

D) 0 E) 1

2. $|3-2\sqrt{3}|+|4-2\sqrt{3}|=?$

A) 0 B) 1 C) 2

D) $\sqrt{2}$ E) $2\sqrt{2}$

3. $|5x-3|=x-9 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\frac{3}{2}\right\}$ B) $\{2\}$ C) $\{3\}$

D) $\left\{-\frac{3}{2}\right\}$ E) Ø

4. $x>0$,

$|-x-3|-|3x+7|+|-8|=0 \Rightarrow x=?$

A) 1 B) 2 C) 3

D) 4 E) 5

5. $|3+2x|=15 \Rightarrow \sum x=?$

A) 9 B) 6 C) 3

D) 0 E) -3

6. **$|3x-4|<5$ eşitsizliğini sağlayan kaç tane tamsayı vardır?**

How many integer values x satisfy inequality $|3x-4|<5$?

A) 1 B) 2 C) 3

D) 4 E) 5

7. $|2x-6|+|4y+20|=0 \Rightarrow x+y=?$

A) -2 B) -1 C) 3

D) 8 E) 14

8. $|2x-4|+|y-5|+|z+2|=0$

$\Rightarrow x+y+z=?$

A) 2 B) 3 C) 4

D) 5 E) 6

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $x > 2$, $|3x + |2 - x|| = 10 \Rightarrow$ **ÇK(SS) = ?**

A) { 2 } B) { 3 } C) { 4 }

D) { −2 ,2 } E) { 2 , 3 }

10. $|3 - 5x| \le 13 \Rightarrow$ **? < x < ?**

A) $-5 \le x < -2$ B) $-3 \le x \le \frac{11}{3}$ C) $-3 \le x \le 3$

D) $-2 \le x \le \frac{16}{5}$ E) $-2 \le x \le 4$

11. $|a^2 - 16| - |a + 4| = 0 \Rightarrow$ **ÇK (SS) = ?**

A) { −2, 3, 4} B) { − 4 , 3 , 5 }

C) {− 2, 3, 6 } D) { −1, 2, 0 }

E) { −1, 0, 1}

12. $|x| - 2 = |x - 2| \Rightarrow$ **ÇK(SS) = ?**

A) R B) $(-\infty, 0)$ C) $(-2, +\infty)$

D) $[2 , +\infty)$ E) Ø

13. $x \in Z$

$2 < |x - 1| \le 5 \Rightarrow \sum x = ?$

A) −15 B) −14 C) 6

D) −6 E) −4

14. $x \in Z$

$|2x + 1| \ge 7 \Rightarrow \sum x = ?$

A) −4 B) −3 C) 0

D) 3 E) 4

15. $x \in R$

$|x - 2| + |2x - 9| = A \Rightarrow$ **min(A) = ?**

A) 0 B) $\frac{1}{2}$ C) 1

D) $\frac{5}{2}$ E) 5

16. $|2x - 5| = |9 - 2x| \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\frac{1}{2}\right\}$ B) $\left\{\frac{1}{2}, 7\right\}$ C) $\left\{\frac{7}{2}\right\}$

D) $\left\{-\frac{7}{2}, 0\right\}$ E) Ø

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	B	E	B	E	C	A	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	D	B	D	C	D	D	C

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $x < -2 \Rightarrow \sqrt{x^2+5x+1+\sqrt{x^2-6x+9}} + x = ?$

A) $x-2$ B) x C) $-x-2$

D) $2-x$ E) -2

2. $2 < x < 6$,

$|x-2| + |x-6| + 2x = 10 \Rightarrow x = ?$

A) 3 B) 4 C) 5

D) 6 E) 8

3. $a < \frac{3}{4} \Rightarrow |4a-3| - |12-4a| = ?$

A) -10 B) -9 C) -3

D) 8 E) 12

4. $-5 < x < -1$,

$|x+1| + |x+5| + y = 7 \Rightarrow y = ?$

A) 3 B) 4 C) 5

D) 6 E) 7

5. $|x| \le 2 \Rightarrow |x+2| + |x-4| = ?$

A) 2 B) 3 C) 4

D) 5 E) 6

6. $\frac{2 \cdot x^4}{|x^3|} + 3x = 20 \Rightarrow \sum x = ?$

A) 4 B) 8 C) 12

D) 16 E) 24

7. $x \in Z$,

$3 < |x+1| < 5 \Rightarrow \sum x = ?$

A) -2 B) -1 C) 0

D) 1 E) 2

8. $|-3x+7| = 5 \Rightarrow \sum x = ?$

A) 6 B) $\frac{4}{3}$ C) $\frac{8}{3}$

D) 12 E) $\frac{14}{3}$

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $|x-1|^{x^2+x-2}=1 \Rightarrow$ **ÇK(SS) = ?**

A) {–2 , 1} B) {2 , –1} C) {–1}
D) {2} E) {– 2, 0,2}

10. $|3x-2|=2x-1 \Rightarrow$ **ÇK(SS) = ?**

A) {1} B) $\left\{\frac{3}{5}\right\}$ C) $\left\{\frac{3}{5},1\right\}$
D) {0,1} E) $\left\{\frac{3}{5},2\right\}$

11. $\frac{x+3}{|x-1|+2}=\frac{1}{2} \Rightarrow$ **ÇK(SS) = ?**

A) {–5 , –1} B) {–5} C) {–1}
D) Ø E) {–1, 5}

12. $|x-5|<|x-3|$, $x \in Z \Rightarrow$ **min(x) = ?**

A) 4 B) 5 C) 6
D) 7 E) 10

13. $x \in Z$

$\left|\frac{13}{5x+12}\right|>1 \Rightarrow \sum x = ?$

A) –10 B) –5 C) 0
D) 5 E) 10

14. $||x+2|-3|=1 \Rightarrow \sum x = ?$

A) 0 B) 2 C) 4
D) –6 E) –8

15. $x \in Z$

$1<|2x-1|<7 \Rightarrow \sum x = ?$

A) 1 B) 2 C) 3
D) 4 E) 5

16. $|2x-1|<3$

$\Rightarrow$ **$|x-2|+|x+3|$ = ?**

A) –2x B) 2x C) 2x + 2
D) 5 E) 4

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	A	B	A	E	A	A	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	C	C	B	A	E	B	D

1. $4x^2 - 4x + 1 + \sqrt{y^2 - 2y + 1} + |z - 4| = 0$
$\Rightarrow$ **x + y + z = ?**

A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) $\frac{7}{2}$
D) $\frac{9}{2}$ E) $\frac{11}{2}$

2. $||x - 1| - 5| = 8 \Rightarrow \sum x = ?$
A) 1 B) 2 C) 3
D) 4 E) 5

3. $x \in Z$,
$4^{|x-2|} < 16 \Rightarrow \sum x = ?$
A) 2 B) 4 C) 6
D) 8 E) 10

4. $x \cdot y < 0$
$A = \frac{\sqrt[4]{x^4 \cdot y^4} + \sqrt{y^2}}{y} \Rightarrow \sum A = ?$
A) $-2x - 1$ B) $-2x$ C) $1 - 2x$
D) $2x + 1$ E) $2x$

5. $x \in R$
$|x - 3| \leq 4 \Rightarrow$ **max($x^2 - 4x + 1$) = ?**
A) 19 B) 22 C) 33
D) 46 E) 53

6. $|x^2 - 6x + 9| > 0 \Rightarrow$ **ÇK(SS) = ?**
A) {3} B) (–3,3) C) $(3,\infty)$
D) Ø E) R-{3}

7. $x^2 - 4x + 5 = A$
min(A) için / for $|1 - x|$ = ?
A) –1 B) 0 C) 1
D) 2 E) 3

8. $\sqrt{x^2 y^2} = -xy$
$xy^2 > 0 \Rightarrow |y - x| + |y| - |x| = ?$
A) –2x B) –2y C) –2x + 2y
D) –2x – 2y E) 0

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $|5 - |x - |1 - x|| = 10 \Rightarrow$ **ÇK(SS) = ?**

A) {–5} B) {–7} C) {–5,5}
D) Ø E) {10}

10. $5^{|x-4|-1} = 1 \Rightarrow \sum x = ?$

A) 2 B) 4 C) 6
D) 8 E) 10

11. $|x| + |2x| + |3x| + ... + |9x| = 90$
$\Rightarrow \prod x = ?$

A) –1 B) –2 C) –3
D) –4 E) –5

12. $|2x - 2| \cdot |x + 1| = 16 \Rightarrow$ **ÇK(SS) = ?**

A) {–1,1} B) {–2,2} C) {–3,3}
D) {–4,4} E) {–5,5}

13. $x \in Z$,
$11 < |2x - 9| < 15 \Rightarrow \sum x = ?$

A) 3 B) 5 C) 7
D) 9 E) 11

14. $|x^2 - x - 6| - |x - 3| = 0 \Rightarrow \sum x = ?$

A) –3 B) 0 C) –1
D) 1 E) 3

15. $\frac{|2x-4|}{x+1} < 0 \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, -1)$ B) (–1,2) C) $(2,\infty)$
D) $(-\infty,4)$ E) (–2,)

16. $|x - 2| < 4 \Rightarrow ? < x^2 < ?$

A) [0,9] B) [0,16) C) [0,36)
D) [–16,36) E) [0,121)

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	B	C	B	B	E	C	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	D	D	C	D	C	A	C

METROPOL YAYINLARI • METROPOL PUBLICATIONS

BÖLÜM CHAPTER 10

TARİHSEL NOT / HISTORICAL NOTE

Maryam Mirzakhani [1977 - 2017]

İranlı matematikçi ve Stanford Üniversitesi matematik profesörüdür. Araştırma konuları arasında Teichmüller teorisi, hiperbolik geometri, ergodik teori ve semlectic geometri yer almaktadır. 13 Ağustos 2014'te Fields Madalyası (matematiğin en prestijli ödülü) ile onurlandırıldı. Böylece, bu ödülle onurlandırılan ilk kadın ve ilk İranlı oldu. Ödül komitesi çalışmalarını "dinamizm ve geometride, Riemann yüzeylerinin ve moduli uzayları" konularında onurladı.

She was an Iranian mathematician and a professor of mathematics at Stanford University. Her research topics included Teichmüller theory, hyperbolic geometry, ergodic theory, and symplectic geometry. On August 13th 2014, she was honored with the Fields Medal (the most prestigious award in mathematics.) Thus, she became the first woman and the first Iranian to be honored with this award at the same time. The award committee cited her work in "the dynamics and geometry of Riemann surfaces and their moduli spaces".

MANTIK
LOGIC

Bu bölüm, 6 çözümlü örnek, 8 çözümlü test sorusu, 32 test sorusu içermektedir.

This chapter includes 6 examples with solutions, 8 tests questions with solutions and 32 test questions.

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MANTIK / LOGIC

- Mantık / Logic ..... 361 - 366
  - Bağlaçların Doğruluk Tablosu / The Correctness Table of Connectives ..... 363 - 366
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BÖLÜM
10
CHAPTER

MANTIK
LOGIC

ÖNEMLİ / IMPORTANT

- *Bir önermenin doğruluk değeri :*
- *Truth-value of one proposition :*

$2^1 = 2$

- *İki önermenin doğruluk değeri :*
- *Truth-value of two propositions :*

$2^2 = 4$

- *Üç önermenin doğruluk değeri :*
- *Truth-value of three propositions :*

$2^3 = 8$

- *n önermenin doğruluk değeri :*

2^n *değişik durum vardır.*

- *Truth-value of n propositions:*

It has 2^n *different situations.*

MANTIK / LOGIC

Mantık: Doğru düşünmenin kurallarını öğreten bir bilim dalıdır. Matematik bilimi de sistemli ve doğru düşünme yeteneğimizi geliştirmeyi amaçlandığından mantık biliminden yararlanır.

Logic: Logic is a field instructing rules of righteous thinking. Mathematics also aims to improve our ability to think correctly and systematically and from this point of view is taking advantage of logic.

Tanım: Doğru ya da yanlış kesin bir hüküm bildiren ifadelere önerme denir.
p, q, r, s ,t ... gibi harflerle gösterilir.

Definition: An expression declerating undoubted right or wrong is called a proposition. And it's shown by letters like p,q,r,s,t, ...

Bir önermenin doğru ya da yanlış olmasına bu önermenin **doğruluk değeri** denir.
To be correct or false of a proposition is called its **truth-value**.

Doğru ise doğruluk değeri / If it's right its truth-value : 1	yazılarak belirtilir.
Yanlış ise doğruluk değeri / If it's wrong its truth-value : 0	is expressed by writing

Etkinlikler / Activities

1.

p	q
1	1
1	0
0	1
0	0

$\Rightarrow 2^2 = 4$

2.

p	q	r
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

$\Rightarrow 2^3 = 8$

Tanım: Doğruluk değeri aynı olan önermelere **denk önerme** denir ve $p \equiv q$ biçiminde gösterilir.

Definition: Propositions with the same truth-value are called **equivalent propositions** and shown by $p \equiv q$.

Etkinlikler / Activities

a. p : Bir hafta yedi gündür / a week has 7 days
q : $2^4 = 16$
} $p \equiv q$

b. p : 4 bir asal sayıdır / 4 is a prime number
q : $13 < 8$
} $p \equiv q$

c. p : $-2 + 7 > 3 - 4$
q : $(-5)\cdot(3) = -15$
} $p \not\equiv q$

Tanım: Bir önermenin doğruluk değerini değiştirerek oluşturulan yeni önermeye bu önermenin olumsuzu (değili) denir. p önermesinin değili p´ sembolü ile gösterilir.

Definition: The new proposition composed by altering the truth-value of an proposition is called adverse of that proposition. Adverse of proposition p is shown by p´.

Etkinlikler / Activities

a. p : 5 asal sayıdır / is prime.
p´ : 5 asal sayı değildir / is not prime.

b. p : $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$

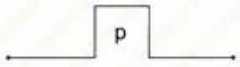
p´ : $\frac{1}{3} + \frac{1}{2} \neq \frac{5}{6}$

Tanım: İki veya daha fazla önermenin birbirine ve, veya, ise, ancak ve ancak gibi bağlaçlarla birleştirilmesiyle elde edilen yeni önermeye BİLEŞİK ÖNERME denir.

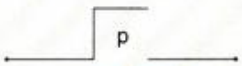
Definition: The new proposition composed by combining two or more propositions with connectives like: and, or, if, and if and only if is called compound proposition.

Bağlaç adı Name of Connective	işareti symbol
veya / or	$\vee$
ve / and	$\wedge$
ise / if	$\Rightarrow$
ancak ve ancak / if and only if	$\Leftrightarrow$

ÖNEMLİ / IMPORTANT

- 

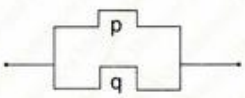
doğruluk değeri / truth-value : p ≡ 1

- 

doğruluk değeri / truth-value : p ≡ 0

- 

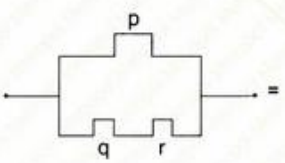
seri bağlama / coupling in series : p ∧ q

- 

paralel bağlama / coupling in paralel :

p ∨ q

PEKİŞTİRME / REINFORCE

✓ 

.....[p ∨ (q ∧ r)]

BAĞLAÇLARIN DOĞRULUK TABLOSU
THE CORRECTNESS TABLE of CONNECTIVES

p	q	p ∨ q	p ∧ q	p ⇒ q	p ⇔ q
1	1	1	1	1	1
1	0	1	0	0	0
0	1	1	0	1	0
0	0	0	0	1	1

şeklindedir.

is like the table above.

Özellik / Property

1. p ∨ p ≡ p,

p ∧ p ≡ p

2. p ∨ q ≡ q ∨ p

p ∧ q ≡ q ∧ p

3. p ∨ (q ∨ r) ≡ (p ∨ q) ∨ r

p ∧ (q ∧ r) ≡ (p ∧ q) ∧ r

4. p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)

p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r)

5. (p ∨ q)′ ≡ p′ ∧ q′

(p ∧ q)′ ≡ p′ ∨ q′

6. p ∨ p′ ≡ 1 (totoloji / tautology)

p ∧ p′ ≡ 0 (çelişki / contradiction)

7.

p	p ∨ 1	p ∧ 1	p ∨ 0	p ∧ 0	p ∨ p′	p ∧ p′
1	1	1	1	0	1	0
0	1	0	0	0	1	0

8. p ⇒ q ≡ p′ ∨ q

9. p ∨ (p ∧ q) ≡ p

p ∧ (p ∨ q) ≡ p

Örnek / Example 1

$(p \wedge q) \vee (p' \wedge q) \equiv ?$

A) p B) q C) 1 D) 0 E) q′

Çözüm / Solution

Yanıt / Answer **B**

$(p \wedge q) \vee (p' \wedge q) \equiv (q \wedge p) \vee (q \wedge p')$

$\equiv q \wedge (p \vee p')$

$\equiv q \wedge 1$

$\equiv q$

Örnek / Example 2

$p' \vee (q \wedge r')' \equiv 0 \Rightarrow (p, q, r) \equiv (?, ?, ?)$

A) (1, 1, 1) B) (1, 0, 1) C) (0, 0, 1) D) (1, 1, 0) E) (0, 0, 0)

Çözüm / Solution

Yanıt / Answer **D**

$p' \vee (q \wedge r')' \equiv 0$

$0 \quad \overline{0}$

$p' \equiv 0 \Rightarrow p \equiv 1 \quad ,(q \wedge r')' \equiv 0$

$q \wedge r' \equiv 1$

$q \equiv 1$

$r' \equiv 1 \Rightarrow r \equiv 0$

Örnek / Example 3

$(p \vee q)' \wedge (r \wedge q') \equiv 1 \Rightarrow (p, q, r) \equiv (?, ?, ?)$

A) (0, 0, 1) B) (0, 0, 0) C) (0, 1, 1) D) (1, 1, 1) E) (1, 0, 1)

Çözüm / Solution

Yanıt / Answer **A**

$(p \vee q)' \wedge (r \wedge q') \equiv 1$

$\underbrace{\quad}_{1} \quad \underbrace{\quad}_{1}$

$(p \vee q)' \equiv 1$

$p \vee q \equiv 0 \Rightarrow p \equiv 0, \quad q \equiv 0$

$r \wedge q' \equiv 1$

$r \wedge 1 \equiv 1 \Rightarrow r \equiv 1$

Örnek / Example 4

$[(p \vee r) \wedge q'] \wedge (r \wedge q) \equiv ?$

A) p B) q C) r D) 1 E) 0

Çözüm / Solution

Yanıt / Answer **E**

$[(p \vee r) \wedge q'] \wedge (r \wedge q) \equiv ?$

$(p \vee r) \wedge [q' \wedge (r \wedge q)] \equiv ?$

$(p \vee r) \wedge [(q' \wedge r) \wedge \underbrace{(q' \wedge q)}_{0}] \equiv 0$

Örnek / Example 5

$q' \vee (p \wedge q')' \equiv ?$

A) p B) q C) p′ D) 1 E) 0

Çözüm / Solution Yanıt / Answer **D**

$q' \vee (p \wedge q')' \equiv ?$

$q' \vee (p' \vee q) \equiv$

$\underbrace{(q' \vee q)} \vee p' \equiv$

1

$1 \vee p' \equiv 1$

ÖNEMLİ / IMPORTANT

- $[\exists x, P(x)]' \equiv \forall x, P'(x)$
- $[\forall x, P(x)]' \equiv \exists x, P'(x)$

Örnek / Example 6

$[(p' \Rightarrow q') \wedge (p \vee q')]' \equiv ?$

A) p B) q C) p′ D) q′ E) $p' \wedge q$

Çözüm / Solution Yanıt / Answer **E**

$$[(p' \Rightarrow q') \wedge (p \vee q')]' \equiv (p' \Rightarrow q')' \wedge (p \vee q')'$$
$$\equiv (p \vee q')' \vee (p' \wedge q)$$
$$\equiv (p' \wedge q) \vee (p' \wedge q)$$
$$\equiv p' \wedge q$$

1. **Aşağıdaki önermelerin kaç tanesinin doğruluk değeri 0 dır?**
The truth value of how many of the following propositions is 0?

I. 2 asal ve çift sayıdır / 2 is a prime and even number.

II. $(-3)^2 = 9$

III. En küçük asal sayı 1 dir / The smallest prime number is 1.

IV. Her doğal sayı bir tamsayıdır,
Each natural number is an integer,

A) 0 B) 1 C) 2 D) 3 E) 4

2. $p \equiv 1$, $q \equiv 0$, $r \equiv 1$
Aşağıdaki ifadelerden hangisi doğrudur?
Which of the followings is true?

A) $(p' \wedge q) \wedge r' \equiv 1$

B) $r \wedge (q' \wedge p) \equiv 0$

C) $[p \wedge (q \wedge r)]' \equiv 0$

D) $(p' \wedge q) \vee r \equiv 0$

E) $[(p' \wedge q) \wedge r]' \equiv 1$

3. p: 4 çift doğal sayıdır / 4 is an even natural number.
q: 21 asal bir sayıdır / 21 is a prime number.
önermeleri için aşağıdaki bileşik önermelerin hangisi doğrudur?
For the propositions above which of the following compound propositions is true?

A) $p \wedge q$ B) $p' \vee q$ C) $p \Rightarrow q$ D) $p \vee q'$ E) $p \Leftrightarrow q$

4. $(p \Rightarrow q) \Rightarrow (r' \wedge s)' \equiv 0$
p, q, r, s sırasıyla doğruluk değerleri hangisidir?
What are the truth-values of p, q, r, and s respectively?

A) (1, 1, 0, 0) B) (1, 0, 0, 1) C) (0, 0, 0, 1) D) (1, 1, 1, 0) E) (1, 1, 1, 1)

5. $[(p \vee q) \Rightarrow (p' \wedge q)] \equiv ?$

A) p B) p′ C) q D) q′ E) 1

6. $(p \wedge q) \Rightarrow (p \vee q) \equiv ?$

A) 0 B) 1 C) p D) q E) $p \vee q$

7. p: Elif akıllıdır / Elif is smart.
q: Akıllı insanlar başarılıdır / Smart people are successful.
<< Elif akıllı değilse, başarısızdır >>
Önermesi aşağıdakilerden hangisidir?
<< If Elif is not smart, then she is not successful>>
is which of the following propsitions?

A) $p' \Rightarrow q$ B) $p \Rightarrow q'$ C) $p' \Rightarrow q'$ D) $p' \vee q$ E) $p \Leftrightarrow q$

8. $[p \Rightarrow (q \vee r')] \equiv 0$
Önermesi için / for the proposition $(p' \vee q) \Rightarrow r \wedge q' \equiv ?$

A) p′ B) r′ C) q′ D) $p \wedge q$ E) $(p \wedge r)'$

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. I. 2 asal ve çift sayıdır
2 is a prime and an even number. (doğru / true)
II. $(-3)^2 = 9$ (doğru / true)
III. En küçük asal sayı 1 dir.
The smallest prime number is 1 (yanlış / false)
IV. Her doğal sayı bir tamsayıdır
Each natural number is an integer (doğru / true)

Yanıt / The Answer is B

---

2. A) $(p' \wedge q) \wedge r' \equiv 1$ (yanlış / false)
B) $r \wedge (q' \wedge p) \equiv 0$ (yanlış / false)
C) $p \wedge (q \wedge r) \equiv 0$ (yanlış / false)
D) $(p' \wedge q) \vee r \equiv 0$ (yanlış / false)
E) $[(p' \wedge q) \wedge r]' \equiv 1$ (doğru / true)

Yanıt / The Answer is E

---

3. $p \equiv 1$, $q \equiv 0$
A) $1 \wedge 0 \equiv 0$ (yanlış / false)
B) $1' \vee 0 \equiv 0$ (yanlış / false)
C) $1 \Rightarrow 0 \equiv 0$ (yanlış / false)
D) $1 \vee 1' \equiv 1$ (doğru / true)
E) $1 \Leftrightarrow 0 \equiv 0$ (yanlış / false)

Yanıt / The Answer is D

---

4. $p \Rightarrow q \equiv 1 \Rightarrow p \equiv 1$, $q \equiv 0$ (olmaz / can't be)
$(r' \wedge s)' \equiv 0 \Rightarrow r \equiv 0, s \equiv 1$ (olur / can be)

Yanıt / The Answer is C

---

5. $p \Rightarrow q \equiv p' \vee q$ olduğundan / as
$(p \vee q)' \vee (p' \wedge q) \equiv$
$(p' \wedge q') \vee (p' \wedge q) \equiv$
$p' \wedge (q' \vee q) \equiv$
$p' \wedge 1 \equiv p'$

Yanıt / The Answer is B

---

6. $p \Rightarrow q \equiv p' \vee q$
$(p \wedge q)' \vee (p \vee q) \equiv$
$p' \vee q' \vee p \vee q \equiv$
$\underbrace{p' \vee p}_{1} \vee \underbrace{q' \vee q}_{1} \equiv$
$1 \vee 1 \equiv 1$

Yanıt / The Answer is B

---

7. Akıllıdır / is Smart $\equiv p$
Akıllı değildi / is not smart $\equiv p'$
Başarılıdır / is successful $\equiv q$
Başarısızdır / is not successful $\equiv q'$
Akıllı değil ise Başarısızdır / if not smart, then not successful
$p' \Rightarrow q'$

Yanıt / The Answer is C

---

8. $[\underbrace{p}_{1} \Rightarrow \underbrace{(q \vee r')}_{0}] \equiv 0$
$q \vee r' \equiv 0$
$q \equiv 0 \quad r' \equiv 0$
$r \equiv 1$
$(p' \vee q) \Rightarrow r \wedge q'$
$(0 \vee 0) \Rightarrow 1 \wedge 1$
$q \equiv 0$
$q' \equiv 1$

Yanıt / The Answer is C

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. **5 önermenin doğruluk değeri için kaç değişik durum vardır?**
How many different situations for truth-value of 5 expressions are possible?

A) 5 B) 8 C) 10
D) 16 E) 32

2. **Aşağıdaki önermelerin hangileri denktir?**
Which of the expressions below is equivalent?

p : 3 + 7 = 21
q : bir yıl 12 aydır / a year is 12 months.
r : en küçük tamsayı –1 dir / the smallest integer is –1.
s : 10 den küçük 3 asal sayı vardır
there are three prime numbers less than 10.

A) $q \equiv r$ B) $p \equiv q \equiv r$ C) $q \equiv s$
D) $p \equiv q$ E) $p \equiv r \equiv s$

3. $p' \wedge (p' \vee q)' \equiv ?$

A) p B) q C) p´
D) 0 E) 1

4. $p \vee (p' \wedge q) \equiv ?$

A) p B) q C) $p \vee q$
D) $p \wedge q$ E) 0

5. $(p \vee q) \wedge (q \vee p') \equiv ?$

A) $p \vee q$ B) $p \wedge q$ C) q
D) p E) p´

6. $p \vee q' \equiv 0$ **ise / if** $(p, q) = ?$

A) (0, 0) B) (0, 1) C) (1, 0)
D) (1, 1) E) (q, p)

7. $p' \wedge (q \wedge r') \equiv 1$ **ise / if** $(p, q, r) = ?$

A) (0, 1, 0) B) (1, 0, 1) C) (0, 0, 1)
D) (1, 1, 0) E) (1, 1, 1)

8. $(q \wedge r) \wedge [q' \wedge (r \vee p)] \equiv ?$

A) p B) q C) 1
D) 0 E) p´

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $[p \Rightarrow (p \wedge q)]' \equiv ?$

A) p B) q C) $p \wedge q'$

D) $p \vee q'$ E) $p \vee q$

10. $p \wedge (q' \Rightarrow p') \equiv ?$

A) $p \wedge q$ B) 1 C) q

D) p E) q'

11. $(p \Rightarrow p) \wedge q \equiv ?$

A) 0 B) 1 C) q

D) p E) q'

12. $(p \vee q)' \wedge p \equiv ?$

A) 0 B) 1 C) p

D) q E) p'

13. **Aşağıdakilerden hangisi yanlıştır?**

Which of the following is wrong?

A) $(1 \vee 0) \wedge 1 \equiv 1$

B) $(1 \wedge 1) \vee 0 \equiv 1$

C) $0 \vee (1 \wedge 0) \equiv 0$

D) $0 \vee (1 \wedge 0)' \equiv 0$

E) $(0 \vee 1) \wedge (0 \wedge 1) \equiv 0$

14. $(p \vee p)' \wedge q \equiv ?$

A) 0 B) 1 C) p

D) p' E) q

15. $(p \Rightarrow q') \vee r \equiv ?$

A) q B) r C) 1

D) r' E) q'

16. $(p \Rightarrow q) \vee r \equiv 0$

$\Rightarrow$ **(p,q,r) = (?,?,?)**

A) (1,0,0) B) (1,0,1) C) (1,1,0)

D) (0,1,0) E) (0,0,1)

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	E	D	C	C	B	A	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	A	C	A	D	A	C	A

1. $p \vee (p \vee q)' \equiv ?$

A) p B) p′ C) q

D) q′ E) p ∨ q′

2. $(p \wedge q) \vee (p \wedge q') \equiv ?$

A) p B) q C) q′

D) p∨q E) p′

3. $(p \vee q) \wedge (p' \wedge q)' \equiv ?$

A) q B) q′ C) p

D) p′ E) p∧q

4. $p \equiv 1,\ q \equiv 0,\ r \equiv 1$

ise / if $[r \wedge (q' \vee p)]' \equiv ?$

A) p B) q C) r

D) q′ E) 1

5. $p \equiv 0,\ q \equiv 0,\ r \equiv 1$

ise / if $(p' \wedge q) \vee (r \wedge p)' \equiv ?$

A) p B) q C) r

D) r′ E) 0

6. $p \wedge (p \Rightarrow q) \equiv ?$

A) p B) q C) 1

D) 0 E) p∧q

7. $p \vee (p \Rightarrow q) \equiv ?$

A) 0 B) 1 C) p

D) q E) p∨q

8. $(p' \vee q) \wedge (p \wedge q') \equiv ?$

A) 0 B) 1 C) p

D) q E) p′

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $[p \Rightarrow (p \wedge q)]' \equiv ?$

A) $p \vee q$ B) $p \wedge q$ C) $p' \wedge q$

D) $p \wedge q'$ E) $p \vee q'$

10. $[(p \wedge q) \Rightarrow q]' \equiv ?$

A) 0 B) q C) p'

D) q' E) $p \vee q$

11. $p \wedge (q' \wedge r) \equiv 1$ **ise / if** **(p,q,r) = ?**

A) (0,0,0) B) (0,1,0) C) (1,0,1)

D) (1,1,0) E) (1,1,1)

12. $p' \vee (q \wedge r)' \equiv 0$ **ise / if** **(p,q,r) = ?**

A) (0,0,0) B) (1,0,1) C) (1,1,0)

D) (0,1,1) E) (1,1,1)

13. $(p \wedge q')' \vee r' \equiv 0$ **ise / if** **(p,q,r) = ?**

A) (0,0,1) B) (1,0,0) C) (1,0,1)

D) (1,1,0) E) (1,1,1)

14. $(p' \Rightarrow q)' \wedge r' \equiv 1$ **ise / if** **(p,q,r) = ?**

A) (0,0,0) B) (1,0,1) C) (0,0,1)

D) (1,1,0) E) (1,1,1)

15. $(p' \wedge q) \Rightarrow (r \wedge q) \equiv 0$ **ise / if** **(p,q,r) = ?**

A) (0,1,0) B) (0,1,1) C) (1,0,0)

D) (0,0,1) E) (1,1,1)

16. $(p \vee q')' \Rightarrow r \equiv 0$ **ise / if** **(p,q,r) = ?**

A) (0,0,1) B) (1,0,0) C) (1,0,1)

D) (1,1,0) E) (0,1,0)

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	A	C	B	C	E	B	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	A	C	E	C	A	A	E

BÖLÜM CHAPTER 11

TARİHSEL NOT / HISTORICAL NOTE

Cantor, Georg (Ferdinand Ludwig Philipp) [1845 – 1918]

St. Petersburg'da doğmuş, hayatının büyük bir kısmını Almanya'da Halle Üniversitesi'nde geçirmiştir. 1873'de rasyonel sayılar kümesinin sayılamaz olduğunu göstermiştir. Aynı zamanda reel sayılar kümesinin de sayılamaz sonsuzlukta olduğunu göstermiştir. Daha sonra sonsuz kümeler kuramını geliştirmiştir.

He was born in St Petersburg, but spent most of his life at the University of Halle in Germany. In 1873, he showed that the set of rational numbers is non-denumerable. He also showed that the set of all real numbers is not. Later he fully developed his theory of infinite sets.

KÜMELER
SETS

Bu bölüm, 21 çözümlü örnek, 12 çözümlü test sorusu, 16 çözümlü YÖS sorusu, 64 test sorusu içermektedir.
This chapter includes 21 examples with solutions, 12 tests questions with solutions, 16 YÖS questions with solutions and 64 test questions.

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BÖLÜM 11 CHAPTER

KÜMELER SETS

ÖNEMLİ / IMPORTANT

- Kümede yer alan nesneler, harfler, sayılar ve insanlar olabilir.
- Kümeler, A,B,C gibi büyük harflerle gösterilir.
- Objects in sets can be letters, numbers, people, etc.
- Sets are represented by capital letters, such as A,B,C,... .

ÖNEMLİ / IMPORTANT

- { } veya Ø sembolleri **boş küme** olarak adlandırılır ve hiçbir eleman içermez.
- The symbols { } or Ø are called the empty set and contain no elements.

$A = \{\} \Rightarrow n(A) = 0$

$A = \{\emptyset\} \Rightarrow n(A) = 1$

$\Rightarrow \emptyset \in A$

ÖNEMLİ / IMPORTANT

- Bir küme sonlu sayıda elemana sahipse **sonlu küme** olarak adlandırılır. Sonsuz kümeler, sayılamayacak kadar çok elemana sahiptir. Örneğin; $Z = \{0, \pm1, \pm2, \ldots\}$ bir sonsuz kümedir.
- A set can have a finite number of elements and so is said to be a **finite set. Infinite sets** are sets which have infinitely many members. For example; $Z = \{0, \pm1, \pm2, \ldots\}$ is an infinite set.

KÜMELER / SETS

Tanım: Küme, nesnelerin açık olarak tanımlanmış bir listesidir. Kümeyi oluşturan nesnelere *kümenin elemanları* denir. Bir A kümesinin eleman sayısı n(A) veya s(A) ile gösterilir.

$\in$: kümenin elemanı

$\notin$: kümenin elemanı değil diye okunur.

$A = \{1, 3, a, b\}$ kümesi için $3 \in A$, fakat $4 \notin A$ dır.

Definition: A set is a well defined collection of objects. Objects forming the set are called elements or members of the set. The number of elements of a set A is represented by n(A) or s(A).

$\in$ reads: is an element of

$\notin$ reads: is not an element of

For the set $A = \{1, 3, a, b\}$ we can say $3 \in A$, but $4 \notin A$.

Örnek / Example 1

$A = \{x \mid x < 6 \text{ ve } x \in N\}$

$\Rightarrow$ **n(A) = ?**

A) 2 B) 3 C) 4 D) 6 E) 8

Çözüm / Solution

Yanıt / Answer **D**

$A = \{0, 1, 2, 3, 4, 5\}$

$n(A) = 6$

Örnek / Example 2

$B = \{0, 1, \{1\}, \emptyset, \{1, 0\}\} \Rightarrow$ **n(B) = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution

Yanıt / Answer **E**

$0 \in B$, $1 \in B$, $\{1\} \in B$, $\emptyset \in B$ ve $\{1, 0\} \in B$

$n(B) = 5$

Örnek / Example 3

A = {a, b, c, {c}, {b, c}, d, e}

I. n(A) = 7

II. e ∈ A

III. {a, b, c} ∉ A

IV. B ∈ A

A kümesine göre yukarıdaki ifadelerden hangileri doğrudur?

Which of the expressions above is true according to set A?

A) I B) I, II C) I, II, III D) III, IV E) II, IV

Çözüm / Solution Yanıt / Answer **C**

I. n(A) = 7 doğru / true

II. e ∈ A doğru / true

III. {a, b, c} ∉ A doğru / true

IV. B ∈ A yanlış / false

KÜMELERİN GÖSTERİMİ / DISPLAYING of SETS

▼ **Liste yöntemi / List method:**

Bir kümenin elemanlarının küme parantezi içerisinde virgülle ayrılarak yazılmasına liste yöntemi denir.

Writing elements of a set inside curly braces separated with commas is called list method.

A = {1, 2, 3, 4, 5}

▼ **Ortak özellikler yöntemi / Common characteristic method:**

Bir kümenin elemanlarının ortak özellikleriyle beraber küme parantezi içerisinde yazılmasına ortak özellik yöntemi denir.

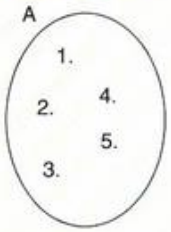
Writing elements of a set with their characteristics inside curly braces is called common characteristic method.

A = {x: x ≤5, x ∈ N*}

▼ **Venn şeması / Venn diagram:**

Bir kümenin elemanlarının kapalı bir geometrik şekil içerisinde gösterilmesine Venn şeması denir.

Displaying elements of a set inside an enclosed geometric figure is called Vtenn diagram.



ALT KÜME / SUBSET

Bir A kümesinin her elemanı, bir B kümesinin de elemanı ise A'ya B'nin alt kümesi denir.

$A \subset B$ biçiminde gösterilir.

If every element of set A is also an element of set B, then we say that set A is a subset of the set B. It is represented by $A \subset B$.

Özellik / Property

Alt Kümenin Özellikleri / Properties of Subset

★ Her küme, kendisinin alt kümesidir ($A \subset A$).

★ Boş küme, her kümenin alt kümesidir ($\emptyset \subset A$).

★ $A \subset B$ ve $B \subset C \Rightarrow A \subset C$

★ $A \subset B$ ve $B \subset A \Rightarrow A = B$

★ n elemanlı bir kümenin alt kümelerinin sayısı 2^n dir.

★ n elemanlı bir kümenin r elemanlı alt kümelerinin sayısı $C(n,r) = \frac{n!}{r! \cdot (n-r)!}$

★ Every set is subset of itself ($A \subset A$).

★ Empty set is subset of every set ($\emptyset \subset A$).

★ $A \subset B$ and $B \subset C \Rightarrow A \subset C$

★ $A \subset B$ and $B \subset A \Rightarrow A = B$

★ The number of subsets of a set with n elements is equal to 2^n.

★ The number of subsets which has r members, of a set which has n members can be found by the equation; $C(n,r) = \frac{n!}{r! \cdot (n-r)!}$

ÖZALT KÜME / PROPER SUBSET

Bir kümenin kendisinden farklı her alt kümesine özalt küme denir.

Each subset of a set in which does not equal to set itself is culled a proper subset.

Özellik / Property

★ n elemanlı bir kümenin özalt kümelerinin sayısı $2^n - 1$dir.

★ The number of proper subsets of a set which has n members is $2^n - 1$.

Örnek / Example 4

A = {1, 2, 3, 4, 5, 6, 7}

A kümesinin kaç tane alt kümesi vardır?

How many subsets of set A are there?

A) 32 B) 64 C) 128 D) 256 E) 512

Çözüm / Solution

Yanıt / Answer **C**

A = {1, 2, 3, 4, 5, 6, 7}

n(A) = 7 ⇒ alt küme sayısı / number of subsets $2^7 = 128$

Örnek / Example 5

A = {1, 2, 3, 4, 5, 6}

A kümesinin 3 elemanlı alt kümelerinin sayısı kaçtır?

What is the number of three-element subsets of set A?

A) 20 B) 22 C) 24 D) 26 E) 28

Çözüm / Solution

Yanıt / Answer **A**

$$C\binom{6}{3} = \frac{6!}{(6-3)!\cdot 3!} = 20$$

Örnek / Example 6

A = {a, b, c, d, e, f}

A kümesinin 4 elemanlı alt kümelerinin kaç tanesinde d eleman olarak bulunmaz?

How many of four-element subsets of set A dont have d as an element?

A) 4 B) 5 C) 6 D) 7 E) 8

Çözüm / Solution

Yanıt / Answer **B**

$$C\binom{5}{4} = \frac{5!}{(5-4)!\cdot 4!} = 5$$

Örnek / Example 7

$A = \{x \mid x \leq 8, x \in N^+\}$

A kümesinin 5 elemanlı alt kümelerinin kaç tanesinde 3 ve 7 eleman olarak bulunur?

How many of four-element subsets of set A have 3 and 7 as an element?

A) 12 B) 16 C) 18 D) 20 E) 24

Çözüm / Solution Yanıt / Answer **D**

$$C\binom{6}{3} = \frac{6!}{(6-3)!\cdot 3!} = 20$$

PEKİŞTİRME / REINFORCE

E = {a, 1, 2, b, c}

✓ A = {a} ⇒ **A′ = ?**

.....[{1,2,b,c}]

✓ A = {1, 2, b} ⇒ **A′ = ?**

.....[{a, c}]

✓ A′ = {b, c, 1} ⇒ **A = ?**

.....[{a, 2}]

✓ A = {1, 2, c} ⇒ **n(A′) = ?**

..... [2]

✓ A′ = {a, b} ⇒ **n(A) = ?**

..... [3]

EVRENSEL KÜME / UNIVERSAL SET

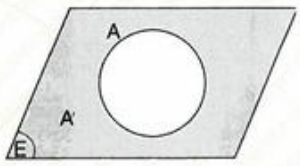
Üzerinde işlem yapılan tüm kümeleri kapsayan kümeye *"evrensel küme"* denir. Evrensel küme E veya U ile gösterilir.

A set comprising all sets, on which operations are being done, is called the *universal set* and represented by E or U.

KÜMELERDE TÜMLEME / COMPLEMENTATION in SETS

E kümesinin A kümesinde olmayan elemanlarının oluşturduğu kümeye *"A kümesinin tümleyeni"* denir. A kümesinin tümleyeni A′ ile gösterilir.

The complement of A is the set of all elements of E which are not in A. The complement of A is denoted by A′.



Özellik / Property

★ E′ = Ø

★ Ø′ = E

★ (A′)′ = A

★ $A \subset B \Leftrightarrow B' \subset A'$

Örnek / Example 8

$E = \{x \mid x < 10 \text{ ve } x \in N\}$, $A = \{0, 2, 4, 6\} \Rightarrow A' = ?$

A) {1, 3, 5, 7, 8, 9} B) {1, 3, 5, 7, 8} C) {1, 2, 3, 5, 7, 8}
D) {1, 2, 3, 5, 7, 8, 9} E) {1, 2, 3, 7, 8, 9}

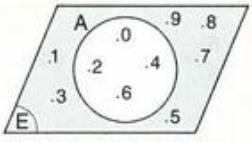
Çözüm / Solution

Yanıt / Answer **A**

$E = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{0, 2, 4, 6\}$

$A' = \{1, 3, 5, 7, 8, 9\}$



Taralı bölge A′ kümesidir.

The shaded area is represented by A′.

PEKİŞTİRME / REINFORCE

✓ $A = \{1, 2, 3, 4, 5\}$,
$B = \{-3, -2, -1\}$
$\Rightarrow A \cup B = ?$

.....[{−3, −2, −1, 1, 2, 3, 4, 5}]

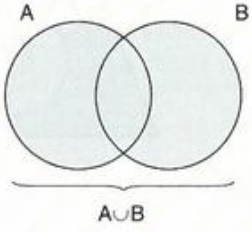
KÜMELERDE BİRLEŞİM / UNION in SETS

A ve B kümeleri için ortak elemanları bir kez kullanılarak A nın ve B nin tüm elemanlarından oluşturulan yeni kümeye A ile B nin *birleşim kümesi* denir.

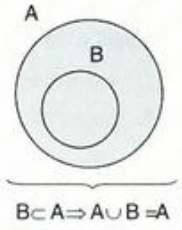
$A \cup B$ ile gösterilir.

$A \cup B$ is the union of sets A and B and is the set made up of all elements which are in A or B.

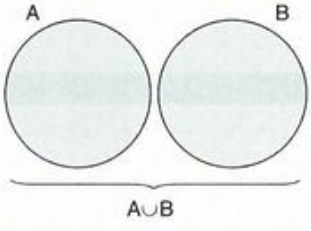
$$A \cup B = \{x \mid x \in A \text{ veya/or } x \in B\}$$



$A \cup B$



$B \subset A \Rightarrow A \cup B = A$



$A \cup B$

PEKİŞTİRME / REINFORCE

✓ A = {a, b, c},
B = {a, b, c, ç, d}
⇒ A∩B = ?, A∪B = ?

..... [A∩B = {a, b, c}]

.....[A∪B = {a, b, c, ç, d}]

Örnek / Example 9

$A = \{x \mid x^2 = 16 , x \in R\}$

$B = \{x \mid 1 \leq x < 5 , x \in Z\}$

⇒ **A ∪ B = ?**

A) {1, 2, 3, 4}

B) {–4, 1, 2, 3}

C) {–4, 1, 2, 3, 4}

D) {1, 2, 3}

E) {1, 2}

Çözüm / Solution Yanıt / Answer **C**

A = {–4, 4}

B = {1, 2, 3, 4}

A∪B = {–4, 1, 2, 3, 4}

KÜMELERDE KESİŞİM / INTERSECTION in SETS

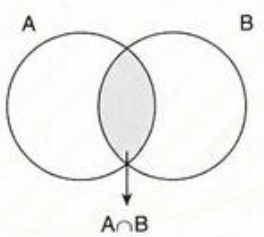
A ve B kümeleri için bu iki kümenin ortak olan elemanlarının oluşturduğu kümeye A ile B nin *kesişim kümesi* denir. A ∩ B ile gösterilir.

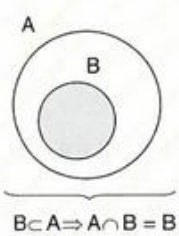
A ∩ B is the intersection of sets A and B and is the set made up of all elements which are in both A and B.

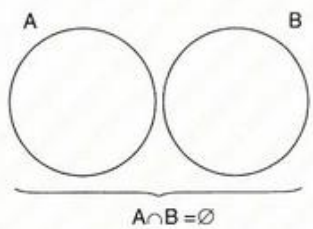
ÖNEMLİ / IMPORTANT

- Ayrık kümeler, hiç ortak elemanı olmayan kümelerdir. Bu kümeler, kesişmeyen çemberler ile gösterilir.
- Disjoint(mutually exclusive) sets are sets which do not have common elements. These sets are represented by non-overlapping circles.

$$A \cap B = \{x \mid x \in A \text{ ve/and } x \in B\}$$







Örnek / Example 10

A = {a, b, c, {a}, {a, b}},

B = {a, {a, c}, {b}, {a, b}}

⇒ **A∩B = ?**

A) {b, {a, c}} B) {a, {b, c}} C) {a, {c, b}}

D) {a, {a, c}} E) {a, {a, b}}

Çözüm / Solution Yanıt / Answer **E**

A = {a, b, c, {a}, {a, b}},

B = {a, {a, c}, {b}, {a, b}} ⇒ A∩B = {a, {a, b}}

Özellik / Property

Birleşim ve Kesişimin Özellikleri / Properties of Union and Intersection

★ $A \cup B = B \cup A$
$A \cap B = B \cap A$

★ $A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

★ $A \cup A = A$ ve $A \cap A = A$

★ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

★ $(A \cup B)' = A' \cap B'$ ve $(A \cap B)' = A' \cup B'$ [De Morgan kuralları]

★ $A \cup \emptyset = A$ ve $A \cap \emptyset = \emptyset$

★ $A \cup E = E$ ve $A \cap E = A$

★ $A \cup A' = E$ ve $A \cap A' = \emptyset$

★ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

★ $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

★ $n(A) + n(A') = n(E)$

PEKİŞTİRME / REINFORCE

A = {a, b, c, d, e} , B = {b, d, f, g}

✓ **A – B = ?**

.....[{a, c, e}]

✓ **B – A = ?**

.....[{f, g}]

✓ **A ∩ B = ?**

.....[{b, d}]

Örnek / Example 11

((A′ ∪ B)′ ∪ (A ∪ B)′ = ?

A) A B) B C) A′ D) B′ E)A∪B

Çözüm / Solution Yanıt / Answer **D**

(A′∪B)′∪ (A ∪ B)′ = [(A′)′ ∩ B′] ∪ [A′ ∩ B′]

= (A ∩ B′) ∪ (A′ ∩ B′)

= (A ∪ A′) ∩ B′

= E ∩ B′

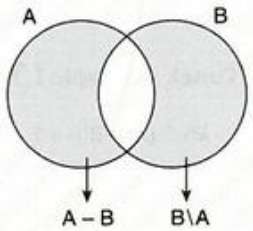
= B′

KÜMELERDE FARK İŞLEMİ / OPERATION of SUBTRACTION in SETS

A ve B kümeleri için A'nın elemanı olup B'nin elemanı olmayan elemanların oluşturduğu kümeye *A fark B* denir. **A–B** ya da **A\B** biçiminde gösterilir.

For the sets A and B, a set composed of elements that exist in A but do not exist in B is called A minus B. It is represented by A–B or A \ B.

A – B = A \ B = {x | x ∈ A ve/and x ∉ B}



PEKİŞTİRME / REINFORCE

A = {1, 2, 3, 4}, B = {5, 6}

✓ **A – B = ?**

.....[{1, 2, 3, 4} = A]

✓ **B – A = ?**

.....[{5, 6} = B]

Özellik / Property

Fark İşleminin Özellikleri / Properties of Subtraction

★ A – B = A ∩ B′

★ A ⊂ B ⇒ A – B = Ø

★ A – B = A – (A ∩ B)

★ E – A = A′

★ A – Ø = A

★ (A – B)′ = (A ∩ B′)′ = A′∪ B

★ (A – B) ∪ B = A ∪ B

PEKİŞTİRME / REINFORCE

A = {a, b, c, d, e} , B = {b, e}

✓ **A – B = ?**

.....[{a, c, d}]

✓ **B – A = ?**

.....[Ø]

Örnek / Example 12

Aşağıdakilerden kaç tanesi doğrudur?

How many of the following are true?

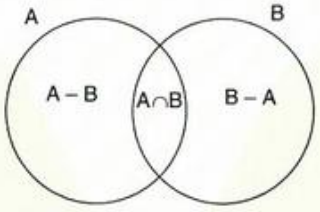
1. $(A-B) \subset B$
2. $A \cap B = \emptyset \Rightarrow A - B = A$
3. $n(A \cup B) = n(B) \Rightarrow A - B = \emptyset$
4. $n(A \cup B) = n(B) \Rightarrow A \supset B$
5. $(A - B) \cup (A \cap B) = A$

A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution

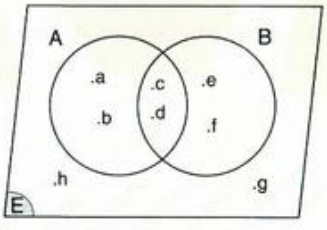
Yanıt / Answer **C**

1. $(A - B) \not\subset B$ **[Yanlış]** [False].
2. $A - B = A$ **[Doğru]** [True].
3. $n(A \cup B) = n(B) \Rightarrow A \subset B$
   $A \subset B \Rightarrow A - B = \emptyset$ **[Doğru]** [True].
4. $n(A \cup B) = n(B) \Rightarrow A \subset B$ **[Yanlış]** [False].
5. $(A - B) \cup (A \cap B) = A$ **[Doğru]** [True].



Örnek / Example 13

$n((A'- B) \cap (A \cup B')) = ?$



A) 0 B) 1 C) 2 D) 3 E) 4

Çözüm / Solution

Yanıt / Answer **C**

$A = \{a, b, c, d\}$,

$A' = \{e, f, g, h\}$,

$B = \{c, d, e, f\}$

$B' = \{a, b, h, g\}$

$A'- B = \{g, h\}$

$A \cup B' = \{a, b, c, d, h, g\}$

$((A'-B) \cap (A \cup B')) = \{g, h\} \Rightarrow n((A'-B) \cap (A \cup B')) = 2$

Örnek / Example 14

Taralı alan = ?

The shaded area = ?

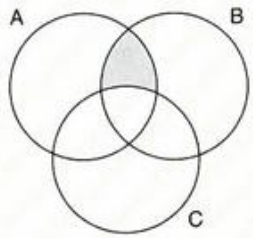
A) $A \cap B \cap C$

B) $(A \cap B) \cup C$

C) $(A \cap B) \setminus C$

D) $C \setminus (A \cap B)$

E) $A \cup B \cup C$



Çözüm / Solution

Yanıt / Answer **C**

Taralı alan / The shaded area = $(A \cap B) \setminus C$

Örnek / Example 15

$A = \{ x: 124 \leq x \leq 216, x = 3k, k \in N\}$

$B = \{ y: 144 \leq y < 224, y = 4k, k \in N\}$

$\Rightarrow$ **n(A∩B) = ?**

A) 6 B) 7 C) 8 D) 9 E) 10

Çözüm / Solution

Yanıt / Answer **B**

$A \cap B = [126, 216] \cap [144, 224) = [144, 216]$

$A \cap B = \{ z: 144 \leq z \leq 216, z = 12k, k \in N\}$

$144 \leq 12k \leq 216$

$\frac{144}{12} \leq k \leq \frac{216}{12}$

$12 \leq k \leq 18$

$\Rightarrow n(A \cap B) = 18 - 12 + 1 = 7$

Örnek / Example 16

$A = \{ x \mid 212 < x \leq 318,\ x = 2k,\ k \in N\}$

$B = \{ y \mid 144 \leq y < 392,\ y = 6k,\ k \in N\}$

$\Rightarrow$ **n(A∪B) = ?**

A) 77 B) 79 C) 81 D) 83 E) 87

Çözüm / Solution

Yanıt / Answer **A**

A: $212 < x \leq 318$

$212 < 2k \leq 318$

$106 < k \leq 159$

$\Rightarrow n(A) = 159 - 107 + 1 = 53$

B: $144 \leq y < 392$

$144 \leq 6k < 392$

$24 \leq k < 65,...$

$\Rightarrow n(B) = 65 - 24 + 1 = 42$

A∩B: $212 < z \leq 318$

$212 < 6k \leq 318$

$35,.. < k \leq 53$

$\Rightarrow n(A \cap B) = 53 - 36 + 1 = 18$

$= 18 n(A \cup B) = n(A) + n(B) - n(A \cap B) = 53 + 42 - 18 = 77$

Örnek / Example 17

$A = \{x: 12 < x < 150,\ x = 2k\}$

$B = \{y: 15 < y\ 240,\ y = 5k\}$

$\Rightarrow$ **n(A´∩B) = ?**

A) 28 B) 29 C) 30 D) 31 E) 32

Çözüm / Solution

Yanıt / Answer **D**

$A' \cap B = B \setminus A$

$n(B) = \frac{235 - 20}{5} + 1 = 44$

$n(A \cap B) = \frac{140 - 20}{10} + 1 = 13$

$n(A' \cap B) = 44 - 13 = 31$

Örnek / Example 18

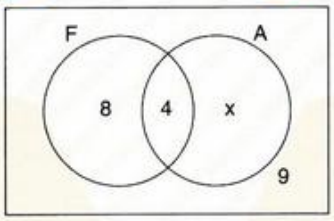
Farsça ve Arapça dillerinin konuşulduğu 26 kişilik bir sınıfta Farsça konuşan 12, her iki dili konuşanlar 4, bu iki dili de konuşamayanlar 9 kişi ise yalnız Arapça konuşanların sayısı kaçtır?

The languages of persian and arabic are spoken in a class of 26 people, if among students of this class 12 people can speak persian, 4 people can speak both languages, and 9 people can't speak eigther of the languages, what is the number of students that can only speak arabic?

A) 4 B) 5 C) 6 D) 7 E) 8

Çözüm / Solution

Yanıt / Answer **B**



$x + 8 + 4 + 9 = 26$

$x = 5$

Örnek / Example 19

22 kişilik bir toplulukta Farsça bilenlerin hepsi aynı zamanda Türkçe bilmektedir. Yalnız Türkçe bilenler 8 kişidir. Türkçe bilenlerin sayısı Farsça bilenlerin sayısının 3 katı ise bu iki dili de bilmeyen kaç kişi vardır?

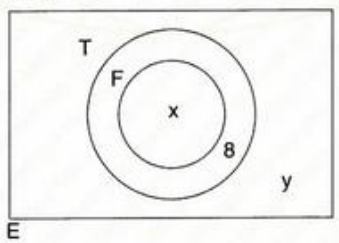
In a group of 22 people, all of the people who know persian also know turkish at the same time. 8 people know only turkish language; if the number of people who know turkish is 3 times the number of people who know persian, how many people don't know eigther of the languages?

A) 8 B) 9 C) 10 D) 11 E) 12

Çözüm / Solution

Yanıt / Answer **C**

$F \subset T$



$n(T) = 3 \cdot n(F)$

$x + 8 = 3x$

$4 = x$

$n(E) = x + 8 + y$

$22 = 4 + 8 + y$

$10 = y$

Örnek / Example 20

Bir sınıfın %70 i İngilizce, % 60 ı Almanca konuşabilmektedir. Sınıfın % 10 u her iki dilide konuşamamaktadır.

İki dilide konuşabilenlerin sayısı 20 olduğuna göre sınıf mevcudu kaçtır?

In a class 70% of students speak in english and 60% of them speak in german. 10 %of students can not speak any of these languages.

Since the amount of students who can not speak any of these languages is 20, how many students are there in the class?

A) 40 B) 45 C) 50 D) 55 E) 60

Çözüm / Solution

Yanıt / Answer **C**

Sınıf mevcudu / Volume of the class: x

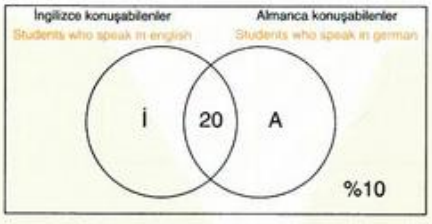
% 70·x + % 60·x + % 10·x = x + 20

%140·x = x + 20

$\frac{140x}{100} - x = 20$

40x = 20·100

x = 50



Örnek / Example 21

Herkesin futbol veya basketbol oynadığı 30 kişilik bir sınıfta futbol oynayanların sayısı 15, futbol veya basketboldan en çok birini oynayanların sayısı 20 dir.

Bu sınıfta sadece futbol oynayanların sayısı kaçtır?

In the class of 30 people where everybody play football or basketball the amount of people who play football is 15, the amount of people who play at most one of football or basketball is 20. How many students play only football in this class?

A) 5 B) 6 C) 7 D) 8 E) 9

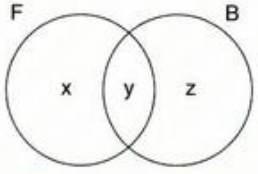
Çözüm / Solution

Yanıt / Answer **A**

x + y + z = 30

x + y = 15 ⇒ z = 15

x + z = 20 ⇒ x = 5



1. A = { a, b, c, d, e},
B = {b, c, e, f }
⇒ **A ∪ B = ?**
A) { a, b, c, d, e }
B) { b, c, e, f }
C) { b, c, e}
D) { a, b, c, d, e, f }
E) { a, d }

---

2. **A = {1, 3, 5 } , B = { x | 0 ≤ x ≤ 7, x ∈ Z } olduğuna göre, A∪B nin özalt küme sayısı kaçtır?**
If A = {1, 3, 5 } and B = { x | 0 ≤ x ≤ 7, x ∈ Z }, what is the number of the proper subsets of A∪B ?
A) 255 B) 127 C) 63
D) 31 E) 15

---

3. A = {0, – 1, 1, 2 } , B = { x | 1 ≤ x < 10, x ∈ Z }
⇒ **A ∩ B = ?**
A) { 0, 1, 2 }
B) { 1, 2 }
C) { 1, 2, 3 }
D) { –1, 0,1 }
E) { 1, 2, 3, 4, 5 }

---

4. A ∩ B = Ø
n(A) = 5, n(B) = 3
⇒ **n(A ∪ B) = ?**
A) 5 B) 6 C) 7
D) 8 E) 9

---

5. **A = {1, 2, 3, 4, 5 } , B = { 3, 5, 7, 9, 11 } olduğuna göre, B – A kümesinin alt küme sayısı kaçtır?**
If A = {1, 2, 3, 4, 5 } and B = { 3, 5, 7, 9, 11 }, what is the number of the subsets of the set B – A?
A) 3 B) 4 C) 8
D) 12 E) 16

---

6. n(A ∩ B) =1, n(B – A) = 12 ve / and n(A ∪ B) = 16
⇒ **n (A) = ?**
A) 8 B) 7 C) 6
D) 5 E) 4

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7. $B \subset A$,

$n(A) = 12$,

$n(A - B) = 8 \Rightarrow$ **n(B) = ?**

A) 3 B) 4 C) 5

D) 6 E) 7

8. $3 \cdot n(A) = 4 \cdot n(B)$,

$\frac{n(A \cap B)}{n(A - B)} = 2$ ve / and

$n(A \cup B) = 26 \Rightarrow$ **n(B) = ?**

A) 4 B) 8 C) 10

D) 12 E) 18

9. $A = \{ 1, 2, \{ a, b \}, d \}$

$B = \{ 1, a, b, \{ d \} \} \Rightarrow$ **A – B = ?**

A) { a, b }

B) { a, b, { d } }

C) { a, b, d }

D) { 2 }

E) { 2, { a, b }, d }

10. $n(A) = 7$,

$n(A \cap B) = 4$

$3 \cdot n(A - B) = n(B)$

$\Rightarrow$ **n(B – A) = ?**

A) 2 B) 3 C) 4

D) 5 E) 6

11. $A = \{a, b, c, d\}$,

$A \cup B = \{a, b, c, d, e, f\}$

$A - B = \{a, b\}$

$\Rightarrow$ **B – (A ∩ B) = ?**

A) { c, d } B) { e, f } C) { c, d, e, f }

D) {a, b } E) Ø

12. **M = { 1, 2, 3, 4, 5, 6, 7, 8 } kümesinin 4 elemanlı alt kümelerinin kaç tanesinde 3 ve 4 birlikte bulunur?**

How many subsets, which have 4 members, of the set M = { 1, 2, 3, 4, 5, 6, 7, 8 } contains 3 and 4?

A) 10 B) 12 C) 15

D) 18 E) 14

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
D	A	B	D	C	E	B	E	E	D	B	C

1. $A \cup B = \{ x \mid x \in A$ veya / or $x \in B \}$

$A \cup B = \{ a, b, c, d, e, f \}$

Yanıt / The Answer is D

---

2. $A = \{1, 3, 5\}$, $B = \{ 0, 1, 2, 3, 4, 5, 6, 7\}$

$A \cup B = \{ 0, 1, 2, 3, 4, 5, 6, 7 \}$

$n(A \cup B) = 8$ ise özalt küme $2^8 - 1 = 255$'dir.

If $n(A \cup B) = 8$, the number of proper subsets is $2^8 - 1 = 255$.

Yanıt / The Answer is A

---

3. $A \cap B = \{ x \mid x \in A$ ve / and $x \in B\}$

$A = \{0, -1, 1, 2\}$,

$B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A \cap B = \{1, 2\}$

Yanıt / The Answer is B

---

4. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 5 + 3 - 0$

$= 8$

Yanıt / The Answer is D

---

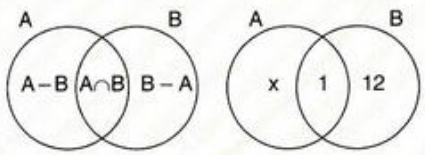
5. $B - A = \{ x \mid x \in B$ ve / and $x \notin A \}$

$B - A = \{ 7, 9, 11 \}$ olduğundan alt küme sayısı $2^3 = 8$ dir.

Since $B - A = \{ 7, 9, 11 \}$, the number of subsets is $2^3 = 8$.

Yanıt / The Answer is C

6.



$x + 1 + 12 = 16$

$x = 3$

$n(A) = 3 + 1$

$= 4$

Yanıt / The Answer is E

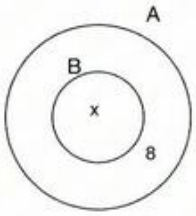
METROPOL YAYINLARI • METROPOL PUBLICATIONS

7. $n(A) = 12$

$x + 8 = 12$

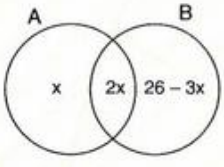
$x = 4$

$n(B) = 4$



Yanıt / The Answer is B

8.



$n(A - B) = x \Rightarrow n(A \cap B) = 2x$

$n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

$26 = x + 2x + n(B - A)$

$n(B - A) = 26 - 3x$

$3 \cdot n(A) = 4 \cdot n(B)$

$3(3x) = 4(26 - x)$

$9x = 104 - 4x$

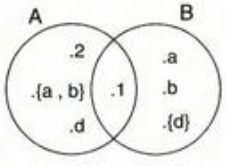
$13x = 104$

$x = 8$

$n(B) = 26 - x = 26 - 8 = 18$

Yanıt / The Answer is E

9.



$A - B = \{ 2, \{a, b\}, d \}$

Yanıt / The Answer is E

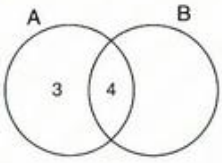
10. $n(B) = 3 \cdot n(A - B)$

$= 3 \cdot 3$

$= 9$

$n(B - A) = 9 - 4$

$= 5$

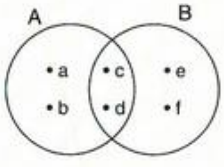


Yanıt / The Answer is D

11. $A \cap B = \{ c, d \}$

$B = \{ c, d, e, f \}$

$B - (A \cap B) = \{e, f\}$



Yanıt / The Answer is B

12. Birlikte bulunması istenen 3 ve 4 sonradan ilave etmek üzere kümeden çıkartılır.

The members 3 and 4 are expelled from the set M. We assume that they already exist in the subsets that we are searching for.

$$\binom{8-2}{4-2} = \binom{6}{2} = \frac{6 \cdot 5 \cdot 4!}{2! \cdot 4!} = 15$$

Yanıt / The Answer is C

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $s(B' - A') = 4$

$s(B - A) = 6$

$s(A) = 9$

ise B kümesinin en çok 2 elemanlı alt kümelerinin sayısı kaçtır?

then what is the number of subsets of B with at most 2 elements?

A) 63 B) 64 C) 65 D) 66 E) 67

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2020]

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2. $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

$A \cup C = \{2, 4, 6, 8, 10\}$

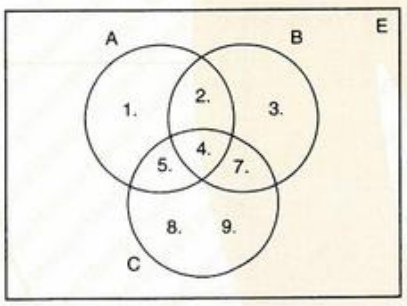
$\Rightarrow$ **$A \cup (B \cap C) = ?$**

A) Ø B) {2} C) {2, 4}

D) {1, 2, 4} E) {2, 4, 6}

[SELÇUK ÜNİVERSİTESİ – YÖS 2019]

3.



$\Rightarrow$ **$A \setminus (B \cap C) = ?$**

A) {1} B) {1,2} C) {1,5} D) {1,6} E) {1,2,5}

[İNÖNÜ ÜNİVERSİTESİ – YÖS 2018]

---

4. $\left.\begin{array}{l} A = \{a,b,c,d,e\} \\ B = \{b,c,k,l,m\} \\ C = A \cap B \end{array}\right\} \Rightarrow$ **$B \cup (A \cap C) = ?$**

A) B B) A C) C

D) {a,d,e,k,l,m} E) {e,m}

[İSTANBUL ÜNİVERSİTESİ – YÖS 2018]

---

5. $n(A \cup B) = 28$

$n(A \setminus B) + n(B \setminus A) = 6 \cdot n(A \cap B)$

$\Rightarrow$ **$n(A \setminus B) + n(B \setminus A) = ?$**

A) 16 B) 18 C) 20 D) 22 E) 24

[İSTANBUL ÜNİVERSİTESİ – YÖS 2018]

---

6. $A \cup A' = E, \quad n(A) + n(A') = n(E)$

$A \setminus B = \{x | x \in A, \ x \notin B\}$

$n(A \cup B) = 29, \ n(A \cap B) = 8$

$n(A) - n(B) = \frac{n(A \setminus B)}{4} \Rightarrow$ **$n(A) = ?$**

A) 10 B) 12 C) 16 D) 18 E) 20

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2017]

---

7. $A = \{x: |x| < 4, \ x \in R\}$

$B = \{x: -2 \le x \le 7, \ x \in Z\}$

$\Rightarrow$ **$s(A \cap B) = ?$**

A) 4 B) 5 C) 6 D) 7 E) 8

[İSTANBUL ÜNİVERSİTESİ – YÖS 2017]

---

8. $A = \{a, b, 3, \{3\}, \square, \{3, \square\}, \{a\}\}$

$B = \{\{2\}, \Delta, a, \{b\}, 3, \square\}$

olduğuna göre A – B ile A∪B'nin eleman sayıları toplamı kaçtır?

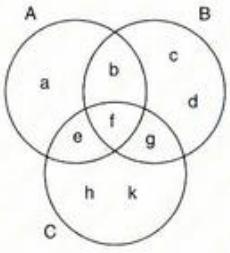
What is the sum of the element numbers of A – B and A∪B?

A) 10 B) 11 C) 12 D) 13 E) 14

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2017]

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9.



$A / B = \{x \mid x \in A \wedge x \notin B\}$

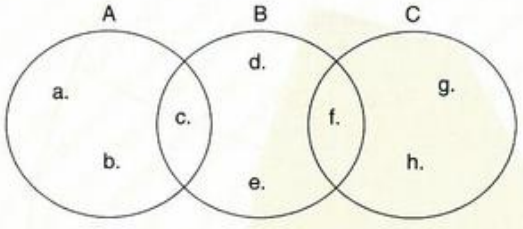
$B / C = \{x \mid x \in B \wedge x \notin C\}$

$\Rightarrow$ **$(A / C) \cap (A / B) = ?$**

A) f B) b C) e D) a E) g

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

10.



$n((B - C) \cap (B - A)) = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

[İSTANBUL ÜNİVERSİTESİ – YÖS 2016]

---

11. $A = \{a, b, 3, \{3\}, \square, \{3, \square\}, \{a\}\}$

$B = \{\{2\}, \Delta, a, \{b\}, 3, \square\}$

$\Rightarrow$ **$n(A - B) + n(A \cup B) = ?$**

A) 10 B) 11 C) 12 D) 13 E) 14

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2015]

---

12. $n(A) = 4$, $n(B) = 8$,

$n(A) \cdot n(A \cup B) = 36$

$\Rightarrow$ **$n(A \cap B) = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

[ANKARA ÜNİVERSİTESİ – YÖS 2015]

13. $A = \{n : n < 75, \quad n = 3k, \quad k \in Z^+\}$

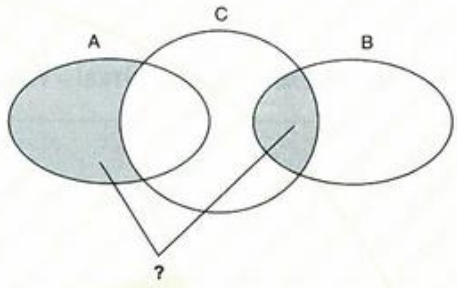
$B = \{n : n < 100, \quad n = 4k, \quad k \in Z^+\}$

$\Rightarrow$ **$n(A \cup B) = ?$**

A) 40 B) 42 C) 43 D) 44 E) 45

[İSTANBUL ÜNİVERSİTESİ – YÖS 2015]

14.



A) $(A \cup B) \cap C'$ B) $A \setminus (C \setminus B)$

C) $(A \setminus C) \cup (C \cap B)$ D) $(A \setminus (B \cap C)$

E) $(A \setminus B) \cup C$

[İSTANBUL ÜNİVERSİTESİ – YÖS 2014]

15.

$$\left.\begin{array}{l} n(A-B) = 3 \cdot n(A \cap B) \\ n(A \cup B) = 17 \\ n(B) = 8 \end{array}\right\} \Rightarrow n(B - A) = ?$$

A) 3 B) 4 C) 5 D) 6 E) 7

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2014]

---

16. $A = \{x : -2 < x < 3, \quad x \in Z\}$

$B = \{y : -3 < y < 2, \quad y \in Z\}$

$\Rightarrow$ **$n(A \cap B) = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

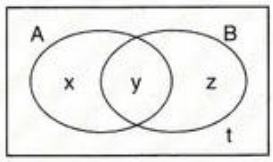
[ANKARA ÜNİVERSİTESİ – YÖS 2013]

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	E	E	A	E	E	C	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	B	E	C	B	C	C	C

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1.



$$\left.\begin{array}{l} x = 4 \\ z = 6 \\ x + y = 9 \end{array}\right\} \Rightarrow s(B) = y + z = 11$$

$$\binom{11}{0} + \binom{11}{1} + \binom{11}{2} = 1 + 11 + 55$$

$$= 67$$

Yanıt / The Answer is E

---

2. $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

$A \cup C = \{2, 4, 6, 8, 10\}$

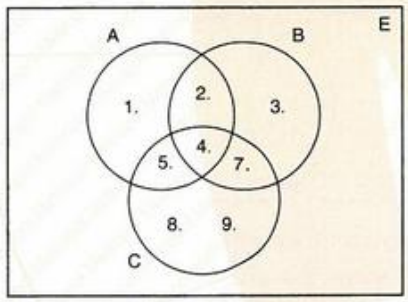
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$= \{1, 2, 3, 4, 5, 6, 7\} \cap \{2, 4, 6, 8, 10\}$

$= \{2, 4, 6\}$

Yanıt / The Answer is E

3.



$A \setminus (B \cap C) =$

$\{1,2,4,5\} \setminus \{4,7\} = \{1,2,5\}$

Yanıt / The Answer is E

---

4. $A = \{a,b,c,d,e\}$

$C = A \cap B = \{b,c\}$

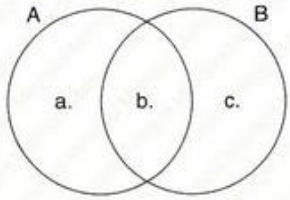
$B = \{b,c,k,l,m\}$

$B \cup (A \cap C) = \{b,c,k,l,m\}$

$= B$

Yanıt / The Answer is A

5.



$n(A \cup B) = a + b + c = 28$

$n(A \setminus B) = a$, $n(B \setminus A) = c$, $n(A \cap B) = b$

$n(A \setminus B) + n(B \setminus A) = 6 \cdot n(A \cap B)$

$a + c = 6 \cdot b$, $\quad a + b + c = 28$, $\quad a + c = 6b$

$6b + b = 28 \Rightarrow 7b = 28$

$b = 4$

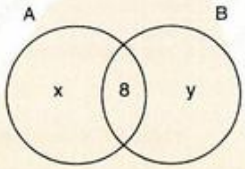
$n(A \setminus B) + n(B \setminus A) = a + c = 28 - b$

$= 28 - 4$

$= 24$

Yanıt / The Answer is E

6.



$n(A \cup B) = 29$

$x + 8 + y = 29$

$x + y = 21$

$4x + 4y = 84$

$3x - 4y = 0$ (+)

$7x = 84$

$n(A) - n(B) = \dfrac{n(A \setminus B)}{4}$

$n(A) = x + 8$

$= 20$

$8 + x - 8 - y = \dfrac{x}{4}$

$4x - 4y = x$

$3x - 4y = 0$

Yanıt / The Answer is E

---

7. $A = \{x: -4 < x < 4, \quad x \in R\}$

$A = (-4,4)$

$B = \{x: -2, -1, 0, 1, 2, 3, 4, 5, 6, 7,\}$

$A \cap B = \{-2, -1, 0, 1, 2, 3,\}$

$\Rightarrow s(A \cap B) = 6$

Yanıt / The Answer is C

---

8. $A - B = \{b, \{3\}, \{3, \square\}, \{a\}\}$

$n(A - B) = 4$

$A \cup B = \{a, b, 3, \{3\}, \square, \{3, \square\}, \{a\}, \{2\}, \Delta, \{b\}\}$

$n(A \cup B) = 10$

$n(A - B) + n(A \cup B) = 14$

Yanıt / The Answer is E

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9. $A \setminus C = \{a, b\}$

$A \setminus B = \{a, e\}$

$(A \setminus C) \cap (A \setminus B) = \{a\}$

Yanıt / The Answer is D

---

10. $B - C = \{c, d, e\}$

$B - A = \{d, e, f\}$

$(B - C) \cap (B - A) = \{d, e\}$

$n((B - C) \cap (B - A)) = 2$

Yanıt / The Answer is B

---

11. $A = \{a, b, 3, \{3\}, \square, \{3, \square\}, \{a\}\}$

$B = \{\{2\}, \Delta, a, \{b\}, 3, \square\}$

$A \setminus B = \{b, \{3\}, \{3, \square\}, \{a\}$

$n(A \setminus B) = 4$

$A \cup B = \{a, b, 3, \{3\}, \square, \{3, \square\}, \{a\}, \{2\}, \Delta, \{b\}\}$

$n(A \cup B) = 10$

$n(A \setminus B) + n(A \cup B) = 4 + 10 = 14$

Yanıt / The Answer is E

---

12. $n(A) \cdot n(A \cup B) = 4 \cdot n(A \cup B) = 36$

$n(A \cup B) = 9$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$9 = 4 + 8 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 3$

Yanıt / The Answer is C

---

13. $n(A) = 24,$

$n(B) = 24,$

$n(A \cap B) = 6$

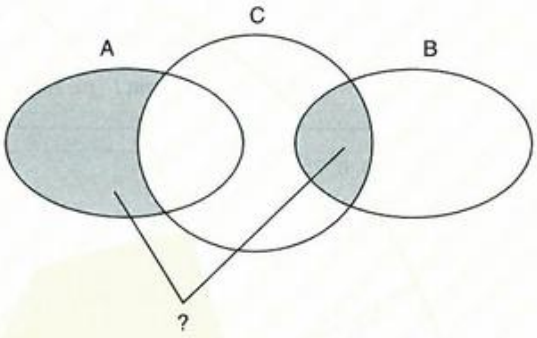
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 24 + 24 - 6$

$= 42$

Yanıt / The Answer is B

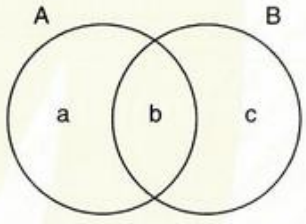
14.



$(A \setminus C) \cup (B \cap C)$

Yanıt / The Answer is C

15.



$n(A \cup B) = a + b + c = 17$

$n(B) = b + c = 8 \Rightarrow a = 9$

$n(A \setminus B) = 3 \cdot n(A \cap B)$

$a = 3 \cdot b$

$3 = b$

$b + c = 8$

$3 + c = 8 \Rightarrow c = 5$

Yanıt / The Answer is C

---

16. $A = \{-1, 0, 1, 2\}$

$B = \{-2, -1, 0, 1\}$

$A \cap B = \{-1, 0, 1\} \Rightarrow n(A \cap B) = 3$

Yanıt / The Answer is C

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1. $A = \{ 1, 3, a, \{a\}, \{a, b\}, c \}$

$B = \{ 1, \{a\}, \{b\}, \{c\} \}$

$\Rightarrow$ **n (A – B) = ?**

A) 4 B) 5 C) 6

D) 7 E) 8

2. $A \cup B = \{a, b, c, d\}$

$A \cup C = \{b, c, d, e\}$

$\Rightarrow$ **$A \cup (B \cap C)$ = ?**

A) {a,b} B) {e} C) {d,e}

D) {d} E) {b,c,d}

3. $A = \{a, b, c, d\}$,

$A \cup B = \{a, b, c, d, e\}$

$A / B = \{a, b\}$

$\Rightarrow$ **B = ?**

A) {e} B) {c,d} C) {c, d, e}

D) {b, d,e} E) {a, e}

4. $A, B \subset E$

$A \subset B \Rightarrow$ **$(A \cup B) \cap (B \cup E)'$ = ?**

A) A B) B C) $A \cup B$

D) $\varnothing$ E) E

5. $A, B \subset E$,

$A = \{a, b, d\}$

$B = \{b, c, e\}$

$\Rightarrow$ **$A \cup (B \cap A')$ = ?**

A) {a,e} B) {a,b,c} C) {d,e}

D) {b,d,e} E) {a,b,c,d,e}

6. **$A \setminus (A \cap B)'$ = ?**

A) $A \cup B$ B) $A \cap B$ C) $A \setminus B$

D) $B \setminus A$ E) $\varnothing$

7. **$(A' \cup B')' \cup (A - B)$ = ?**

A) A B) B C) $\varnothing$

D) E E) A'

8. **$[(A / B) \cup (A \cap B)]'$ = ?**

A) $\varnothing$ B) E C) A

D) A' E) B'

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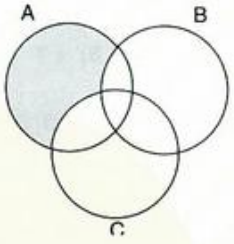
9. **M = {a, 2, b, 3, c, 4} kümesinin 3 elemanlı alt kümelerinin kaç tanesinde 2 elemanı bulunur, 4 elemanı bulunmaz?**

How many subsets, which has 3 members, of M = {a, 2, b, 3, c, 4} does not contain 4 as a member but contains 2?

A) 2 B) 3 C) 6 D) 8 E) 10

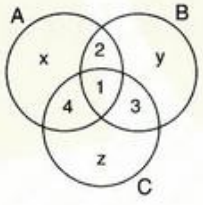
10. **Taralı Alan = ?**

The shaded area = ?



A) (B \ C) \ A
B) (A \ B) ∩ C
C) (A ∩ B) \ C
D) (A \ B) \ C
E) (A \ C) ∩ B

11. $2n(A) = n(B)$
$n(A) = n(C) + 2$
$n(A \cup B \cup C) = 27$
⇒ n(A) = ?



A) 7 B) 10 C) 12 D) 14 E) 25

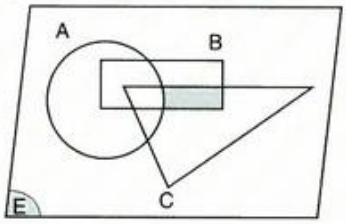
12. **A, B ⊂ E, n(A) + n(B) = 22, n(A \ B) + n(B \ A) = 12 olduğuna göre, A ∩ B kümesinin özalt küme sayısı kaçtır?**

If n(A) + n(B) = 22 and n(A \ B) + n(B \ A) = 12 where A, B⊂E what is the number of the subsets of A ∩ B?

A) 4 B) 16 C) 31 D) 32 E) 44

13. **Taralı alan = ?**

The shaded area = ?



A) A∩B∩C
B) (B∪C)∩A'
C) (A∪B)∩C
D) (A'∪C)∩B
E) A'∩B∩C

14. $A = \{ x \mid 200 < x < 400; x = 4n, n \in N\}$ **⇒ n(A) = ?**

A) 48 B) 49 C) 50 D) 51 E) 52

15. $A = \{x \mid |2x - 3| \leq 7, \quad x \in Z\}$
$B = \{x \mid |x + 1| > 2, \quad x \in R\}$
⇒ n(A − B) = ?

A) 3 B) 4 C) 5 D) 6 E) 7

16. $A = \{ x \mid 25 \leq x \leq 452, x = 3k, k \in Z^+\}$
$B = \{ y \mid 5 \leq y \leq 299, y = 2k, k \in Z^+\}$
⇒ n(A∪B) = ?

A) 198 B) 216 C) 244 D) 260 E) 316

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	E	C	D	E	B	A	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	D	B	C	E	B	B	C

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $n(A \cup B) = 3\, n(A \cap B)$,

$n(A \cup B) = 15$,

$n(A - B) = 3$

$\Rightarrow$ **n(B – A) = ?**

A) 4 B) 5 C) 7

D) 9 E) 11

2. $A = \{1, 2, 3\}$,

$A \cap B = \{1, 3\}$,

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$\Rightarrow$ **B = ?**

A) {1, 2, 3} B) {4, 5, 6} C) {1, 3}

D) {1, 3, 4, 5, 6} E) {2, 4, 5, 6}

3. $A \subset E$ ve / and $B \subset E$

$n(A) = 11$,

$n(B) = 13$ ve / and

$n(A') = 11 \Rightarrow$ **n(B') = ?**

A) 9 B) 10 C) 11

D) 12 E) 13

4. $B = \{1, 2, 3, 4\}$

$A - (A - B) = \{2, 3\}$

$\Rightarrow$ **B – A = ?**

A) {1, 2, 3} B) {1, 2} C) {1, 4}

D) {2, 3, 4} E) {1, 3, 4}

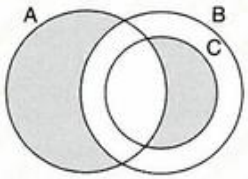
5. $[(B \cap A)' \cap (B' \cup A)]' = ?$

A) B B) $A \cap B$ C) $A \cup B$

D) $A \setminus B$ E) A

6. **Taralı Alan = ?**

The shaded area = ?

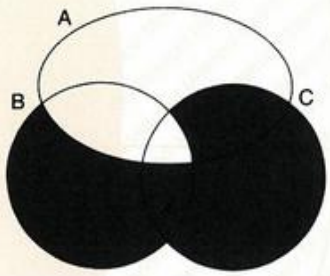


A) (A / B) ∪ (B/C) B) (A / B) ∪ (C / B)

C) (A / B) ∪ (C/A) D) (A∪B∪C) / (A∩B)

E) (A∩B∪C) ∩ (A∩B)

7.



Taralı alan / The shaded area = ?

A) (B∪C′) – (A∩B) B) (B∪C)∩(A′∪B′)

C) A – (B∪C) D) A∩(B – C)

E) (B∪C)∩(A – B)

8. **[(A∪C) \ B] ∩ A = ?**

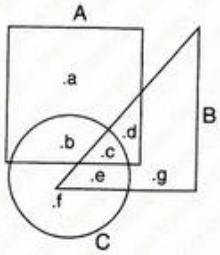
A) {a}

B) {d, c}

C) {a, b}

D) {e, f}

E) {b}



METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $(B \cap A') - B = ?$

A) A B) A′ C) B

D) B′ E) ∅

10. $(B \cap A)' - (A \cap B') = ?$

A) A B) A′ C) B

D) B′ E) A∩B

11. $n(A) = 6$,

$n(B) = 12$

$\Rightarrow$ **max(n(A∪B)) + min(n(A∪B)) = ?**

A) 32 B) 31 C) 30

D) 29 E) 28

12. $n(A) = 10$

$n(B) = 8$

$\Rightarrow$ **max(n(A∪B)) + min(n(A∪B)) = ?**

A) 10 B) 18 C) 20

D) 28 E) 38

13. $\frac{n(A \cup B)}{n(A \cap B)} = 4$,

$n(B \setminus A) = 12$,

$n(A \setminus B) = 9$

$\Rightarrow$ **n(A∪B) = ?**

A) 25 B) 28 C) 30

D) 35 E) 40

14. **A = {a, b, c, d, e} kümesinin alt kümelerinin kaç tanesinde a ve b bulunmaz?**

How many of the subsets of the set A = {a, b, c, d, e} does not contain a and b?

A) 8 B) 12 C) 16

D) 24 E) 28

15. **M = {1, 2, 3, 4, 5, 6, 7, 8} kümesinin alt kümelerinin kaç tanesinde asal sayı bulunmaz?**

How many of the subsets of the set M = {1, 2, 3, 4, 5, 6, 7, 8}, does not contain any prime number?

A) 4 B) 6 C) 8

D) 12 E) 16

16. $E = \{x : 50 \leq x \leq 300, x \in N\}$

$A = \{x \in E : x = 4k, k \in Z\}$

$B = \{x \in E : x = 3k, k \in Z\}$

$\Rightarrow$ **n(A∪B) = ?**

A) 120 B) 123 C) 124

D) 125 E) 126

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	D	A	C	A	C	B	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	B	C	D	B	A	E	E

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $A \cap B = A$, $n(A) = 4$, $n(B - A) = 6 \Rightarrow$ **n (B) = ?**

A) 2 B) 4 C) 6 D) 8 E) 10

2. **$(-3, 5] \cup [2, 7) = ?$**

A) $(-3, 7)$ B) $[-3, -7)$ C) $(-3, 7]$
D) $[-3, 7)$ E) $(2, 5)$

3. $n(A) = 3x$, $n(B) = 4x - 5$, $n(A \cap B) = x$
$n(A \cup B) = 67 \Rightarrow$ **$n(B' \cap A) = ?$**

A) 12 B) 24 C) 28 D) 34 E) 35

4. $A \subset E$, $B \subset E$,
$n(A \cup B) = 22$,
$n(A \cap B') = 6$,
$n(B) = 2 \cdot n(A) \Rightarrow$ **n(A) = ?**

A) 7 B) 8 C) 9 D) 10 E) 11

5. **H = {1, 2, 3, 5, 6, 7, 9} kümesinin 2 elemanlı alt kümelerinin kaç tanesinde çift sayı bulunmaz?**

How many subsets, which has 2 members, of the set H = {1, 2, 3, 5, 6, 7, 9} does not contain any even numbers?

A) 4 B) 5 C) 8 D) 10 E) 15

6. **A = {a, b, c, d} kümesinin alt kümelerinin kaç tanesinde a bulunur?**

How many of the subsets of the set A = {a, b, c, d} contains a?

A) 2 B) 4 C) 8 D) 16 E) 24

7. **A = {a, b, c, d, e} kümesinin alt kümelerinin kaç tanesinde a veya b bulunur?**

How many of subsets of set A = {a, b, c, d, e} have element a or b in it?

A) 8 B) 12 C) 16 D) 24 E) 32

8. $\left.\begin{array}{l} n(A) + n(B') = 15 \\ n(A') + n(B) = 19 \end{array}\right\} \Rightarrow$ **n(E) = ?**

A) 15 B) 17 C) 19 D) 21 E) 23

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $A = (-3, 5]$, $B = (2, 9)$ $\Rightarrow$ **A ∩ B = ?**

A) (2,5] B) (−3,9) C) (−2,5] D) (3,5] E) (3,9)

10. $n(A \cup B) = 5n(B \setminus A) = 4n(A \cap B)$

$n(A) = 64$ $\Rightarrow$ **n(B) = ?**

A) 16 B) 20 C) 24 D) 32 E) 36

11. $\frac{n(A-B)}{n(B-A)} = \frac{2}{3}$ ve / and $n(A \cup B) = 42$

$\Rightarrow$ **min(n(A∩B)) = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

12. $\frac{n(A \cup B)}{n(A \setminus B)} = \frac{9}{2}$, $\frac{n(A \cup B)}{n(B \setminus A)} = \frac{5}{3}$

$n(A \cap B) = 16$ $\Rightarrow$ **n(A ∪ B) = ?**

A) 90 B) 86 C) 82 D) 76 E) 74

13. $A = \{x \mid -3 < 1 - 2x < 4,\ x \in R\}$

$B = \{x \mid 1 < 3x + 2 \le 11,\ x \in Z\}$

$\Rightarrow$ **n(A∩B) = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

14. $A = \{x \mid 20 \le x < 200,\ x = 6k,\ k \in Z\}$ $\Rightarrow$ **n(A) = ?**

A) 26 B) 27 C) 28 D) 29 E) 30

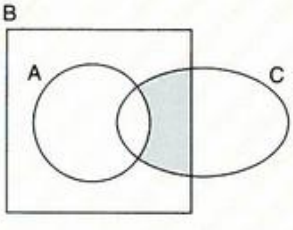
15. $n(A \setminus B) = 22$, $n(A' \cap B) = 7$

$n(A) = 2 \cdot n(B)$ $\Rightarrow$ **n(A ∩ B) = ?**

A) 5 B) 6 C) 7 D) 8 E) 9

16. Taralı alan / The shaded area = ?

A) (A ∩ B) \ C
B) (A ∩ C) \ B
C) (B ∩ C) \ A
D) (A ∪ B) \ C
E) (A ∪ C) \ B



YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	A	B	B	D	C	D	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	E	B	A	B	E	D	C

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $n(A) = 3x + 5$

$n(B) = 2x + 4$

$n(A \cap B) = x + 3$

$n(A \cup B) = 46 \Rightarrow$ **x = ?**

A) 6 B) 8 C) 10

D) 12 E) 14

2. $n(A \cup B) - n(A \cap B) = 27$

$n[(A \cup B)'] = 8$

$n(E) = 40$

$\Rightarrow$ **$n(A \cap B) = ?$**

A) 3 B) 5 C) 7

D) 8 E) 10

3. $n(A \cap B)' = 30$

$n(A') = 14$

$\Rightarrow$ **$n(B' \cap A) = ?$**

A) 10 B) 12 C) 14

D) 16 E) 18

4. $n(A') = 22$, $n(B') = 18$

$n(A) + n(B) = 20$

$\Rightarrow$ **$n(B) = ?$**

A) 12 B) 20 C) 25

D) 30 E) 35

5. **$[(A' \cup B) \cap (B' \cup A)] \cap (A \cup B) = ?$**

A) A' B) A C) $A \cap B$

D) Ø E) $(A \cup B)'$

6. $A = \{ x | 0 < x < 100, \ x = 2k + 1, \ k \in Z \}$

$\Rightarrow$ **$n(A) = ?$**

A) 48 B) 49 C) 50

D) 51 E) 52

7. $A = \{ x | 27 < x < 96, \ x = 2 \ (mod\ 3), \ x \in Z \}$

$B = \{ x | 34 < x < 99, \ x = 3 \ (mod\ 4), \ x \in Z \}$

$\Rightarrow$ **$n(A \cap B) = ?$**

A) 4 B) 5 C) 6

D) 7 E) 8

8. $A = \{ x | 18 < x < 48, \ x = 6k, \ k \in Z \}$

$B = \{ x | 34 < x < 99, \ x = 8k, \ k \in Z \}$

$\Rightarrow$ **$n(A \cup B) = ?$**

A) 5 B) 6 C) 7

D) 8 E) 12

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $[B' \cap (A' \cap B)]' = ?$

A) A' B) B C) $A \cap B$

D) $\emptyset$ E) E

10. $n(A) = 2 \cdot n(B) + 4$

$n(A \setminus B) = 8$

$n(A \cup B) = 10$

$\Rightarrow$ **$n(B \setminus A) = ?$**

A) 1 B) 2 C) 3

D) 4 E) 5

11. $A = \{ x \mid x^2 < 101, \ x \in Z \}$

$\Rightarrow$ **$n(A) = ?$**

A) 10 B) 11 C) 19

D) 20 E) 21

12. $n(A \setminus B) = 2 \cdot n(A \cap B)$

$n(B \setminus A) = 2 \cdot n(A)$

$n(A \cup B) = 45$

$\Rightarrow$ **$n(A \cap B) = ?$**

A) 2 B) 3 C) 4

D) 5 E) 6

13. $n(A) + n(B) - 2n(A \cap B) = 18$

$n(A \cup B) = 24,$

$n(A \setminus B) = 6 \Rightarrow$ **$n(A) = ?$**

A) 6 B) 8 C) 10

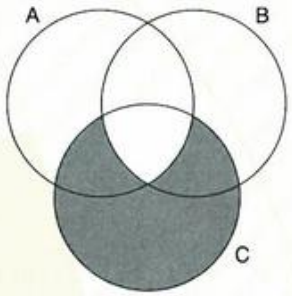
D) 12 E) 14

14. $(A' \cup B') \cap (A' \cap B') = ?$

A) A B) B C) $A \cap B$

D) $(A \cup B)'$ E) $A' \cap B'$

15.



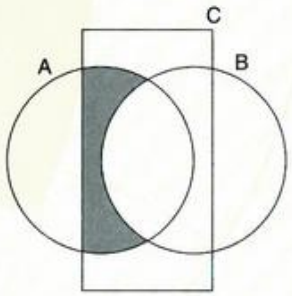
$\Rightarrow$ **Taralı alan / The shaded area = ?**

A) $(A \cap B) \setminus C$ B) $C \setminus (A \cap B)$

C) $(A \cap B) \cup (B \cap C)$ D) $(A \cap C) \setminus B$

E) $C \setminus (A \cup B)$

16.



$\Rightarrow$ **Taralı alan / The shaded area = ?**

A) $(A \cap B) \setminus C$ B) $(A \cup C) \setminus B$

C) $(A \cap C) \setminus B$ D) $A \cap C \cap B$

E) $(A \cap C) \setminus B$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	B	D	A	C	C	C	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	B	E	D	D	D	B	E

METROPOL YAYINLARI • METROPOL PUBLICATIONS

BÖLÜM CHAPTER 12

TARİHSEL NOT / HISTORICAL NOTE

Grigori Perelman [1977 -]

Bir çok kişi tarafından Dünya'nın yaşayan en zeki insanı olarak kabul edilen Grigori Perelman 2000 yılında Clay Matematik Enstitüsünün o güne kadar çözülemeyen ve "Bin Yılın Soruları" olarak görülen milenyum sorularından 1 milyon dolar ödüllü "Poincare Varsayımı" nı 2002 yılında çözmüş ve 2006 yılında Fields ödülüne layık görülmüştür.

Fakat Grigori Perelman hem soru başına koyulan 1 milyon dolarlık ödülü hem de hak ettiği Fields ödülünü reddetmiş ve tarihe bu şekilde geçmiş ilk matematikçidir.

He is regarded as the smartest living person in the world by a lot of people, at the year of 2002 he solved one of the so called "Millennium Questions" that was introduced by Clay Institute, questions which were not resolved until that day and was awarded 1 million dollars in year 2000; he deserved the Fields award in 2006.

However, Grigori Perelman not only rejected the fields award, but also he rejected the 1 million dollars award which was awarded per each question, so his name was written as the first mathematician who didn't accept this award.

BAĞINTI
RELATION

Bu bölüm, 7 çözümlü örnek, 10 çözümlü test sorusu, 16 test sorusu içermektedir.

This chapter includes 7 examples with solutions, 10 tests questions with solutions and 16 test questions.

BÖLÜM / CHAPTER 12 ..... 405 - 416

BAĞINTI / RELATION

- Bağıntı / Relation ..... 407 - 410
  - Sıralı İkili / Ordered Pair ..... 407 - 408
  - İki Kümenin Kartezyen Çarpımı / Cartesian Product of Two Sets ..... 408 - 409
  - Bağıntı / Relation ..... 409 - 410
- ▲ Çözümlü Test / Test with Solutions ..... 411 - 414
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BÖLÜM
12
CHAPTER

BAĞINTI
RELATION

BAĞINTI / RELATION

SIRALI İKİLİ / ORDERED PAIR

x ve y gibi iki elemanın, sırası önemli olmak koşulu ile (x,y) elemanı biçiminde gösterimine sıralı ikili denir.

If two elements like x and y are written as (x , y), with the order of them being important, the result is called an ordered pair.

(Asal sayı, tek sayı) ikililerinden bazıları

(2,5), (3,5), (5,5), (7,15) gibi

Examples of (prime number, odd number) ordered pairs:

(2,5), (3,5), (5,5), (7,15)

Özellik / Property

★ $(a,b) = (c,d) \Leftrightarrow a = c \wedge b = d$

Örnek / Example 1

$(x,4) = (5, y + 2) \Rightarrow x \cdot y = ?$

A) 2 B) 4 C) 8 D) 10 E) 12

Çözüm / Solution

Yanıt / Answer **D**

$(x,4) = (5, y + 2)$

$x = 5, \quad y + 2 = 4$

$y = 2$

$x \cdot y = 5 \cdot 2 = 10$

Örnek / Example 2

$(2x - 1, 7) = (x, 2y - 3) \Rightarrow x + y = ?$

A) 4 B) 5 C) 6 D) 7 E) 8

Çözüm / Solution Yanıt / Answer **C**

$(2x - 1) = x \qquad 7 = 2y - 3$

$x = 1 \qquad 10 = 2y$

$5 = y$

$x + y = 6$

İKİ KÜMENİN KARTEZYEN ÇARPIMI
CARTESIAN PRODUCT of TWO SETS

A ve B iki küme olsun.

Birinci bileşeni A kümesinden ikinci bileşeni B kümesinden alınarak oluşturulan tüm sıralı ikililere **A kartezyen çarpım B** denir, $AxB = \{(x,y) \mid x \in A \wedge y \in B\}$ şeklinde gösterilir.

Let A and B be two sets,

set of all the ordered pairs in which their fist and second components are taken from first and second sets respectively is called A cartesian product B and is shown by $AxB = \{(x,y) \mid x \in A \wedge y \in B\}$.

Tanım / Definition: $AxB = \{(x,y) \mid x \in A \wedge y \in B\}$

Örnek / Example 3

$A = \{ x \mid 1 < x < 4, \quad x \in N\}$

$B = \{ x \mid 1 < y \leq 6, \quad y = 2k, \quad x \in N\} \Rightarrow$ **AxB = ?**

A) {(2,4), (2,6), (3,2), (3,4), (3,6)}

B) {(2,2), (2,4), (2,6), (3,2), (3,4), (3,6)}

C) {(2,2), (2,4), (3,2), (3,4)}

D) {(2,2), (2,4), (3,2), (3,4), (3,6)}

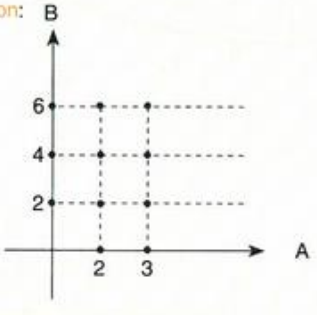
E) {(2,2), (2,4), (2,6), (3,4), (3,6)}

Çözüm / Solution Yanıt / Answer **B**

$A = \{2,3\} \quad B = \{2,4,6\}$

$AxB = \{(2,2), (2,4), (2,6), (3,2), (3,4), (3,6)$

grafik ile gösterimi / diagram demonostration:



Özellik / Property

- ★ $A \times B \neq B \times A$
- ★ $n(A \times B) = n(B \times A) = n(A) \cdot n(B)$
- ★ $A \times (B \cup C) = (A \times B) \cup (A \times C)$
- ★ $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Örnek / Example 4

$A = \{ x \mid 7 \leq x \leq 30, \quad x = 3k, \quad x \in N\}$

$\Rightarrow$ **n(AxA) = ?**

A) 36 B) 49 C) 54 D) 64 E) 81

Çözüm / Solution

Yanıt / Answer **D**

$n(A) = 8$

$n(A) \cdot n(A) = 8 \cdot 8 = 64$

BAĞINTI / RELATION

Tanım: A ve B kümeleri için, AxB nin her alt kümesine A dan B ye bir bağıntı denir. A dan B ye bağıntı sayısı $2^{n(A) \cdot n(B)}$ dir.

Definition: From set A to set B, each subset of AxB is called a relation from A to B. The number of relations from A to B are $2^{n(a) \cdot n(b)}$.

Örnek / Example 5

$A = \{1,2,3\}$, $B = \{4,5\}$ ise / if

A dan B ye bağıntı sayısı kaçtır?

What is the number of relations from A to B?

A) 3 B) 6 C) 18 D) 32 E) 64

Çözüm / Solution

Yanıt / Answer **E**

$n(A \times B) = n(A) \cdot n(B)$

$= 2 \cdot 3$

$= 6$ ve/and $2^6 = 64$

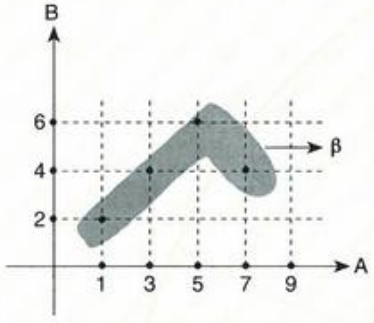
Örnek / Example 6

A = {1,3,5,7,9}, B = {2,4,6}

β = {(1,2), (3,4), (5,6), (7,4)}

β bağıntısının grafiğini çizelim / draw the diagram of relation β.

Çözüm / Solution



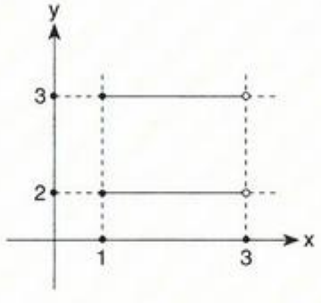
Örnek / Example 7

A = { x| 1 ≤ x < 3, x ∈ R}

B = { y| 1 < y ≤ 3, x ∈ Z} ⇒ **AxB nin grafiğini çiziniz /** draw the diagram of AxB.

Çözüm / Solution

A = [1,3), B = {2,3}



1. $(2a - b, 3a - c, b + c) = (5, 11, 4)$

$\Rightarrow$ **a + b + c = ?**

A) 5 B) 6 C) 7

D) 8 E) 9

2. $(2x - y, 3^x) = (x + y, 27)$

$\Rightarrow$ **x + y = ?**

A) 3 B) 4 C) $\frac{9}{2}$

D) 5 E) $\frac{11}{2}$

3. $a,b \in Z^+$

$(a + b, 2c) = (\frac{c}{a-b}, 34)$

$\Rightarrow$ **b = ?**

A) 7 B) 8 C) 9

D) 10 E) 12

4. $A = \{1, 2, 3\}, B = \{2, 3\}, C = \{1, 2\}$

$\Rightarrow$ **n[Ax(B $\cap$ C)] = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

5. $A = \{B, A, T, U\}$ $\Rightarrow$ **AxAxA kartezyen çarpımın birinci bileşeni B olan kaç elemanı vardır?**

From cartesian product AxAxA, how many of its elements have B as the first component?

A) 4 B) 6 C) 8

D) 12 E) 16

6. $n[(BxA) \cup (CxA)] = 24$

$\Rightarrow$ **max [n(A)] = ?**

A) 8 B) 9 C) 10

D) 12 E) 16

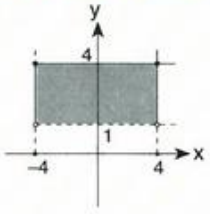
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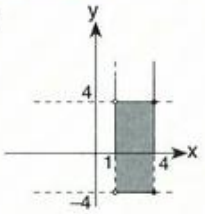
7. $A = \{ x \mid |x| \le 4, \quad x \in R \}$

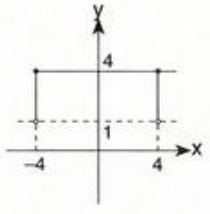
$B = \{ y \mid 1 < y \le 4, \quad y \in R \}$

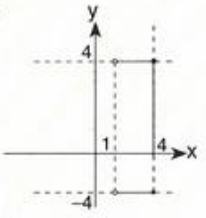
AxB nin grafiği aşağıdakilerden hangisidir?

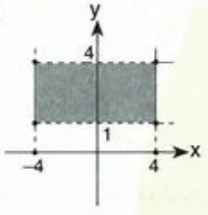
Which one of the following is the diagram of AxB?

A) 

B) 

C) 

D) 

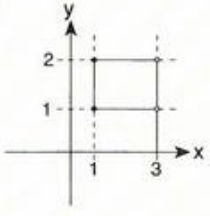
E) 

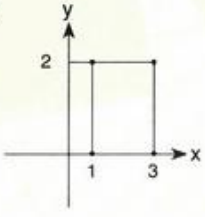
8. $A = \{ x \mid 1 \le x < 3, \quad x \in R \}$

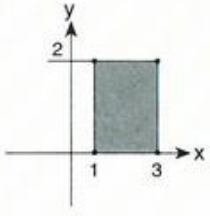
$B = \{ y \mid 0 < y \le 2, \quad y \in Z \}$

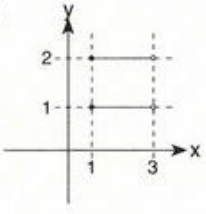
AxB nin grafiği aşağıdakilerden hangisidir?

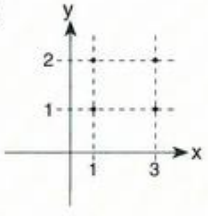
Which one of the following is the diagram of AxB?

A) 

B) 

C) 

D) 

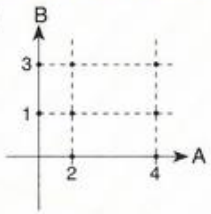
E) 

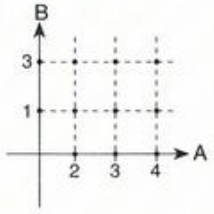
9. $A = \{ x \mid 2 \le x \le 4, \quad x \in Z \}$

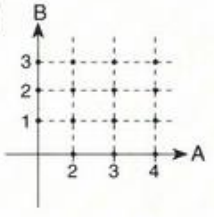
$B = \{1,3\}$

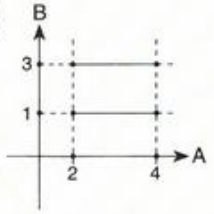
AxB nin grafiği aşağıdakilerden hangisidir?

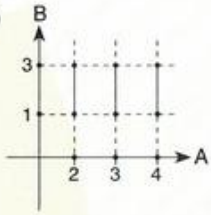
Which one of the following is the diagram of AxB?

A) 

B) 

C) 

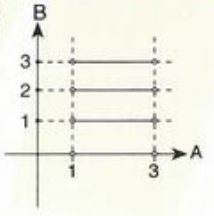
D) 

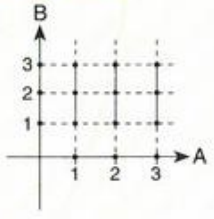
E) 

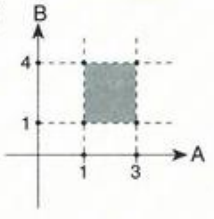
10. $A = (1,3], \quad B = \{ x \mid |x - 2| \le 1, \quad x \in R \}$

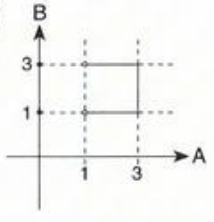
AxB nin grafiği aşağıdakilerden hangisidir?

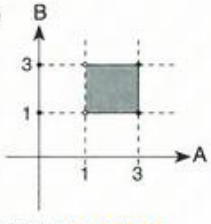
Which one of the following is the diagram of AxB?

A) 

B) 

C) 

D) 

E) 

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10
D	C	B	C	E	D	A	D	B	E

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1. $(2a-b, 3a-c, b+c) = (5,11,4)$

$2a-b=5$

$+\quad b+c=4$

$2a+c=9$

$2a+c=9$

$+\quad 3a-c=11$

$5a=20$

$a=4$

$2\cdot4-b=5 \qquad b+c=4$

$3=b \qquad c=1$

$a+b+c=8$

Yanıt / The Answer is D

2. $(2x-y, 3^x) = (x+y, 27)$

$3x=27 \qquad 2\cdot3-y=3+y$

$x=3 \qquad 6-3=2y$

$\frac{3}{2}=y$

$x+y=3+\frac{3}{2}=\frac{9}{2}$

Yanıt / The Answer is C

3. $(a+b, 2c) = \left(\frac{c}{a-b}, 34\right)$

$2c=34 \qquad a+b=\frac{c}{a-b}$

$c=17$

$a^2-b^2=17$

$(a-b)(a+b)=17$

$a-b=1$

$+\quad a+b=17$

$2a=18$

$a=9$

$b=8$

Yanıt / The Answer is B

4. $n[Ax(B\cap C)] = n(A)\cdot n(B\cap C)$

$=3\cdot1$

$=3$

Yanıt / The Answer is C

5. $\frac{1}{\{B\}}\cdot(4)\cdot(4)=16$

Yanıt / The Answer is E

6. $n[(BxA)\cup(CxA)] = 24$

$n[(AxB)\cup(AxC)] = 24$

$n[Ax(B\cup C)]=24$

$12\cdot 2$

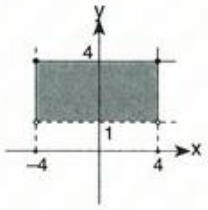
$\max[n(A)]=12$

Yanıt / The Answer is D

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7. $|x| \le 4 \quad -4 \le x < 4$

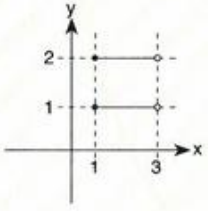
$1 < y \le 4$



Yanıt / The Answer is A

8. $1 \le x < 3, \quad x \in R$

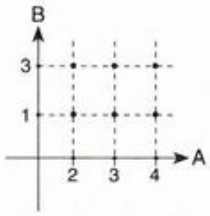
$0 < y \le 2, \quad y \in Z$



Yanıt / The Answer is D

9. $2 \le x \le 4, \quad x \in R$

$B = \{1,3\} \quad y \in Z$

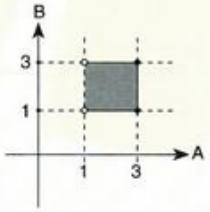


Yanıt / The Answer is B

10. $A = (1,3], \quad B = |x - 2| \le 1$

$-1 \le x - 2 \le 1$

$1 \le x \le 3$



Yanıt / The Answer is E

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1. $(9a,\ 6b-10) = (18,\ b)$

$\Rightarrow$ **a + b = ?**

A) 11 B) 7 C) 5

D) 4 E) 2

2. $A = \{a, b\}$, $B = \{2, 3, 5\}$

$\Rightarrow$ **$n(A \times B) + n(B^2) = ?$**

A) 6 B) 8 C) 7

D) 12 E) 15

3. $(2x-1, -3) = (y+1, x+y)$

$\Rightarrow$ **x − y = ?**

A) $\frac{7}{3}$ B) $\frac{8}{3}$ C) 3

D) $\frac{10}{3}$ E) $\frac{13}{3}$

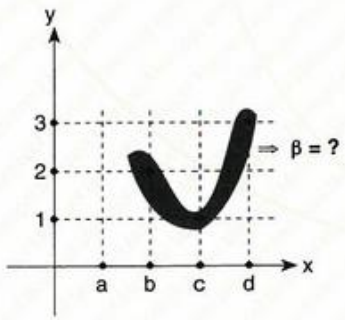
4. $A \times B = \{(a,1), (a,3), (b,1), (b,3), (c,1), (c,3)\}$

$\Rightarrow$ **$n[Ax(A \cup B)] = ?$**

A) 6 B) 8 C) 10

D) 12 E) 15

5.



A) $\beta = \{(b,2), (c,2), (d,3)\}$

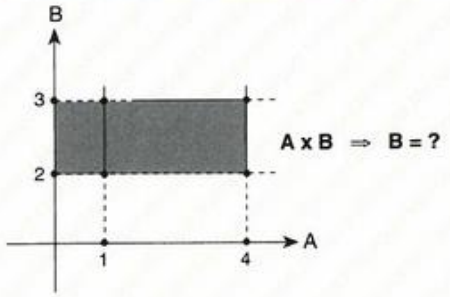
B) $\beta = \{(b,2), (c,1), (d,2)\}$

C) $\beta = \{(b,2), (c,1), (d,3)\}$

D) $\beta = \{(a,1), (b,2), (c,3)\}$

E) $\beta = \{(a,2), (b,2), (c,1)\}$

6.



A) [2,3) B) [2,3) C) [2,3]

D) (1,2) E) [1,2]

7. $A \times B = \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\}$

$\Rightarrow$ **$(A \cup B) = ?$**

A) {0,1,2} B) {1,2,3} C) {2,3,4}

D) {1,2,3,4} E) {0,1,2,3,4}

8. $A = \{0,1,2,3\}$, $B = \{2,3,4\}$, $C = \{4,5,6\}$

$\Rightarrow$ **$n[Ax(B \cup C)] = ?$**

A) 10 B) 15 C) 18

D) 20 E) 25

9. $(2a+1,\ b-a) = (7, -5)$

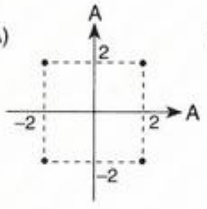
$\Rightarrow$ **a + b = ?**

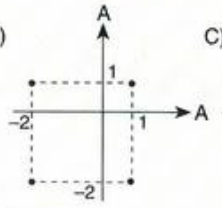
A) −2 B) −1 C) 0

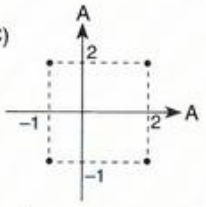
D) 1 E) 2

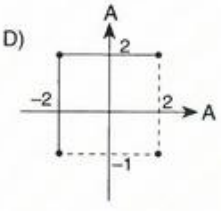
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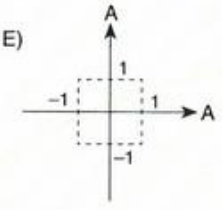
10. A = {−2,1} ⇒ **AxA grafiği / graph = ?**

A) 

B) 

C) 

D) 

E) 

11. A = {x| 2 ≤ x < 8, x ∈ Z}

B = {x| |2x − 5| < 7, x ∈ Z}

C = {2,3,6,7}

⇒ **n[(A∩B)x(A∩C)] = ?**

A) 16 B) 24 C) 36

D) 48 E) 54

12. A = {1,2}, B = {a}, C = {d}

⇒ **A x B x C = ?**

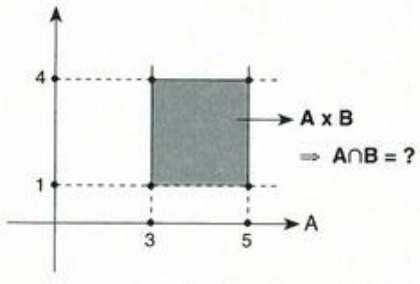
A) {(1,2), (1,a), (1,d)}

B) {(1,2,a}

C) {(1,2,a,d)}

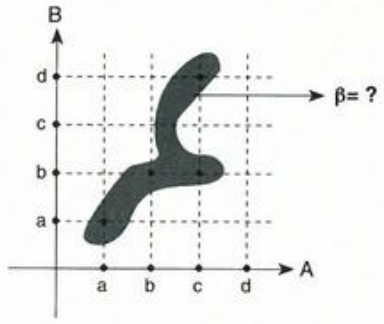
D) {(1,a), (2,d)}

E) {(1,a,d), (2,a,d)

13. 

A) [3,4] B) [1,5] C) [1,2]

D) [4,5] E) [1,4}

14. 

A) β = {(a,a), (b,b), (c,c), (c,d)}

B) β = {(a,a), (b,b), (c,b), (c,c)}

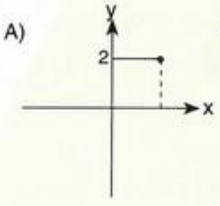
C) β = {(a,a), (b,b), (c,b), (c,d)}

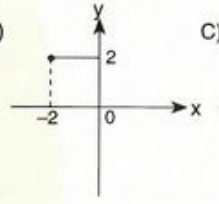
D) β = {(b,b),(c,c), (c,d), (d,d)}

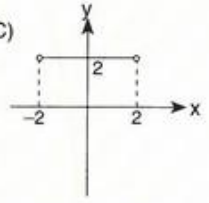
E) β = {(b,b), (b,c), (c,d), (d,c)}

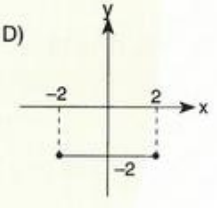
15. β = {(x,y)| |x| ≤ 2, y = 2}

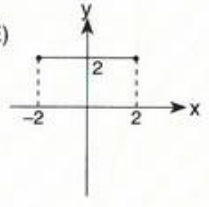
⇒ **β grafiği / graph = ?**

A) 

B) 

C) 

D) 

E) 

16. AxB = {(0,0), (0,1), (1,0), (1,1), (2,0), (2,1)}

⇒ **A∩B = ?**

A) Ø B) {1,2} C) {0,1}

D) {0,2} E) {0,1,2}

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	E	A	E	C	C	E	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	B	A	E	A	C	E	C

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BÖLÜM CHAPTER 13

TARİHSEL NOT / HISTORICAL NOTE

Bessel Friedrich Wilhelm. [1784 – 1846]

Alman gök bilimci ve matematikçi. Bessel fonksiyonlarını ortaya çıkarmakla matematiğe büyük katkıda bulunmuştur. Bessel fonksiyonları, diferansiyel denklemlerin elde edilmesinde ve muhtemelen temel fonksiyonlardan sonra mühendislik ve fizik alanında en sık kullanılan fonksiyonlardır.

German astronomer and mathematician who made a major contribution to mathematics in the development of what are now called Bessel functions. These functions satisfy certain differential equations, and probably are the most commonly occurring functions in physics and engineering after the elementary functions.

FONKSİYONLAR
FUNCTIONS

Bu bölüm, 26 çözümlü örnek, 12 çözümlü test sorusu, 16 çözümlü YÖS sorusu, 128 test sorusu içermektedir.
This chapter includes 26 examples with solutions, 12 tests questions with solutions, 16 YÖS questions with solutions and 128 test questions.

BÖLÜM / CHAPTER 13 ..... 417 - 458

FONKSİYONLAR / FUNCTIONS

- Fonksiyonlar / Functions ..... 419 - 434
  - Fonksiyon Çeşitleri / Types of Functions ..... 421 - 430
  - Fonksiyon Grafikleri / Graphs of Functions ..... 431 - 434
- Çözümlü Test / Test with Solutions ..... 435 - 438
- YÖS Soruları ve Çözümleri / YÖS Questions and Solutions ..... 439 - 442
- Konu Testleri / Practice Tests ..... 443 - 458

BÖLÜM
13
CHAPTER

FONKSİYONLAR
FUNCTIONS

ÖNEMLİ / IMPORTANT

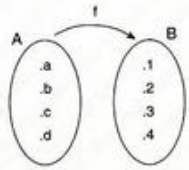
Bir fonksiyonun tanım kümesi, fonksiyonun tanımlı olduğu aralıktaki değerleridir. Fonksiyonun görüntü kümesi, fonksiyon uygulandığında elde edilen sonuçlardır.

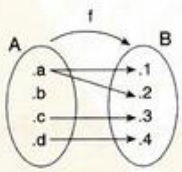
The domain of a function is the set of all the values for which the function is defined. The range of a function is the set of all values that are the outputs, or results, of applying the function.

FONKSİYONLAR / FUNCTIONS

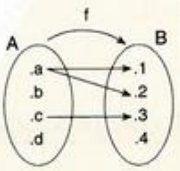
Tanım: A' dan B' ye bir f bağıntısı A' nın her elemanını B' de bir ve yalnız bir elemanla eşliyorsa ve A'da eşlenmeyen eleman kalmıyorsa f, A' dan B' ye bir fonksiyondur denir ve f: A → B ile gösterilir. A kümesine *tanım kümesi*, B kümesine *değer kümesi* denir.

Definition: If a relation f from the set A to set B makes an assignment to each element in the set A and assigns one element of A to one and only one element in the set B then f is called *a function*. Set A is called the *domain* and set B is called the *range* of the function.

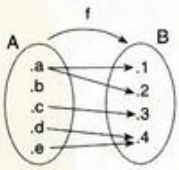




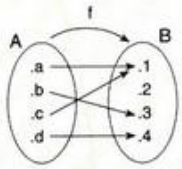
Fonksiyon değil
Not a function



Fonksiyon değil
Not a function



Fonksiyon değil
Not a function



Fonksiyon
A function

ÖNEMLİ / IMPORTANT

- $\sqrt[n]{g(x)}$ biçimindeki fonksiyonların tanım kümesi,
  - **a.** n tek ise, R'dir.
  - **b.** n çift ise, $g(x) \geq 0$ dır.
- $\frac{a}{b}$ biçimindeki fonksiyonların tanım kümesi bulunurken, önce payda sıfıra eşitlenir ve paydayı sıfır yapan değer bulunur. tanım kümesi, R'den paydayı sıfır yapan değer çıkarılır.
- Polinom şeklindeki fonksiyonların tanım kümesi R'dir.

- The domain set of functions in $\sqrt[n]{g(x)}$ form;
  - **a.** If n is odd, it is R.
  - **b.** If n is even, then $g(x) \geq 0$.
- The domain set of functions in $\frac{a}{b}$ form; you should first find the values of which make denominator equal to zero and the function undefined. The domain of the function is R minus these vales.
- The domain set of polynomial functions is R.

Etkinlikler / Activities

a. $f: A \to B, f(x) = \sqrt{4-x} \Rightarrow A, B = ?$

Kök derecesi çift olduğundan 4 – x, 0'dan büyük veya eşit olmalıdır.

$4 - x \geq 0 \Rightarrow 4 \geq x \Rightarrow$ Tanım kümesi = A = $(-\infty, 4]$

$4 - x \geq 0 \Rightarrow \sqrt{4-x} \geq 0 \Rightarrow$ görüntü kümesi = B = $[0, \infty) = R^+ \cup \{0\}$

a. $f: A \to B, f(x) = \sqrt{4-x} \Rightarrow A, B = ?$

Since the root index is even, 4 – x should be equal to or greater than 0.

$4 - x \geq 0 \Rightarrow 4 \geq x \Rightarrow$ Domain = A = $(-\infty, 4]$

$4 - x \geq 0 \Rightarrow \sqrt{4-x} \geq 0 \Rightarrow$ Range = B = $[0, \infty) = R^+ \cup \{0\}$

Örnek / Example 1

f: A → B

A = {–2, –1, 0, 1},

f(x) = 2x + 3 ⇒ **B = ?**

A) {–1, 1, 3, 4} B) {–1, 1, 3, 5} C) {–1, 0, 1, 3}

D) {–1, 0, 3, 4} E) {–1, 1, 3, 6}

Çözüm / Solution Yanıt / Answer **B**

x = –2 ⇒ y = 2·(–2) + 3 = –1

x = –1 ⇒ y = 2·(–1) + 3 = 1

x = 0 ⇒ y = 2·0 + 3 = 3

x = 1 ⇒ y = 2·1 + 3 = 5

TOPLAMA ve ÇIKARMA İŞLEMİ / ADDITION and SUBTRACTION

f : A → R, g : A → R

(f ± g)(x) = f(x) ± g(x)

Etkinlikler / Activities

a. (f + g)(3) = f(3) + g(3)

b. f(x) = 2x – 2

g(x) = 3x + 3 ⇒ (f + g)(3) = 5·3 + 1 = 16

f(3) = 2·3 – 2 = 4

g(3) = 3·3 + 3 = 12 } ⇒ f(3) + g(3) = (f + g)(3)

PEKİŞTİRME / REINFORCE

f(x) = 2x – 1, g(x) = 3x + 1

✓ **(f + g) (2) = ?**

..... [10]

✓ **(f – g) (1) = ?**

..... [–3]

✓ **(f + g) (1) = ?**

..... [5]

✓ **(f – g) (0) = ?**

..... [–2]

ÇARPMA ve BÖLME İŞLEMİ / MULTIPLICATION and DIVISION OPERATION

$(f \cdot g)(x) = f(x) \cdot g(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

Etkinlikler / Activities

a. f(x) = x – 1

g(x) = 2x + 3 ⇒ (f·g)(3) = (x – 1)·(2x + 3)

= (3 – 1)·(2·3 + 3)

= 18

b. $f(x) = x^2 - 2$ $g(x) = 3x + 1 \Rightarrow \left(\frac{f}{g}\right)(1) = \frac{x^2-2}{3x+1} = \frac{1^2-2}{3\cdot 1+1} = -\frac{1}{4}$

PEKİŞTİRME / REINFORCE

f(x) = x + 3, g(x) = 2x – 4

✓ **(f · g) (2) = ?**

..... [0]

✓ **(f · g) (–1) = ?**

..... [–12]

✓ $\frac{f}{g}(0) = ?$

..... $\left[-\frac{3}{4}\right]$

✓ **2·f(3) + 3·g(1) = ?**

..... [6]

FONKSİYON ÇEŞİTLERİ / TYPES of FUNCTIONS

1. Birim (Etkisiz) Fonksiyon (Identity Function)
2. Sabit Fonksiyon (Constant Function)
3. Ters Fonksiyon (Inverse Function)
4. Bileşke Fonksiyon (Composite Function)
5. Permütasyon Fonksiyonu (Permutation Function)
6. Doğrusal Fonksiyon (Linear Function)

BİRİM (ETKİSİZ) FONKSİYON / IDENTITY FUNCTION

$f(x) = x \Rightarrow$ f birim fonksiyondur. $f(x) = x$ ile gösterilir.

$f(x) = x \Rightarrow$ f is identity function. It is denoted by $f(x) = x$.

Örnek / Example 2

$f(x) = (4 - a)x + 2b + 6$ fonksiyonu birim fonksiyon ise, **a + b kaçtır?**

If $f(x) = (4 - a)x + 2b + 6$ is an identity function, **what is a + b?**

A) 0 B) 1 C) 2 D) 3 E) 4

Çözüm / Solution

Yanıt / Answer **A**

$4 - a = 1$ ve $2b + 6 = 0$

$a = 3$ ve $b = -3$

$\Rightarrow a + b = 3 + (-3) = 0$

PEKİŞTİRME / REINFORCE

$f(x) = x$

✓ $I(x) = (a - 2)x \Rightarrow$ **a = ?**

..... [3]

✓ $I(x) = (b - 3)x \Rightarrow$ **b = ?**

..... [4]

✓ $I(x) = (a - 3)x^2 + x \Rightarrow$ **a = ?**

..... [3]

Örnek / Example 3

$f(x) = (a + 5)x^2 + (b - 3)x + a + c$ fonksiyonu birim fonksiyon ise, **a + b + c kaçtır?**

If $f(x) = (a + 5)x^2 + (b - 3)x + a + c$ is an identity function, **what is a + b + c ?**

A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution

Yanıt / Answer **D**

$a + 5 = 0$ $b - 3 = 1$ $a + c = 0$

$a = -5$ $b = 4$ $c = -a = 5$

$\Rightarrow a + b + c = -5 + 4 + 5 = 4$

SABİT FONKSİYON / CONSTANT FUNCTION

$f(x) = c, c \in R \Rightarrow$ f sabit fonksiyondur.

$f(x) = c, c \in R \Rightarrow$ f is a constant function.

Örnek / Example 4

$f(x) = (2a - b + 5)x^2 + 2bx + 1$ fonksiyonu sabit fonksiyon ise, **a + b kaçtır?**

If $f(x) = (2a - b + 5)x^2 + 2bx + 1$ is a constant function, **find is a + b?**

A) $\frac{9}{2}$ B) $\frac{5}{2}$ C) 0 D) $-\frac{9}{2}$ E) $-\frac{5}{2}$

Çözüm / Solution

Yanıt / Answer **E**

$f(x) = (2a - b + 5)x^2 + 2bx + 1$

$2a - b + 5 = 0$ and $2b = 0 \Rightarrow 2a - 0 + 5 = 0$

$a = -\frac{5}{2} \Rightarrow a + b = -\frac{5}{2} + 0 = -\frac{5}{2}$

Örnek / Example 5

$f(x) = \frac{3x + a}{4x - 8}$ sabit fonksiyon ise, **a kaçtır /** is a constant function, **what is a?**

A) –8 B) –7 C) –6 D) –5 E) –4

Çözüm / Solution

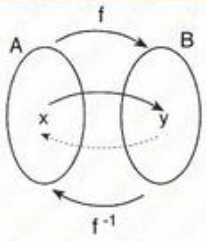
Yanıt / Answer **C**

$$\frac{3}{4} = \frac{a}{-8}$$
$$-24 = 4a$$
$$a = -6$$

TERS FONKSİYON / INVERSE FUNCTION

$f : A \to B$;

$f^{-1} : B \to A$



$f(x) = y \Rightarrow f^{-1}(y) = x$

ÖNEMLİ / IMPORTANT

Bir fonksiyonun tersini bulmak için,

- x yalnız bırakılır.
- y yerine x, x yerine $f^{-1}(x)$ yazılır.

To the find the inverse of a function,

- isolate x.
- write x instead of y and $f^{-1}(x)$ instead of x.

Özellik / Property

★ $(f^{-1})^{-1}(x) = f(x)$

★ $f(x) = I(x) \Rightarrow f^{-1}(x) = f(x)$

ÖNEMLİ / IMPORTANT

- f^{-1} fonksiyonunun grafiği, f fonksiyonun $y = x$ doğrusuna göre simetriğidir.
- The graph of f^{-1} is a reflection of the graph of f in the line $y = x$.

Örnek / Example 6

$y = f(x) = 2x - 5 \Rightarrow f^{-1}(x) = ?$

A) $\frac{x+5}{2}$ B) $\frac{x-5}{2}$ C) $-\frac{x+5}{2}$ D) $\frac{x+4}{2}$ E) $-\frac{x+4}{2}$

Çözüm / Solution

Yanıt / Answer **A**

$y = 2x - 5$

$y + 5 = 2x$

$x = \frac{y+5}{2} \Rightarrow f^{-1}(x) = \frac{x+5}{2}$

ÖNEMLİ / IMPORTANT

- $f(x) = y \Rightarrow f^{-1}(y) = x$

Örnek / Example 7

$y = f(x) = \frac{2x-4}{x+4} \Rightarrow f^{-1}(x) = ?$

A) $\frac{4x+4}{2+x}$ B) $\frac{4x+4}{2-x}$ C) $-\frac{4x+4}{2-x}$ D) $\frac{4x+4}{x-3}$ E) $-\frac{4x+4}{x+2}$

Çözüm / Solution

Yanıt / Answer **B**

$y = \frac{2x-4}{x+4} \Rightarrow xy + 4y = 2x - 4$

$4y + 4 = 2x - xy$

$4y + 4 = x(2 - y)$

$x = \frac{4y+4}{2-y}$

$\Rightarrow f^{-1}(x) = \frac{4x+4}{2-x}$

ÖNEMLİ / IMPORTANT

- Bir fonksiyonun tersinin tersi, kendisidir.
- The inverse of the inverse of a function is that function itself.

$(f^{-1})^{-1} = f$

Örnek / Example 8

$f\left(\frac{x-2}{2x}\right)=2x+5 \Rightarrow f^{-1}(3)=?$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

Çözüm / Solution

Yanıt / Answer **C**

$f\left(\frac{x-2}{2x}\right)=2x+5$

$\frac{x-2}{2x}=f^{-1}(2x+5) \quad \left[\begin{matrix} y=f(x)\Leftrightarrow f^{-1}(y)=x \\ 2x+5=3 \Rightarrow \quad x=-1 \end{matrix}\right]$

$x=-1 \Rightarrow \frac{-1-2}{2\cdot(-1)}=f^{-1}(-2+5)$

$\frac{-3}{-2}=f^{-1}(3)$

$f^{-1}(3)=\frac{3}{2}$

Örnek / Example 9

$f(x)=x^2-2x \Rightarrow f^{-1}(x)=?$

A) $\sqrt{x-1}+1$ B) $\sqrt{x+1}-1$ C) $\sqrt{x-1}-1$
D) $\sqrt{x+1}+1$ E) $\sqrt{x-2}+2$

Çözüm / Solution

Yanıt / Answer **D**

$y=x^2-2x$
$y=(x-1)^2-1$
$y+1=(x-1)^2$
$x-1=\pm\sqrt{y+1}$
$x=\pm\sqrt{y+1}+1 \Rightarrow f^{-1}(x)=\sqrt{x+1}+1$

PEKİŞTİRME / REINFORCE

✓ $f(x+1)=3x-4 \Rightarrow f^{-1}(1)=?$
..... $\left[\frac{8}{3}\right]$

✓ $f(2x-4)=3x+6 \Rightarrow f^{-1}(0)=?$
..... [–8]

✓ $f(3x+1)=2x-4 \Rightarrow f^{-1}(6)=?$
..... [16]

✓ $f(x-1)=3x+3 \Rightarrow f(x)=?$
..... [3x + 6]

✓ $f(3x-4)=6x-5 \Rightarrow f(x)=?$
..... [2x + 3]

ÖNEMLİ / IMPORTANT

• İkinci dereceden bir fonksiyonun tersi alınacaksa fonksiyon önce tam kare ifadeye çevrilir.

• When inversing seeond degree function, we need to convert the function to a compete square first.

ÖNEMLİ / IMPORTANT

• Üçüncü dereceden bir fonlsiyonun tersi alınacaksa fonksiyon önce tam küp ifadeye çevrilir.

• When inversing a third degree function we need to convert the function to a complete cube first.

PEKİŞTİRME / REINFORCE

✓ $f(x)=x^2-4 \Rightarrow f^{-1}(x)=?$
..... $[\sqrt{x+4}]$

✓ $f(x)=x^2+2x \Rightarrow f^{-1}(x)=?$
..... $[\sqrt{x+1}-1]$

✓ $f(x)=x^3-3x^2+3x \Rightarrow f^{-1}(x)=?$
..... $[\sqrt[3]{x-1}+1]$

BİR FONKSİYONUN TERSİNİ BULMADA PRATİK FORMÜLLER
PRACTICAL FORMULAS to FIND the INVERSE of a FUNCTION

1. $f(x)=ax+b \Rightarrow f^{-1}(x)=\frac{x-b}{a}$

2. $f(x)=\frac{ax+b}{cx+d} \Rightarrow f^{-1}(x)=\frac{-dx+b}{cx-a}$

3. $f(x)=a^x \Rightarrow f^{-1}(x)=\log_a x$

PEKİŞTİRME / REINFORCE

✓ $f(x)=2x-1 \Rightarrow f^{-1}(x)=?$

..... $\left[\frac{x+1}{2}\right]$

✓ $f(x)=\frac{3x-4}{2x+1} \Rightarrow f^{-1}(x)=?$

..... $\left[\frac{-x-4}{2x-3}\right]$

✓ $f(x)=\frac{x}{2x-1} \Rightarrow f^{-1}(x)=?$

..... $\left[\frac{x}{2x-1}\right]$

✓ $f(x)=\frac{3x+4}{2x} \Rightarrow f^{-1}(x)=?$

..... $\left[\frac{4}{2x-3}\right]$

✓ $f(x)=\frac{4}{3x-1} \Rightarrow f^{-1}(x)=?$

..... $\left[\frac{x+4}{3x}\right]$

Örnek / Example 10

$f^{-1}(2x+4)=x \Rightarrow f\left(\frac{2}{x}-1\right)=?$

A) $\frac{2}{x}-2$ B) $\frac{2}{x}+2$ C) $\frac{4}{x}+2$ D) $\frac{4}{x}+4$ E) $\frac{8}{x}+2$

Çözüm / Solution

Yanıt / Answer **C**

$f^{-1}(2x+4)=x \Rightarrow f(x)=2x+4$

$f\left(\frac{2}{x}-1\right)=2\cdot\left(\frac{2}{x}-1\right)+4$

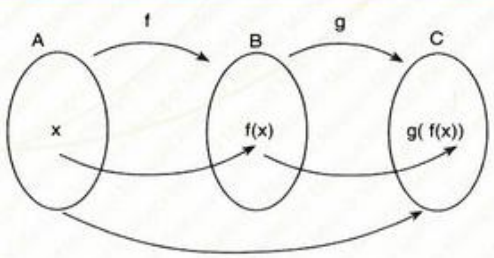
$f\left(\frac{2}{x}-1\right)=\frac{4}{x}-2+4$

$f\left(\frac{2}{x}-1\right)=\frac{4}{x}+2$

ÖNEMLİ / IMPORTANT

- (fog)(x): f fonksiyonundaki x'lerin yerine g(x) konulur.
- (gof)(x): g fonksiyonundaki x'lerin yerine f(x) konulur.
- fog(x): Insert g(x) instead of x in f(x).
- gof(x): Insert f(x) instead of x in g(x).

BİLEŞKE FONKSİYON / COMPOSITE FUNCTION



(gof) (x)

(gof) (x) = g [f (x)]

Örnek / Example 11

$f(x) = 2x - 4$, $g(x) = 3x-1 \Rightarrow$ **(fog)(x) = ?**

A) $6x - 4$ B) $6x - 5$ C) $6x - 6$

D) $6x - 8$ E) $6x - 10$

Çözüm / Solution

Yanıt / Answer **C**

$(fog)(x) = f(g(x))$

$= f(3x - 1)$

$= 2\cdot(3x - 1) - 4$

$= 6x - 2 - 4$

$= 6x - 6$

Özellik / Property

★ $fog \neq gof$

★ $(fog)oh = fo(goh)$

★ $(fog)^{-1}(x) = (g^{-1}of^{-1})(x)$

★ $(f^{-1}of)(x) = (fof^{-1})(x) = I(x)$

★ $(foI)(x) = (Iof)(x) = f(x)$

★ $(fog)(x) = h(x) \Rightarrow g(x) = f^{-1}(h(x))$

★ $(fog)(x) = h(x) \Rightarrow f(x) = h(g^{-1}(x))$

Örnek / Example 12

$f(x) = 3x - 1$,

$(gof)(x) = 9x + 5$

$\Rightarrow$ **g(x) = ?**

A) $3x + 4$ B) $3x + 6$ C) $3x + 8$

D) $3x - 6$ E) $3x - 8$

Çözüm / Solution

Yanıt / Answer **C**

$f(x) = 3x - 1$, $(gof)(x) = 9x + 5$

$(gof)(x) = 9x + 5 \Rightarrow g(x) = 9 \cdot f^{-1}(x) + 5$

$$g(x) = 9 \cdot \frac{x+1}{3} + 5$$

$= 3(x + 1) + 5$

$= 3x + 8$

PEKİŞTİRME / REINFORCE

$f(x) = 3x - 4$, $g(x) = 4x + 6$

✓ **(fog)(x) = ?**

..... **[12x + 14]**

✓ **(gof)(x) = ?**

..... **[12x – 10]**

ÖNEMLİ / IMPORTANT

• Bir fonksiyonun tersi kendisine eşit ise bu fonksiyon birim fonksiyon olmak zorunda değildir.

• If a function's reverse equels the function itself, the function doesnt need to be identity function.

Örnek / Example 13

$f(x)=\frac{2x+2}{3x-a}$

$(fofofofof)(x) = f(x) \Rightarrow$ **a = ?**

A) –2 B) –1 C) 0 D) 1 E) 2

Çözüm / Solution Yanıt / Answer **E**

$(fofofofof)(x) = f(x) \quad f(x) = f^{-1}(x)$

$\frac{2x+2}{3x-a}=\frac{ax+2}{3x-2}$

$\Rightarrow a = 2$

Örnek / Example 14

$f(x)=\begin{cases}2x+5 \ ; \ x<-1\\ 2-x^2 \ ; \ x\geq 1\end{cases}$

$g(x) = 3 - x \Rightarrow$ **(fog)(5) = ?**

A) –2 B) –1 C) 0 D) 1 E) 2

Çözüm / Solution Yanıt / Answer **D**

$(fog)(5) = f(g(5)) = f(3-5)$

$= 2 \cdot (-2) + 5$

$= 1$

PERMÜTASYON FONKSİYON / PERMUTATION FUNCTION

Tanım ve değer kümeleri aynı olan fonksiyondur.

A function that its range set is same as its domain set.

$$f=\begin{pmatrix}a & b & c & d\\ d & c & b & a\end{pmatrix} \Rightarrow \begin{cases}f(a)=d, \ f^{-1}(d)=a\\ f(b)=c, \ f^{-1}(c)=b\\ f(c)=b, \ f^{-1}(b)=c\\ f(d)=a, \ f^{-1}(a)=d\end{cases}$$

Örnek / Example 15

$f=\begin{pmatrix}1&2&3&4\\4&3&1&2\end{pmatrix} \Rightarrow f^{-1}=?$

A) $f=\begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix}$ B) $f=\begin{pmatrix}1&2&3&4\\4&1&3&2\end{pmatrix}$ C) $f=\begin{pmatrix}1&2&3&4\\4&2&1&3\end{pmatrix}$

D) $f=\begin{pmatrix}1&2&3&4\\3&4&2&1\end{pmatrix}$ E) $f=\begin{pmatrix}1&2&3&4\\3&2&4&1\end{pmatrix}$

Çözüm / Solution

Yanıt / Answer **D**

$$f=\begin{pmatrix}1&2&3&4\\4&3&1&2\end{pmatrix} \Rightarrow \begin{cases}f(1)=4 \Rightarrow f^{-1}(4)=1\\f(2)=3 \Rightarrow f^{-1}(3)=2\\f(3)=1 \Rightarrow f^{-1}(1)=3\\f(4)=2 \Rightarrow f^{-1}(2)=4\end{cases} \Rightarrow f^{-1}=\begin{pmatrix}1&2&3&4\\3&4&2&1\end{pmatrix}$$

Örnek / Example 16

$f=\begin{pmatrix}a&b&c&d\\b&c&d&a\end{pmatrix}, g=\begin{pmatrix}a&b&c&d\\c&b&d&a\end{pmatrix} \Rightarrow fog=?$

A) $f=\begin{pmatrix}a&b&c&d\\b&c&d&a\end{pmatrix}$ B) $f=\begin{pmatrix}a&b&c&d\\d&c&a&b\end{pmatrix}$ C) $f=\begin{pmatrix}a&b&c&d\\d&a&c&b\end{pmatrix}$

D) $f=\begin{pmatrix}a&b&c&d\\c&d&a&b\end{pmatrix}$ E) $f=\begin{pmatrix}a&b&c&d\\b&c&a&d\end{pmatrix}$

Çözüm / Solution

Yanıt / Answer **B**

$$fog=\underbrace{\begin{pmatrix}a&b&c&d\\b&c&d&a\end{pmatrix}}_{f} o \underbrace{\begin{pmatrix}a&b&c&d\\c&b&d&a\end{pmatrix}}_{g}=\begin{pmatrix}a&b&c&d\\d&c&a&b\end{pmatrix}$$

ÖNEMLİ / IMPORTANT

- $\begin{array}{ccccc}\underline{g}&\underline{f}&&\underline{fog(x)}\\ a\to c & c\to d & \Rightarrow & a\to d\\ b\to b & b\to c & \Rightarrow & b\to c\\ c\to d & d\to a & \Rightarrow & c\to a\\ d\to a & a\to b & \Rightarrow & d\to b\end{array}$

DOĞRUSAL FONKSİYONU / LINEAR FUNCTION

a ve b sabit sayılar olmak üzere, y = ax + b şeklindeki fonksiyonlardır.

Provided that a and b are constants, y = ax + b is a linear function.

Örnek / Example 17

$f(x) + f(x + 1) = 2x + 4 \Rightarrow f\left(\frac{1}{2}\right) = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution

Yanıt / Answer **B**

$f(x) + f(x + 1) = 2x + 4$

$f(x) = ax + b$

$ax + b + a\cdot(x + 1) + b = 2x + 4$

$2ax + 2b + a = 2x + 4$

$a = 1, \quad b = \frac{3}{2}$

$f(x) = x + \frac{3}{2} \Rightarrow f\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$

Örnek / Example 18

$f(x) + 2 \cdot f\left(\frac{1}{x}\right) = 2x + 4 \Rightarrow f(2) = ?$

A) 1 B) 0 C) −1 D) $\frac{2}{3}$ E) $\frac{1}{3}$

Çözüm / Solution

Yanıt / Answer **D**

$x = 2 \Rightarrow f(2) + 2f\left(\frac{1}{2}\right) = 8$

$x = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) + 2f(2) = 5$ (−2 ile çarpılır)

$f(2) - 4f(2) = 8 - 10$

$-3f(2) = -2$

$f(2) = \frac{2}{3}$

PEKİŞTİRME / REINFORCE

✓ $2f(x) + f(-x) = 2x + 3 \Rightarrow f(1) = ?$

..... [3]

✓ $f(x + y) = f(x) \cdot f(y)$,

$f(2) = 3 \Rightarrow f(4) = ?$

..... [9]

✓ $f(x\cdot y) = f(x) + f(y)$,

$f(3) = 2 \Rightarrow f(27) = ?$

..... [6]

Örnek / Example 19

$f(x) = ax + b$

$f(1) = 8,\ f(2) = f(-1) + 6$

$\Rightarrow$ **$f(-2) = ?$**

A) -2 B) -1 C) 0 D) 1 E) 2

Çözüm / Solution

Yanıt / Answer **E**

$f(x) = ax + b$

$f(2) = f(-1) + 6 \Rightarrow 2a + b = -a + b + 6$

$3a = 6$

$a = 2$

$f(1) = 8 \Rightarrow a + b = 8$

$2 + b = 8$

$b = 6$

$\Rightarrow f(x) = 2x + 6$

$f(-2) = 2\cdot(-2) + 6 = 2$

Örnek / Example 20

$f(x) = ax + b,$

$f(x + 1) - f(x) = -1,$

$f(1) = 3 \Rightarrow$ **$f(-1) = ?$**

A) -1 B) 0 C) 2 D) 3 E) 5

Çözüm / Solution

Yanıt / Answer **E**

$f(x) = ax + b,$

$f(x + 1) - f(x) = a\cdot(x + 1) + b - ax - b$

$= ax + a + b - ax - b$

$-1 = a$

$f(1) = 3 \Rightarrow a + b = 3$

$b = 4$

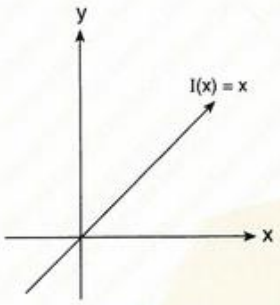
$f(x) = -x + 4$

$f(-1) = 5$

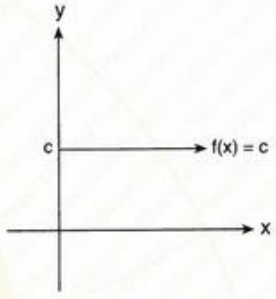
FONKSİYON GRAFİKLERİ / GRAPHS of FUNCTIONS

Bir fonksiyonun tanım kümesinin elemanlarının x ekseninden görüntü kümesinin elemanlarının y ekseninden yazılarak oluşturulan noktaların dik izdüşümlerinin kesişim noktaları birleştirilmesiyle oluşan şekil fonksiyon grafiği denir.

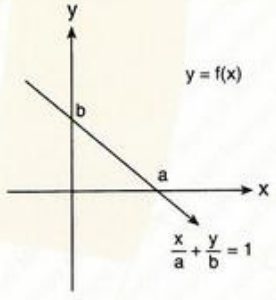
The figure formed by combining cross points of orthographic projeetions of the element of domain of a function written in x axis, with the elements of range of the same function written in y axis is called the graph of the function.



Birim fonksiyon grafiği / Identity function graph



Sabit fonksiyon grafiği / Constant function graph

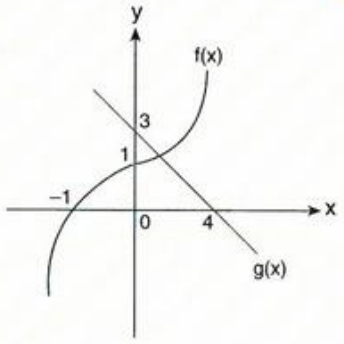


Doğrusal fonksiyon grafiği / Linear function graph

Örnek / Example 21

(fog)(4) + (gof)(–1) = ?

A) 1
B) 2
C) 4
D) 6
E) 8



Çözüm / Solution

Yanıt / Answer **C**

(–1, 0) ⇒ f(–1) = 0, (0, 1) ⇒ f(0) = 1,
(4, 0) ⇒ g(4) = 0, (0, 3) ⇒ g(0) = 3
⇒ (fog) (4) = f(g(4)) = f(0) = 1 , (gof) (–1) = g(f(–1)) = g(0) = 3
(fog)(4) + (gof)(–1) = 1 + 3 = 4

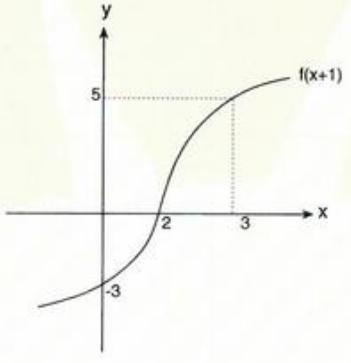
ÖNEMLİ / IMPORTANT

- (a, b) noktası f(x) in grafiği üzerinde bir nokta ise, f(a) = b dir.
- If the point (a,b) is on the graph of f(x), f(a) = b.

Örnek / Example 22

$$\frac{f^{-1}(5)+f(1)}{f^{-1}(-3)+f(3)} = ?$$

A) –1
B) 0
C) 1
D) 2
E) 3



Çözüm / Solution

Yanıt / Answer **C**

$f^{-1}(5) = 3 + 1 = 4$ [$f(x) = 5 \Rightarrow f^{-1}(5) = x$]

f(1): x + 1 = 1

x = 0 ⇒ f(1) = –3

$f^{-1}(-3) = 0 + 1 = 1$

f(3): x + 1 = 3

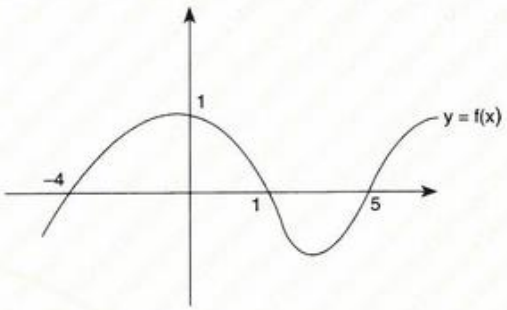
x = 2 ⇒ f(3) = 0

$$\frac{f^{-1}(5)+f(1)}{f^{-1}(-3)+f(3)} = \frac{4-3}{1+0} = 1$$

Örnek / Example 23

(fofof)(–4) = ?

A) –3
B) –2
C) 0
D) 2
E) 3



Çözüm / Solution

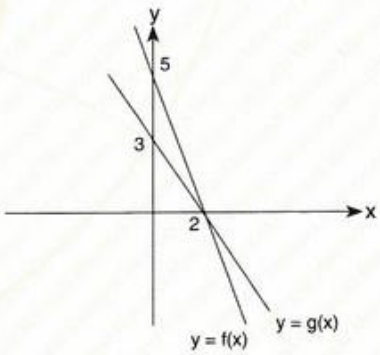
Yanıt / Answer **C**

(fofof)(–4) = fofof(–4)
= fof(0)
= f(1) = 0

Örnek / Example 24

(fog)(–2) = ?

A) –10
B) –6
C) 0
D) 6
E) 10



Çözüm / Solution

Yanıt / Answer **A**

$\frac{x}{2}+\frac{y}{3}=1$
$3x+2y=6$
$g(x)=\frac{6-3x}{2}$

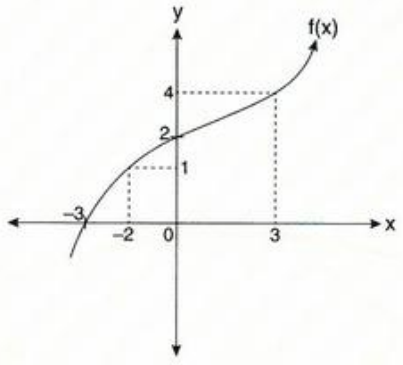
$g(-2)=6$

$\frac{x}{2}+\frac{y}{5}=1$
$5x+2y=10$
$f(x)=\frac{10-5x}{2}$

$fog(-2)=f(g(-2))$
$f(6)=\frac{-20}{2}=-10$

Örnek / Example 25

$f(a - f(3)) = 3 + f(-2) \Rightarrow a = ?$



A) 3

B) 4

C) 5

D) 6

E) 7

Çözüm / Solution

Yanıt / Answer **E**

$\left.\begin{matrix} f(-2) = 1 \\ f(3) = 4 \end{matrix}\right\} \Rightarrow f(a - 4) = 3 + 1$

$f(a - 4) = 4$

$f(a - 4) = f(3)$

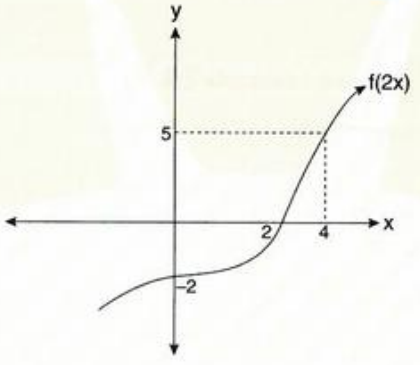
$\Rightarrow a - 4 = 3$

$a = 7$

Örnek / Example 26

$y = f(2x)$

$g(x) = \frac{3x - 2}{5} \Rightarrow \frac{(gof)(0)}{(f^{-1}og)(9)} = ?$



A) $-\frac{1}{5}$

B) 0

C) $\frac{1}{5}$

D) $\frac{1}{3}$

E) $\frac{1}{2}$

Çözüm / Solution

Yanıt / Answer **C**

$f(0) = -2 \Rightarrow (gof)(0 = g(-2) = -\frac{8}{5}$

$g(9) = 5 \Rightarrow (f^{-1}og)(9) = f^{-1}(5)$

$f(2x) = 5 \Rightarrow f(8) = 5$

$f^{-1}(5) = 8$

$\frac{(gof)(0)}{(f^{-1}og)(9)} = -\frac{8}{\frac{5}{8}} = -\frac{1}{5}$

1. $f(x) = x^2 - 3x + 5 \Rightarrow \mathbf{f(1) = ?}$

A) -1 B) 0 C) 1

D) 2 E) 3

2. $f : R \to R$,

$f(x) = 3 - x \Rightarrow \mathbf{f^{-1}(x) = ?}$

A) $3 - x$ B) $3 + x$ C) $-x - 3$

D) $\frac{4x+1}{3}$ E) $\frac{4x-1}{3}$

3. $f : R \to R$, $f(x) = \frac{3x+1}{4} \Rightarrow \mathbf{f^{-1}(x) = ?}$

A) $4x - 3$ B) $\frac{3x-1}{4}$ C) $\frac{3x+1}{4}$

D) $\frac{4x+1}{3}$ E) $\frac{4x-1}{3}$

4. $f : R - \left\{-\frac{4}{5}\right\} \to R - \left\{\frac{2}{5}\right\}$,

$f(x) = \frac{2x-3}{5x+4} \Rightarrow \mathbf{f^{-1}(x) = ?}$

A) $\frac{-4x-3}{5x+2}$ B) $\frac{-4x+3}{5x-2}$ C) $\frac{4x+3}{2-5x}$

D) $\frac{4x+3}{5x+2}$ E) $\frac{4x-3}{2+5x}$

5. $f(3x + 5) = 6x - 1 \Rightarrow \mathbf{f(x) = ?}$

A) $2x - 11$ B) $2x - 12$ C) $2x + 2$

D) $2x + 5$ E) $2x + 10$

6. $f\left(\frac{2x-4}{3x}\right) = 2x - 1 \Rightarrow \mathbf{f(2) = ?}$

A) -1 B) -2 C) -3

D) -4 E) 5

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7. $f: R \setminus \{3\} \to R \setminus \{2\}$

$f(x) = \frac{2x+m}{x-3}$, $f^{-1}(4) = 1 \Rightarrow$ **m = ?**

A) 4 B) 6 C) 8

D) −10 E) − 8

8. f,g: R → R,

(fog)(x) = 6x + 3,

g(x) = 3x−1 ⇒ **f(x) = ?**

A) 2x + 1 B) 2x + 3 C) 2x + 5

D) 2x + 6 E) 2x + 8

9. f, g: R → R,

$f(x) = \frac{3x+4}{7}$, $g(x) = \frac{7x-4}{3} \Rightarrow$ **(fog)(x) = ?**

A) x B) x + 1 C) x − 1

D) x + 2 E) 1

10. $\frac{f(x)}{2 \cdot f(x) - x} = \frac{1}{1-x} \Rightarrow f^{-1}(x) = ?$

A) $-\frac{1}{x+1}$ B) $\frac{1}{x+1}$ C) $\frac{x}{x-1}$

D) $\frac{-x}{x-1}$ E) $\frac{-x}{x+1}$

11. $f: R - \left\{-\frac{2}{3}\right\} \to R - \left\{\frac{2}{3}\right\}$

$f(x) = \frac{2x+1}{3x+2}$, $(fof)(x) = \frac{ax+4}{bx+7} \Rightarrow$ **a + b = ?**

A) 12 B) 13 C) 15

D) 17 E) 19

12. f, g, h: R → R

f (x) = 2x ,

g(x) = 3x + 1,

$h(x) = \frac{x}{3} \Rightarrow$ **(gofoh)(1) = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
E	A	E	C	A	C	D	C	A	D	E	C

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1. $f(x) = x^2 - 3x + 5$

$f(1) = 1^2 - 3 \cdot 1 + 5$

$= 3$

Yanıt / The Answer is E

2. $f(x) = 3 - x \Rightarrow f^{-1}(x) = 3 - x$

Yanıt / The Answer is A

3. $f(x) = \frac{3x+1}{4} \Rightarrow f^{-1}(x) = \frac{4x-1}{3}$

Yanıt / The Answer is E

4. $f(x) = \frac{2x-3}{5x+4} \Rightarrow f^{-1}(x) = \frac{-4x-3}{5x-2}$

$= \frac{4x+3}{2-5x}$

Yanıt / The Answer is C

5. $f(3x + 5) = 6x - 1$

$f(x) = 6 \cdot \left(\frac{x-5}{3}\right) - 1$

$= 2(x - 5) - 1$

$= 2x - 11$

Yanıt / The Answer is A

6. $\frac{2x-4}{3x} = 2$

$2x - 4 = 6x$

$4x = -4$

$x = -1$

$\Rightarrow 2x - 1 = 2 \cdot (-1) - 1 = -3$

Yanıt / The Answer is C

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7.

$f(x)=\frac{2x+m}{x-3}$

$f^{-1}(4)=1 \Rightarrow f(1)=4$

$\frac{2\cdot 1+m}{1-3}=4$

$2+m=-8$

$m=-10$

Yanıt / The Answer is D

8. $g(x)=3x-1$

$(fog)(x)=f[g(x)]=6x+3$

$f(3x-1)=6x+3$

$f(x)=6\left(\frac{x+1}{3}\right)+3$

$=2x+5$

Yanıt / The Answer is C

9. $f(x)=\frac{3x+4}{7}$, $g(x)=\frac{7x-4}{3}$

$fog(x)=f[g(x)]=\frac{3\cdot\left(\frac{7x-4}{3}\right)+4}{7}$

$=\frac{7x-4+4}{7}=x$

Yanıt / The Answer is A

10. $\frac{f(x)}{2\cdot f(x)-x}=\frac{1}{1-x}$

$(1-x)\cdot f(x)=2\cdot f(x)-x$

$f(x)-x\cdot f(x)-2f(x)=-x$

$f(x)\cdot[-x-1]=-x$

$f(x)=\frac{x}{x+1} \Rightarrow f^{-1}(x)=\frac{-x}{x-1}$

Yanıt / The Answer is D

11. $f(x)=\frac{2x+1}{3x+2} \Rightarrow (fof)(x)=\frac{2\cdot\left(\frac{2x+1}{3x+2}\right)+1}{3\cdot\left(\frac{2x+1}{3x+2}\right)+2}$

$=\frac{7x+4}{12x+7}$

$=\frac{ax+4}{bx+7}$

$a=7,\ b=12 \Rightarrow a+b=7+12=19$

Yanıt / The Answer is E

12. $f(x)=2x,\ g(x)=3x+1,\ h(x)=\frac{x}{3}$

$(gofoh)(1)=gof(h(1))$

$=gof\left(\frac{1}{3}\right)$

$=g\left(\frac{2}{3}\right)$

$=3\cdot\frac{2}{3}+1=3$

Yanıt / The Answer is C

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1. $f: R \to R,\ f(x) = 5^{x+3}$

ise / if $f(a+b-1) = ?$

A) $25f(a+b)$ B) $5f(a+b)$ C) $f(a+b)$

D) $\frac{f(a+b)}{5}$ E) $\frac{f(a+b)}{25}$

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2020]

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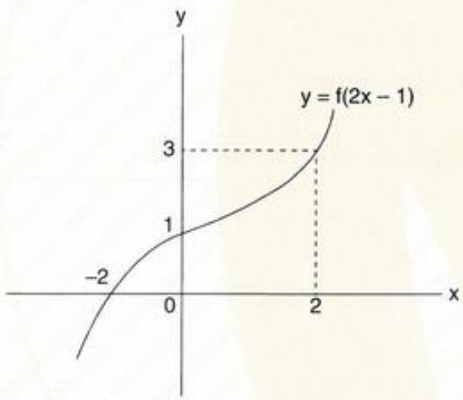
2. $2\cdot f(x) + f(-x) = 3x^2 - x$

$f(2) = ?$

A) -2 B) -1 C) 0 D) 1 E) 2

[İSTANBUL AYDIN ÜNİVERSİTESİ – YÖS 2019]

3.



$$\frac{f^{-1}(3)+f(3)}{f(-5)+f^{-1}(1)} = ?$$

A) -9 B) -6 C) -3 D) -1 E) 0

[CUMHURİYET ÜNİVERSİTESİ – YÖS 2019]

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4. $f(x) = 2x + m \Rightarrow$ **$(fofof)(x) = ?$**

A) $6x + m$ B) $8x + m$ C) $6x + 6m$

D) $8x + 6m$ E) $8x + 7m$

[İNÖNÜ ÜNİVERSİTESİ – YÖS 2018]

5. $f: R \to R$

$f(x) = x^2 - 4 \Rightarrow$ **$f^{-1}(x) = ?$**

A) $\sqrt{x+4}$ B) $\sqrt{x-4}$ C) $\sqrt{x} + 4$

D) $\sqrt{x} - 4$ E) f fonksiyonun tersi yoktur

f^{-1} does not exist

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2018]

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6. $f(x^2 - 6x + 2) = \frac{x-1}{x+3} \Rightarrow$ **$f(-7) = ?$**

A) 2 B) $\frac{4}{5}$ C) $\frac{1}{3}$

D) 0 E) $-\frac{7}{2}$

[ANADOLU ÜNİVERSİTESİ – YÖS 2017]

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7. $f(x-1) = \frac{2}{3} + f(x),\ f(0) = 36 - f(21) \Rightarrow$ **$f(0) = ?$**

A) 20 B) 25 C) 30

D) 35 E) 40

[İSTANBUL ÜNİVERSİTESİ – YÖS 2017]

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8. $f: R \to R$

$f(x) = 2^{x+3} \Rightarrow$ **$f(m+n-2) = ?$**

A) $\frac{f(m+n)}{2}$

B) $\frac{f(m)\cdot(n)}{32}$

C) $f(m) \cdot f(n)$

D) $2^{f(m)-f(n)}$

E) $[f(m)]^{f(n)-1}$

[KARADENİZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

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9. $f(x) = 5^{2x-1}$

$\Rightarrow$ **$(fof^{-1})(x) + (f^{-1}of)(x) = ?$**

A) x B) $2x$ C) 5^x

D) 5^{x-1} E) 1

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2015]

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10. $f(x) = x^2 + 1$,

$g(x) = 2^{x+1}$

$\Rightarrow$ **$(fof^{-1}og^{-1})(16) = ?$**

A) 1 B) 2 C) 3

D) 4 E) 5

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2014]

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11. $f : N \to R$

$$\left.\begin{array}{l} f(2x+1) = f(2x-1) + x \\ f(7) = 3 \end{array}\right\} \Rightarrow \mathbf{f(1) = ?}$$

A) -3 B) -2 C) 2

D) 1 E) 3

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2014]

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12. $f(x) = \dfrac{2x}{x-1}$ $\Rightarrow$ **$f(x-1) = ?$**

A) $\dfrac{f(x)-4}{f(x)-3}$ B) $\dfrac{-f(x)+2}{f(x)-4}$ C) $\dfrac{4}{4-f(x)}$

D) $\dfrac{f(x)+4}{2 \cdot f(x)+2}$ E) $\dfrac{-f(x)+2}{f(x)-2}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2014]

13. $f(x) = mx^2 + 6x - n$,

$f(x_1) = f(x_2) = 0$

$x_1 = x_2 \Rightarrow$ **$m \cdot n = ?$**

A) -11 B) -10 C) -9

D) -8 E) -7

[İSTANBUL ÜNİVERSİTESİ – YÖS 2014]

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14. $f(2x+a) = \dfrac{x+6}{x+4}$

$f(3) = 3 \Rightarrow$ **$a = ?$**

A) 9 B) 5 C) 3

D) -1 E) -3

[İSTANBUL ÜNİVERSİTESİ – YÖS 2014]

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15. $f(x) = \dfrac{x+1}{3}$ ve (and) $g(x) = \dfrac{1}{x}$ $\Rightarrow$ **$(gof)(x) = ?$**

A) $\dfrac{1+x}{3x}$ B) $\dfrac{3x}{x+1}$ C) $\dfrac{3}{x+1}$

D) $\dfrac{3x+1}{x}$ E) $\dfrac{3x+1}{x+1}$

[AKDENİZ ÜNİVERSİTESİ – YÖS 2014]

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16. $f(x^2 + 2x + 4) = 3x^2 + 6x + 7 \Rightarrow$ **$f^{-1}(4) = ?$**

A) -5 B) $-\dfrac{5}{3}$ C) -1

D) 3 E) 4

[İSTANBUL ÜNİVERSİTESİ – YÖS 2013]

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	E	B	E	E	C	B	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	C	A	C	C	A	C	D

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1. $f(x) = 5^{x+3}$

$f(a+b-1) = 5^{a+b-1+3}$

$f(a+b-1) = 5^{a+b+2} = 5^{a+b}\cdot 25$

$f(a+b) = 5^{a+b+3} = 5^{a+b}\cdot 125$

$\Rightarrow f(a+b-1) = \dfrac{f(a+b)}{5}$

Yanıt / The Answer is D

---

2. $x = 2$

$\Rightarrow 2\cdot f(2) + f(-2) = 10 \ldots (1)$

$x = -2 \Rightarrow$

$2\cdot f(-2) + f(2) = 14 \ldots (2)$

(1) & (2)

$$\begin{array}{rr} & -2 / 2\cdot f(2) + f(-2) = 10 \\ + & 2\cdot f(-2) + f(2) = 14 \\ \hline & -3\cdot f(2) = -6 \\ & f(2) = 2 \end{array}$$

Yanıt / The Answer is E

---

3. $y = f(2x-1)$

$$\frac{f^{-1}(3)+f(3)}{f(-5)+f^{-1}(1)} = \frac{3+3}{0-1} = -6$$

Yanıt / The Answer is B

---

4. $f(x) = 2x + m$

$(fofof)(x) = 2[2(2x+m)+m] + m$

$= 2[4x+3] + m$

$= 8x + 7m$

Yanıt / The Answer is E

---

5. $f:R \to R$

$f(x) = x^2 - 4$

$y + 4 = x^2$

$\sqrt{y+4} = x$

$\sqrt{x+4} = f^{-1}(x)$

$0 \le x + 4$

$-4 \le x$

Görüntü kümesi R olduğundan, tersi fonksiyon değildir.

Since its range is R, f^{-1} does not exist.

Yanıt / The Answer is E

---

6. $x^2 - 6x + 2 = -7$

$x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$

$x = 3$

$f(3 - 6\cdot 3 + 2) = \dfrac{3-1}{3+3}$

$f(-7) = \dfrac{2}{6} = \dfrac{1}{3}$

Yanıt / The Answer is C

7.

$$\begin{array}{lll} & f(x-1) - f(x) = \frac{2}{3} \\ x = 1 & f(0) - f(1) = \frac{2}{3} \\ x = 2 & f(1) - f(2) = \frac{2}{3} \\ \vdots & \vdots \quad \vdots \quad \vdots & \vdots \\ x = 21 & f(20) - f(21) = \frac{2}{3} \\ & + \\ \hline & f(0) - f(21) = \frac{2}{3}\cdot 21 \end{array}$$

$f(0) - [36 - f(0)] = 14$

$2f(0) = 50$

$f(0) = 25$

Yanıt / The Answer is B

---

8. $f(m+n-2) = 2^{m+n-2+3} = 2^{m+n+1}$

$$\left.\begin{array}{l} f(m) = 2^{m+3} \\ f(n) = 2^{n+3} \end{array}\right\} \frac{f(m)\cdot f(n)}{32} = \frac{2^{m+3}\cdot 2^{n+3}}{2^5} = \frac{2^{m+n+6}}{2^5} = 2^{m+n+1}$$

Yanıt / The Answer is B

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9. $\underbrace{(fof^{-1})(x)}_{x}+\underbrace{(f^{-1}of)(x)}_{x}=x+x=2x$

Yanıt / The Answer is B

---

10. $\underbrace{fof^{-1}}_{i}\circ g^{-1}(16)=?$

$g^{-1}(16)=?$

$g(x)=2^{x+1}$

$g^{-1}(2^{x+1})=x \Rightarrow 2^{x+1}=16$

$x+1=4$

$x=3$

Yanıt / The Answer is C

---

11. $f(2x+1)-f(2x-1)=x$

$x=3 \Rightarrow f(7)-f(5)=3$

$x=2 \Rightarrow f(5)-f(3)=2$

$x=1 \Rightarrow f(3)-f(1)=1$

$+$

$f(7)-f(1)=6$

$3-6=f(1)$

$-3=f(1)$

Yanıt / The Answer is A

---

12. $f(x)=\frac{2x}{x-1} \Rightarrow f(x-1)=\frac{2x-2}{x-2}$

$x\cdot f(x)-f(x)=2x$

$x\cdot(f(x)-2)=f(x) \Rightarrow x=\frac{f(x)}{f(x)-2}$

$f(x-1)=\frac{2x-2}{x-2}$

$=\frac{2\cdot\left(\frac{f(x)}{f(x)-2}\right)-2}{\frac{f(x)}{f(x)-2}-2}$

$=\frac{2\cdot f(x)-2\cdot f(x)+4}{f(x)-2\cdot f(x)+4}=\frac{4}{4-f(x)}$

Yanıt / The Answer is C

---

13. $\Delta=b^2-4ac$

$0=36-4\cdot m\cdot(-n)$

$-4m\cdot n=36$

$mn=-9$

Yanıt / The Answer is C

---

14. $f(2x+a)=\frac{x+6}{x+4}$

$\frac{x+6}{x+4}=3 \Rightarrow x+6=3x+12$

$-6=2x$

$x=-3$

$2x+a=2\cdot(-3)+a=3 \Rightarrow a=9$

Yanıt / The Answer is A

---

15. $(gof)(x)=g\left(\frac{x+1}{3}\right)$

$=\frac{1}{\frac{x+1}{3}}$

$=\frac{3}{x+1}$

Yanıt / The Answer is C

---

16. $f(x^2+2x+4)=3(x^2+2x+4)-5$

$f(x)=3x-5$

$f^{-1}(x)=\frac{x+5}{3}$

$f^{-1}(4)=\frac{4+5}{3}=3$

Yanıt / The Answer is D

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1. $f(2x + 5) = 3x - 4 \Rightarrow \mathbf{f(3) = ?}$

A) -7 B) -6 C) 0 D) 6 E) 7

2. $f(2x + 3) = x^3 - 3x - 2$

$\Rightarrow \mathbf{f(-1) = ?}$

A) -4 B) -2 C) 0 D) 4 E) 8

3. $f(x) = 2x^2 - x + a$,

$f(0) + f(1) = 13 \Rightarrow \mathbf{a = ?}$

A) 3 B) 4 C) 5 D) 6 E) 7

4. $f(x) = -x + 2 \Rightarrow \mathbf{f(x - 1) = ?}$

A) $1 - f(x)$ B) $2 - f(x)$ C) $f(x)$ D) $f(x) + 1$ E) $f(x) + 2$

5. $f(x) = \frac{2x-7}{3x-6} \Rightarrow \mathbf{f^{-1}(x) = ?}$

A) $\frac{-2x+7}{3x+6}$ B) $\frac{3x-6}{2x-7}$ C) $\frac{6x-7}{3x-2}$ D) $\frac{x-7}{3x-2}$ E) $\frac{3x-7}{2x-6}$

6. $y = f(x)$

$2xy + 4 - x + 3y = 0$

$\Rightarrow \mathbf{f^{-1}(x) = ?}$

A) $\frac{x-4}{2x+3}$ B) $\frac{3x+4}{1-2x}$ C) $\frac{x-4}{3x}$ D) $\frac{2x-1}{3x+4}$ E) $\frac{2x+3}{x-4}$

7. $f(x) = \frac{3x-7}{x-2a}$

$f(x) = f^{-1}(x) \Rightarrow \mathbf{a = ?}$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) 1 D) $\frac{3}{2}$ E) $\frac{5}{2}$

8. $f(-x) = 2 \cdot f(x) + 6 \Rightarrow \mathbf{f(3) = ?}$

A) -6 B) -2 C) 0 D) 2 E) 6

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9. $(gof)(x+1) = 3x-2$

$g(x+4) = 2x-1$

$\Rightarrow$ **f(8) = ?**

A) 11 B) 14 C) 19

D) 21 E) 29

10. $f: R \to R$,

$f(2x+1) = 2ax+2$,

$f^{-1}(4) = 3 \Rightarrow$ **a = ?**

A) 0 B) 1 C) 2

D) 3 E) 4

11. $f(x) = ax+3$,

$f^{-1}(2) = 3$

$\Rightarrow$ **$f^{-1}(5)$ = ?**

A) –2 B) –4 C) –6

D) –8 E) –10

12. $f(x) = \frac{3x+b}{x+4}$, $f^{-1}(5) = 6 \Rightarrow$ **b = ?**

A) 12 B) 18 C) 24

D) 26 E) 32

13. $f(2^{x+1} - 3x - 1) = x+1$

$\Rightarrow$ **$f^{-1}(4)$ = ?**

A) 6 B) 7 C) 8

D) 9 E) 10

14. $f\left(\frac{x+1}{x-2}\right) = 4x-1 \Rightarrow$ **f(x) = ?**

A) $\frac{7x+5}{x-1}$ B) $\frac{8x-5}{x-2}$ C) $\frac{5x+3}{x-1}$

D) $\frac{x-1}{7x+5}$ E) $\frac{1}{x}$

15. $f(x) = 3x+1$, $g(x) = \frac{3x}{2}$, $h(x) = \frac{x-1}{4}$

$\Rightarrow$ **(gofoh)(2) = ?**

A) $\frac{11}{8}$ B) $\frac{21}{8}$ C) $\frac{11}{3}$

D) $\frac{21}{3}$ E) 7

16. $f: [-3, \infty) \to [-1, \infty)$

$f(x) = x^2 + 6x + 8 \Rightarrow$ **$f^{-1}(x)$ = ?**

A) $\sqrt{x+3}$ B) $\sqrt{x+3}-1$ C) $\sqrt{x+1}-1$

D) $\sqrt{x+1}-2$ E) $\sqrt{x+1}-3$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	A	D	D	C	B	D	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	B	C	E	A	A	B	E

1. $f(x)=\begin{cases}|x|, & -2\le x<2\\ 3x, & 2\le x<4\\ x^2-1, & 4\le x\end{cases} \Rightarrow \dfrac{f(-2)+f(4)+1}{f(2)} = ?$

A) 0 B) 1 C) 2
D) 3 E) 4

2. $f(2x+1) = 2x-1 \Rightarrow f(-2) = ?$

A) –5 B) –4 C) –3
D) –2 E) –1

3. $f(2x-1) = 4x+3 \Rightarrow f^{-1}(x) = ?$

A) $\frac{x-5}{2}$ B) $\frac{x+5}{2}$ C) $\frac{2}{x-5}$
D) $\frac{2}{x+5}$ E) $\frac{x-5}{x+5}$

4. $f(x) = x^2+x-1 \Rightarrow f(x-1) = ?$

A) x^2-x-1
B) x^2-1
C) x^2+x-1
D) x^2-x+1
E) x^2-3x-1

5. $a > 0$, $f(x) = x^2+2x+a$,
$f(a) = 4 \Rightarrow a = ?$

A) –4 B) 0 C) 1
D) 2 E) 3

6. $f(x) = 2x$, $g(x) = x-1$
$\Rightarrow (fog)(x) = ?$

A) x – 2 B) 2x – 1 C) 2x – 2
D) 2x – 4 E) 2x + 2

7. $(gof)(x) = -3\cdot f(x) \Rightarrow g(x) = ?$

A) –6x + 3 B) –2x + 1 C) –3x
D) –2x E) x

8. $g(x) = 6x-4$
$(f^{-1}og)^{-1}(x+1) = 3x+4$
$\Rightarrow f(x) = ?$

A) 6x + 4 B) 9x – 4 C) 18x + 2
D) 18x – 2 E) 18x – 4

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9. $f(1) = 5$,
$f(x + 1) = f(x) + 4$
$\Rightarrow$ **f (5) = ?**

A) 15 B) 16 C) 18
D) 21 E) 24

10. $f(x) = f(x - 1) + x$,
$f(0) = 7 \Rightarrow$ **f(20) = ?**

A) 212 B) 215 C) 216
D) 217 E) 220

11. $f(x) = \frac{x}{x+1} \Rightarrow$ **f(x − 1) = ?**

A) $\frac{f(x)+1}{2 \cdot f(x)}$ B) $\frac{f(x)+2}{2 \cdot f(x)}$ C) $\frac{2 \cdot f(x)+1}{2 \cdot f(x)}$
D) $\frac{2 \cdot f(x)+1}{f(x)}$ E) $\frac{2 \cdot f(x)-1}{f(x)}$

12. $f(x + 5) = x^2 - 2x$
$g(3 - x) = 2x + 7$
$\Rightarrow$ **(fog)(−1) = ?**

A) 5 B) 18 C) 46
D) 60 E) 80

13. $f(2x - 3) = \frac{1}{x} + 1 \Rightarrow$ **f(x) = ?**

A) $\frac{x+5}{x+3}$ B) $\frac{x-5}{x+3}$ C) $\frac{x+5}{x-3}$
D) $\frac{x-5}{x-3}$ E) $\frac{x+5}{x}$

14. $f(2x - 1) = 6x - 9 \Rightarrow$ **f(x) = ?**

A) 2x − 3 B) 3x − 6 C) 3x + 6
D) 6x − 3 E) 6x + 9

15. $g(x) = 5x + 4$
$(fog)(x) = (gof)(x) \Rightarrow$ **f(−1) = ?**

A) −1 B) 0 C) 1
D) 3 E) 5

16. $f : R - \{3\} \longrightarrow R - \{4\}$

$f(x) = \frac{(a-2)x-5}{2x-(b+1)} \Rightarrow$ **$f^{-1}(x)$ = ?**

A) $\frac{3x-5}{2x-6}$ B) $\frac{6x-5}{2x-8}$ C) $\frac{4x-5}{2x-3}$
D) $\frac{8x-5}{2x-3}$ E) $\frac{6x-5}{2x-10}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	B	A	A	C	C	C	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	D	E	E	A	B	A	B

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1. $f\left(\frac{2x-1}{x+3}\right) = 2x+1 \Rightarrow \mathbf{f(4) = ?}$

A) 18 B) 20 C) −12 D) −11 E) 0

2. $\left.\begin{array}{l} f(x) = x+2 \\ f^{-1}(A) = \{1,2,3\} \end{array}\right\} \Rightarrow \mathbf{A = ?}$

A) {0, 1, 2}
B) { − 1, 0, 1}
C) {3, 4, 5}
D) {−3, − 4, −5}
E) {−1, − 2, −3}

3. $f(3x+1) = 5x-3 \Rightarrow \mathbf{f^{-1}(2) = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

4. $f(x) = 4x^2 + 8x + 4 \Rightarrow \mathbf{f\left(\frac{x-2}{2}\right) = ?}$

A) x^2
B) $x^2 + 4$
C) $x^2 + 4x + 8$
D) $x^2 - 6x + 4$
E) $x^2 - 8$

5. $f(2x-1) = \frac{x^2-1}{3} \Rightarrow \mathbf{f(x) = ?}$

A) $\frac{x^2+x-2}{4}$ B) $\frac{x^2+2x-3}{6}$ C) $\frac{x^2+2x-3}{12}$

D) $\frac{x^2-x+3}{6}$ E) $\frac{x^2-2x+1}{4}$

6. $f : N^+ \to R,\ f(n) = \frac{1}{n} \cdot f(n+1),\ f(1) = 1 \Rightarrow \mathbf{f(17) = ?}$

A) 8! B) 10! C) 12! D) 14! E) 16!

7. $f(x) = 4x - 3$,
$(fog)(x) = 8x + 12$
$\Rightarrow \mathbf{g^{-1}(3) = ?}$

A) $-\frac{3}{8}$ B) $-\frac{1}{2}$ C) −1 D) 1 E) $\frac{1}{8}$

8. $f(x) = 3x - 8$
$g(x) = 2x + 3$
$(fog)(x) = 6x + k \Rightarrow \mathbf{k = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

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9. $f(3x+1) = 9x^2 - 3x \Rightarrow \mathbf{f(x^2) = ?}$

A) $x^4 + 3x^2 + 2$
B) $x^4 - 3x^2 - 2$
C) $x^4 - 3x^2 + 2$
D) $x^4 + 3x^2 - 2$
E) $x^4 + 3x^2 + 1$

10. $x^2y + yx - y - 3 = 0 \Rightarrow \mathbf{y = f(x) = ?}$

A) $\frac{3}{x^2+x-1}$
B) $\frac{-3}{x^2+x-2}$
C) $x^2 + x + 1$
D) $-x^2 + x - 1$
E) 1

11. $\frac{1}{x}+\frac{1}{y}=1 \Rightarrow \mathbf{y = f(x) = ?}$

A) $\frac{x}{x-1}$ B) $\frac{-x}{x+1}$ C) $\frac{x+1}{x-1}$ D) $\frac{x-1}{x}$ E) $\frac{x-1}{x+1}$

12. $f(5x-2) = 10x + 2$
$\Rightarrow \mathbf{f^{-1}(8) = ?}$

A) -1 B) 0 C) 1
D) 3 E) 5

13. $f(x) = \frac{3}{x-1} \Rightarrow \mathbf{f(x+1) = ?}$

A) $\frac{f(x)}{3+f(x)}$ B) $\frac{2f(x)}{2+f(x)}$ C) $\frac{3f(x)}{3+f(x)}$
D) $\frac{3f(x)}{3-f(x)}$ E) $\frac{4f(x)}{4-f(x)}$

14. $f: R \to R$
$f(x) = \frac{2x \cdot f(x-1)}{x+1}$, $f(1) = 1 \Rightarrow \mathbf{f(7) = ?}$

A) $\frac{33}{4}$ B) 32 C) 16
D) 40 E) $\frac{48}{7}$

15. $f(x) = \frac{3x+5}{x+1}$, $g(x) = \frac{2x-1}{3x+1}$
$(f^{-1}og)(x) = 1 \Rightarrow \mathbf{x = ?}$

A) $-\frac{1}{2}$ B) -1 C) 0
D) 1 E) $\frac{1}{2}$

16. $x \in Z$,
$f(x) = \frac{x}{4} + 2$, $g(x) = \frac{2}{x} - 1$,
$(f^{-1}og)(x) = 3x - 14 \Rightarrow \mathbf{x = ?}$

A) 5 B) 4 C) 3
D) 2 E) 1

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	C	D	A	C	E	A	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	A	A	C	C	C	A	D

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1. $f(x) = \frac{2x+a}{x+3}$

$f^{-1}(3) = 12 \Rightarrow$ **a = ?**

A) −21 B) −14 C) 9
D) 18 E) 21

2. $f\left(\frac{4x+1}{x}\right) = x^2+2x+3 \Rightarrow$ **f(3) = ?**

A) 2 B) 3 C) 4
D) 5 E) 6

3. $f\left(\frac{x-4}{2}\right) = 4x-6 \Rightarrow$ **$f^{-1}(8x+3)$ = ?**

A) $\frac{7x-8}{7}$ B) $\frac{8x-7}{8}$ C) $\frac{8x-8}{7}$
D) $\frac{7x-7}{8}$ E) $7x-\frac{1}{8}$

4. $f(x^2+x) = 5x^2+5x-11 \Rightarrow$ **$f^{-1}(4)$ = ?**

A) −3 B) −2 C) 2
D) 3 E) 4

5. $f\left(\frac{2x^2-2}{x-1}\right) = \frac{x-2}{x+3} \Rightarrow$ **$f^{-1}(2)$ = ?**

A) −15 B) −14 C) −13
D) −12 E) −11

6. $f(x+1) = 2x-1$
$g(x-1) = x^2+1$
$\Rightarrow$ **(gof)(−1) = ?**

A) 10 B) 14 C) 16
D) 17 E) 20

7. $f(x) = \begin{cases} -2x+1, & x>0 \\ x^2-1, & x\le 0 \end{cases} \Rightarrow$ **(fofof)(−2) = ?**

A) 22 B) 23 C) 24
D) 25 E) 26

8. $f(x) = x-1$
$g(x) = \frac{x+1}{2}$
$h(x) = x^2+1 \Rightarrow$ **(fogoh)(x) = ?**

A) x^2-2 B) x^2 C) $\frac{x^2}{2}$
D) x^2+1 E) x^2+2

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9. $f(x) = 2 \cdot f\left(\frac{1}{x}\right) + 3x - 1 \Rightarrow \mathbf{f(3) = ?}$

A) $-\frac{14}{3}$ B) -4 C) $-\frac{10}{3}$ D) $-\frac{8}{3}$ E) -2

10. $f(4x) + f(2x + 2) = x + 5 \Rightarrow \mathbf{f(2) + f(3) + f(4) = ?}$

A) 8 B) $\frac{17}{2}$ C) 9 D) $\frac{19}{2}$ E) 10

11. $f(x) = x^4 - 8x^3 + 24x^2 - 32x + 20 \Rightarrow \mathbf{f(x + 2) = ?}$

A) $x^4 + 4$ B) $x^4 + 20$ C) $x^4 - 4$ D) $x^2 + 4$ E) $x^2 - 4$

12. $f(x) = x^2 - 1$

$g(x) = 4x + 3$

$\Rightarrow \mathbf{f(g(f^{-1}(-1))) = ?}$

A) 0 B) 4 C) 8 D) 14 E) 16

13. $x = \frac{2 \cdot f(x) + 1}{1 - f(x)} \Rightarrow \mathbf{f^{-1}(x) = ?}$

A) $\frac{2x}{1+x}$ B) $\frac{2x-1}{1-x}$ C) $\frac{2x+1}{1-x}$ D) $\frac{2x}{1-x}$ E) $\frac{1-2x}{x+1}$

14. $f(x) = \frac{2x}{x-1} \Rightarrow \mathbf{f(x + 1) = ?}$

A) $\frac{f(x)}{4}$ B) $\frac{f(x)-4}{4 \cdot f(x)}$ C) $\frac{f(x)+4}{f(x)}$ D) $\frac{4 \cdot f(x) - 4}{f(x)}$ E) $\frac{f(x)-4}{f(x)}$

15. $f(x) = 6 \cdot f(x - 2)$

$f(7) = 12 \Rightarrow \mathbf{f(3) = ?}$

A) $\frac{1}{15}$ B) $\frac{1}{12}$ C) $\frac{1}{9}$ D) $\frac{1}{6}$ E) $\frac{1}{3}$

16. $(fog)(x) = 3 \cdot g(x) - 2$,

$h(x) = x^2 + 1$

$\Rightarrow \mathbf{(h^{-1}of)^{-1}(-3) = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	A	B	D	B	D	C	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	B	A	C	C	D	E	D

1. $f(4) = -15$

$f(x - 1) = f(x) + 5$

$\Rightarrow$ **f(30) = ?**

A) −145 B) −115 C) −75 D) −60 E) −45

2. $f(x) = x^2 - x + 1$

$\Rightarrow$ **f(1−x) − f(x) = ?**

A) 0 B) 1 C) 1 − x D) $x^2 - 1$ E) $x^2 + 1$

3. $f\left(\frac{x}{2} + 3\right) = 4x - 7 \Rightarrow$ **(fof)(4) = ?**

A) 6 B) 2 C) −10 D) −18 E) −23

4. $f(x) = 2x - 1$,

$(fog)(x) = 2x^2 - 4x + 3$

$\Rightarrow$ **g(x) = ?**

A) $x^2 - 2x + 1$ B) $x^2 + 2x$ C) $x^2 + 2x - 1$ D) $2x^2 - x$ E) $x^2 - 2x + 2$

5. $(fog)(x) = \frac{x}{3x-4}$, $f(x) = \frac{x+1}{x-1} \Rightarrow$ **g(1) = ?**

A) 0 B) 1 C) 2 D) 3 E) 4

6. $f(x) = 2x^2 + 3$,

$g(x) = 3x + 4$,

$(gof)(-1) = f(2x) \Rightarrow$ **x = ?**

A) $\{-\sqrt{2}, \sqrt{2}\}$ B) $\{-1\}$ C) $\left\{\frac{5}{3}\right\}$ D) $\{-1, 1\}$ E) $\{2\}$

7. $f(x) = 2x + 11$,

$g(x) = 3x - 7$

$\Rightarrow$ **$(fog^{-1})(x)$ = ?**

A) $\frac{3x-29}{4}$ B) $\frac{2x+47}{3}$ C) $\frac{x-16}{21}$ D) $\frac{4x-19}{3}$ E) $\frac{5-21x}{19}$

8. $(fog^{-1})(x) = 1 - 3x$

$f(x) = \frac{x+1}{2} \Rightarrow$ **g(x) = ?**

A) $\frac{x-1}{3}$ B) $\frac{x+1}{6}$ C) $\frac{1-x}{3}$ D) $\frac{2-x}{6}$ E) $\frac{1-x}{6}$

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9. $f(x) = 2x + 3$,
$g(x) = \frac{1}{6x-2} \Rightarrow$ **$(fog)^{-1}(1) = ?$**

A) $\frac{1}{6}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$
D) $\frac{2}{3}$ E) $\frac{5}{6}$

10. $f(x) = 2^{x+1} \Rightarrow$ **$f^{-1}(4) = ?$**

A) 1 B) 2 C) 3
D) 4 E) 5

11. $f(x) = \frac{1}{x+2}$, $g(x) = x^2 \Rightarrow$ **$(gof^{-1})(2) = ?$**

A) $\frac{2}{3}$ B) $\frac{5}{4}$ C) $\frac{9}{4}$
D) $\frac{7}{2}$ E) $\frac{10}{3}$

12. $f(x) = 4x + 3$,
$(gof)(x) = x^2 + 1$
$\Rightarrow$ **$g(15) = ?$**

A) –10 B) –9 C) –8
D) 8 E) 10

13. $f: R \to R$, $f(\sqrt[3]{x}) = x^2 + x + 1$
$\Rightarrow$ **$f(2) = ?$**

A) 43 B) 59 C) 61
D) 73 E) 81

14. $f(2x-1) = 6x - 2$,
$g^{-1}(x) = x - 2$
$\Rightarrow$ **$(fog)(x) = ?$**

A) $3x + 1$ B) $3x + 5$ C) $3x + 7$
D) $6x - 1$ E) $6x + 2$

15. $g(x) = x^2 + 4x + 1$,
$(gof)(x) = x^2 - 6x + 6$
$\Rightarrow$ **$f(x) = ?$**

A) $x + 3$ B) $x + 4$ C) $x - 4$
D) $x + 5$ E) $x - 5$

16. $f, g: R \to R$
$f(x) = 2x + 1$,
$g(x) = \frac{x}{3} + 2$, $(foh)^{-1}(x) = g(x) \Rightarrow$ **$h(3) = ?$**

A) 6 B) 4 C) 3
D) 2 E) 1

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	A	E	E	A	A	B	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	A	C	E	D	C	E	E

1. $f(x) = x - 2$,
$g(x) = x^2 - 4$
$\Rightarrow$ **$(fog^{-1})^{-1}(x) = ?$**

A) x^2 B) $x^2 + 4$ C) $x^2 - 4x$
D) $x^2 + 4x$ E) $(x + 2)^2$

2. $f : R \to R$,
$g : R \to R$
$(fog^{-1})(x) = \frac{3x+1}{2}$, $g(x) = 2x + 1 \Rightarrow$ **$f^{-1}(x) = ?$**

A) $\frac{x+2}{3}$ B) $\frac{x-2}{3}$ C) $\frac{2x+1}{2}$
D) $\frac{2x-1}{2}$ E) $\frac{3x+2}{2}$

3. $f(x) = -2x + 1$,
$g^{-1}(x) = \frac{x+3}{x-2} \Rightarrow$ **$(gof^{-1})(4) = ?$**

A) -2 B) -1 C) 0
D) 1 E) 2

4. $f(x) = 2x - 3$,
$(gof)(x) = 6x + 5 \Rightarrow$ **$g(x) = ?$**

A) $3x - 4$ B) $2x + 3$ C) $3x + 14$
D) $3x - 14$ E) $3x + 4$

5. $f(x) = 3x - 4$,
$(gof^{-1})(x) = x + 2 \Rightarrow$ **$g(5) = ?$**

A) -13 B) -12 C) 12
D) 13 E) 14

6. $f(x) = x - 2$,
$(fog)(x) = x^2 + x + 1 \Rightarrow$ **$g(x) = ?$**

A) $x^2 - 3x + 7$
B) $2x^2 + 2x - 7$
C) $x^2 + x + 3$
D) $x^2 + x - 1$
E) $x^2 - 2x + 1$

7. $f(x - 1) + f(x) = 2$
$f(1) = 3 \Rightarrow$ **$f(3) = ?$**

A) -1 B) 1 C) 2
D) 3 E) 4

8. $f(x) = x^x \Rightarrow$ **$f^{-1}(4) = ?$**

A) 5 B) 4 C) 3
D) 2 E) 1

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9. $f(x) = \frac{x+a}{2x+1}$, $(fof)(x) = \frac{3x+2}{4x+3} \Rightarrow$ **a = ?**

A) – 1 B) 0 C) 1
D) 2 E) 3

10. $(f^{-1}og)(x) = x + 4$

$f(2x + 1) = 3x + 4 \Rightarrow$ **g(3) = ?**

A) 9 B) 10 C) 11
D) 12 E) 13

11. $f(x) = \frac{x-2}{3}$, $(fog)(x) = x - 1 \Rightarrow$ **g(x) = ?**

A) $\frac{3x-3}{2}$ B) $3x - 4$ C) $3x$
D) $3x - 1$ E) $\frac{3x-19}{2}$

12. $f(x) = ax + b$,
$g(x) = 3x - 2$,
$(fog)(x) = 6x + 1 \Rightarrow$ **a · b = ?**

A) 2 B) 5 C) 10
D) 15 E) 20

13. $(fog^{-1})(x) = \frac{2x+1}{3}$, $f(x) = 4x - 3 \Rightarrow$ **g(1) = ?**

A) – 1 B) 0 C) 1
D) 2 E) 3

14. $(gof^{-1})(x) = 3x$,
$f(x) = 2x + 4$
$\Rightarrow$ **g(6) = ?**

A) 5 B) 6 C) 7
D) 8 E) 48

15. $f(x) = \frac{ax+3}{bx-2}$, $g^{-1}(x) = x - 2$
$(f^{-1}og)(x) = \frac{2x+7}{3x-7} \Rightarrow$ **a · b = ?**

A) 12 B) 27 C) 39
D) 42 E) 45

16. $\left.\begin{array}{l}(fog)(x) = 2x \\ (goh)(x) = 3x+1\end{array}\right\} \Rightarrow \frac{f(4)}{h(1)} = ?$

A) 3 B) 2 C) 1
D) 0 E) –1

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	B	C	C	D	C	D	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	E	D	C	C	E	C	B

1. $f\left(\frac{ax-3}{2}\right) = x + 5,\ f^{-1}(16) = 4 \Rightarrow \mathbf{a = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

2. $f(x) = \frac{5x-2}{a}$ ve / and, $f^{-1}(x) = \frac{x+b}{5} \Rightarrow \mathbf{a \cdot b = ?}$

A) −3 B) −2 C) −1 D) 1 E) 2

3. $f(3x + 1) = 9x^2 - 4 \Rightarrow \mathbf{f(2x) = ?}$

A) $36x^2 - 4$

B) $5x^2 - 4x + 1$

C) $4x^2 - 3$

D) $4x^2 - 4x - 3$

E) $x^2 - 2x - 3$

4. $f(x) = 3x + 1$

$g(x) = \frac{2x+1}{x+3}$

$g^{-1}of(a) = -4 \Rightarrow \mathbf{a = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

5. $x \cdot f(x + 1) = 2x^2 + x + a - 1,$

$f(2) = -3 \Rightarrow \mathbf{f(3) = ?}$

A) 2 B) 3 C) 5 D) 6 E) 9

6. $f(0) = 1,$

$f(1) = 4,$

$f(n + 2) = f(n) - 2 \cdot f(n + 1) \Rightarrow \mathbf{f(3) = ?}$

A) −36 B) −24 C) −18 D) 18 E) 24

7. $f(x) = x - 3,\ (gof)(x) = \frac{x+2}{x-2} \Rightarrow \mathbf{g(x) = ?}$

A) $\frac{x+3}{x}$ B) $\frac{x+5}{x+1}$ C) $\frac{x+2}{x+5}$ D) $\frac{x+1}{x+5}$ E) $\frac{x-1}{x+3}$

8. $f(5-x) = \begin{cases} 3-x, & x \le 0 \\ ax+1, & x > 0 \end{cases}$

$f(0) = -4 \Rightarrow \mathbf{a = ?}$

A) −2 B) −1 C) 0 D) 1 E) 2

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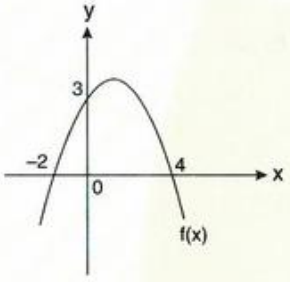
9. $f(x) = \frac{x-2}{x+a}$, $g(x) = \frac{2x+1}{x-1}$

$(f^{-1}og)(4) = 9 \Rightarrow a = ?$

A) $\frac{16}{3}$ B) $\frac{9}{2}$ C) 1

D) $-\frac{16}{3}$ E) $-\frac{20}{3}$

10.

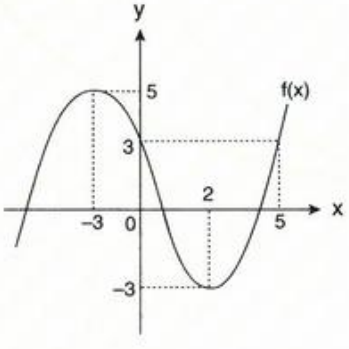


$$g(x) = \begin{cases} x-3, & f(x) < 0 \\ -x+6, & f(x) > 0 \end{cases} \Rightarrow (gog)(5) = ?$$

A) 2 B) 3 C) 4

D) 5 E) 6

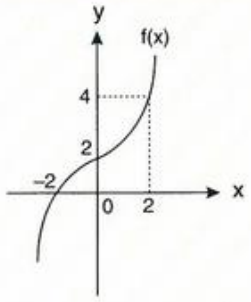
11. $(fof)(2) = ?$



A) 10 B) 9 C) 8

D) 5 E) 0

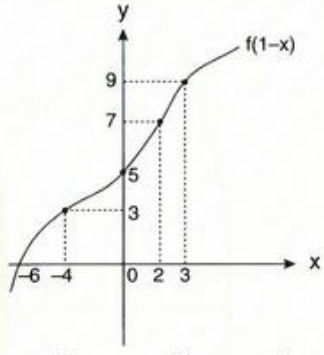
12. $\frac{f^{-1}(0)+f(-2)}{(fof)(0)} = ?$



A) -2 B) -1 C) $-\frac{1}{2}$

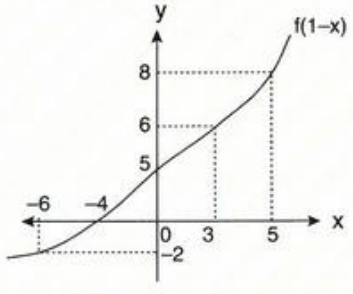
D) $\frac{1}{2}$ E) 2

13. $\frac{f^{-1}(7)+f^{-1}(9)}{f(5)+f(-2)} = ?$



A) $-\frac{3}{4}$ B) $-\frac{1}{4}$ C) $\frac{2}{3}$ D) $\frac{3}{2}$ E) $\frac{3}{4}$

14. $\frac{f(7)-f(-4)}{f^{-1}(5)+f^{-1}(0)} = ?$



A) $-\frac{5}{3}$ B) $-\frac{7}{3}$ C) $\frac{5}{3}$ D) $\frac{7}{3}$ E) 3

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12	13	14
A	E	D	B	A	D	B	B	E	C	D	C	B	A

1. $f: R \to R$,

$f(x) = x + \sqrt[3]{x} + 6 \Rightarrow$ **f (8) · f (–1) = ?**

A) – 32 B) – 16 C) 24

D) 56 E) 64

2. $f: R \to R$,

$3 \cdot f(x) - f(x) = x^2 + 4x + 9 \Rightarrow$ **f (3) = ?**

A) 9 B) 12 C) 15

D) 24 E) 36

3. $f(x^2 + 3x + 7) = 3x^2 + 9x - 5 \Rightarrow$ **f (x) = ?**

A) $f(x) = 3x + 13$

B) $f(x) = \frac{1}{3}x - 19$

C) $f(x) = 3x - 26$

D) $f(x) = 6x - 13$

E) $f(x) = 4x - 33$

4. $f(x^2 + 5x + 11) = 2x^2 + 10x - 13 \Rightarrow$ **f(5) = ?**

A) 15 B) 5 C) – 15

D) – 20 E) – 25

5. $f(x \cdot y) = f(x) + f(y)$,

$f\left(\frac{2}{3}\right) = 4 \Rightarrow$ **f** $\left(\frac{16}{81}\right)$ **= ?**

A) 8 B) 12 C) 16

D) 32 E) 64

6. $f(x + y) = f(x) \cdot f(y)$

$f\left(\frac{1}{2}\right) = \frac{3}{2} \Rightarrow$ **f(2) = ?**

A) $\frac{9}{4}$ B) $\frac{9}{8}$ C) $\frac{27}{4}$

D) $\frac{27}{16}$ E) $\frac{81}{16}$

7. **Aşağıdakilerden hangisi R → R'ye bir fonksiyondur?**

Which of the following is a function from R to R?

A) $x = 3$

B) $y^2 - x^3 = 3$

C) $|y| = 6 - |x|$

D) $y^2 - x^4 = 6$

E) $y^5 - x^3 + 9 = 0$

8. $(fog)(x) = g^3(x) + m^3$, $f^{-1}(35) = 2 \Rightarrow$ **m = ?**

A) – 3 B) – 1 C) 5

D) 4 E) 3

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9. $f\left(\frac{x}{x^2-2x+3}\right) = x-2 \Rightarrow \mathbf{f^{-1}(x+1) = ?}$

A) $\frac{x+2}{x^2+2x+3}$ B) $\frac{x+3}{x^2+4x+6}$ C) $\frac{x+5}{(x+2)^2}$

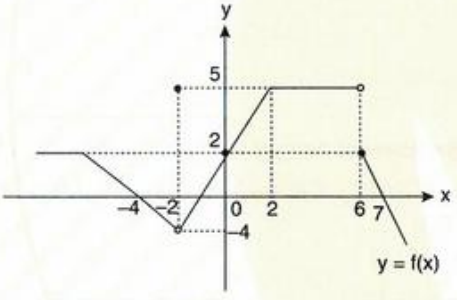
D) $\frac{x+1}{x^2+2x+4}$ E) $\frac{x+4}{x^2+6x+8}$

10. $f\left(\frac{x-1}{x+1}\right) = \frac{x+1}{1-x} \Rightarrow \mathbf{f\left(\frac{-x}{x+2}\right) = ?}$

A) $\frac{x-1}{x+2}$ B) $\frac{x+2}{x-3}$ C) $\frac{x+2}{x}$

D) $-\frac{x+3}{x}$ E) $\frac{x}{x+2}$

11.



$y = f(x) \Rightarrow \mathbf{\frac{(fof)(-4)+(fof)(2)}{(fof)(-2)+f(6)} = ?}$

A) $\frac{7}{2}$ B) 1 C) 4

D) $\frac{9}{2}$ E) $\frac{11}{3}$

12. $f: R-\{-2\} \to R-\{4\}$,

$f(x) = \left(\frac{(a-2)x+9}{3x-b-2}\right) \Rightarrow \mathbf{f^{-1}(x) = ?}$

A) $\frac{4x+3}{x-2}$ B) $\frac{-2x+3}{x-4}$ C) $\frac{x-3}{x+4}$

D) $\frac{3x-5}{x-8}$ E) $\frac{2x-7}{x-4}$

13. $f(2^{2x-1}) = 2x+5 \Rightarrow \mathbf{f(8) = ?}$

A) 6 B) 7 C) 8

D) 9 E) 10

14. $f: R-\left\{\frac{1}{2}\right\} \to R-\{a\}$

$f(x) = \frac{4x+5}{2x-1} \Rightarrow \mathbf{a = ?}$

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$

D) 1 E) 2

15. $f\left(\frac{x+6}{5}\right) = 2x^3-1 \Rightarrow \mathbf{f^{-1}(-17) = ?}$

A) $\frac{3}{5}$ B) $\frac{4}{5}$ C) $\frac{6}{5}$

D) $\frac{-4}{5}$ E) $\frac{8}{5}$

16. $f(m, n) = 3m + 2n$

$g(m) = m^2 - 4m \Rightarrow \mathbf{(gof)(1,-1) = ?}$

A) −3 B) −2 C) 0

D) 2 E) 3

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	C	C	E	C	E	E	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	C	A	B	D	E	B	A

BÖLÜM CHAPTER 14

TARİHSEL NOT / HISTORICAL NOTE

Borel Felix Edouard Justin Emile [1871 - 1956]

Borel Felix Edouard Justin Emile Fransız bir matematikçi ve politikacıydı. Bir matematikçi olarak, ölçü teorisi ve olasılık alanlarında kurucu çalışmaları ile tanınıyordu.

Borel Felix Edouard Justin Emile was a French mathematician and politician. As a mathematician, he was known for his founding work in the areas of measure theory and probability.

İŞLEM ve MODÜLER ARİTMETİK
OPERATION and MODULAR SYSTEMS

Bu bölüm, 35 çözümlü örnek, 12 çözümlü test sorusu, 16 çözümlü YÖS sorusu, 48 test sorusu içermektedir.

This chapter includes 35 examples with solutions, 12 tests questions with solutions, 16 YÖS questions with solutions and 48 test questions.

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- Modüler Aritmetik / Modular Systems ..... 470 - 478
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- ▲ Çözümlü Test / Test with Solutions ..... 479 - 482
- ▲ YÖS Soruları ve Çözümleri / YÖS Questions and Solutions ..... 483 - 486
- ▲ Konu Testleri / Practice Tests ..... 487 - 492

BÖLÜM
14
CHAPTER

İŞLEM VE MODÜLER ARİTMETİK
OPERATION and MODULAR SYSTEMS

İŞLEM / OPERATION

Tanım: A ≠ ∅ olmak üzere A x A kümesinden A'ya tanımlanan her fonksiyona, A'da *ikili işlem* ya da kısaca *işlem* denir.

İşlemler ⊕, ●, ✳, □ ,Δ gibi simgelerle gösterilir.

Definition: Let A ≠ ∅ , each function defined from A x A to A is called as a *binary operation* or simply *operation*.

Operations are shown by symbols like ⊕,●, ✳, ■ ,Δ.

PEKİŞTİRME / REINFORCE

$x \circ y = 2x - 3y$

✓ $2 \circ 3 = ?$ ..... [−5]

✓ $2 \circ (-1) = ?$ ..... [7]

✓ $1 \circ 3 = ?$ ..... [−7]

Örnek / Example 1

$x \Delta y = x^y + x \cdot y \Rightarrow 3 \Delta 2 = ?$

A) 6 B) 9 C) 12 D) 15 E) 18

Çözüm / Solution

Yanıt / Answer **D**

$x \Delta y = x^y + x \cdot y$

$\left.\begin{matrix} x = 3 \\ y = 2 \end{matrix}\right\} \Rightarrow 3 \Delta 2 = 3^2 + 3 \cdot 2 = 9 + 6 = 15$

Örnek / Example 2

$\frac{1}{a} \Delta \frac{1}{b} = \frac{3ab}{a-b} \Rightarrow 3 \Delta 6 = ?$

A) $\frac{1}{6}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) 1

Çözüm / Solution

Yanıt / Answer **E**

$\frac{1}{a} = 3 \Rightarrow a = \frac{1}{3}$

$\frac{1}{b} = 6 \Rightarrow b = \frac{1}{6}$

$\frac{1}{a} \Delta \frac{1}{b} = \frac{3ab}{a-b} \Rightarrow \frac{1}{\frac{1}{3}} \Delta \frac{1}{\frac{1}{6}} = \frac{3 \cdot \frac{1}{3} \cdot \frac{1}{6}}{\frac{1}{3} - \frac{1}{6}}$

$\Rightarrow 3 \Delta 6 = \frac{\frac{1}{6}}{\frac{2-1}{6}} = \frac{\frac{1}{6}}{\frac{2-1}{6}} = \frac{1}{6} \cdot \frac{6}{1} = 1$

Örnek / Example 3

$\forall x, y \in R - \{0\}$

$x \square y = \frac{x}{y} - \frac{y}{x}$,

$(a + 1) \square a = \frac{3}{a} \Rightarrow \mathbf{a = ?}$

A) -3 B) -2 C) -1 D) 0 E) 1

Çözüm / Solution

Yanıt / Answer **B**

$x = a + 1$ ve $y = a \Rightarrow$

$$(a + 1) \square a = \frac{a+1}{a} - \frac{a}{a+1}$$

$$\frac{3}{a} = \frac{(a+1)^2 - a^2}{a(a+1)}$$

$$3a + 3 = a^2 + 2a + 1 - a^2$$

$$3a + 3 = 2a + 1$$

$$a = -2$$

Örnek / Example 4

$\forall x,y \in R - \{1\}$

$x \Delta y = x + y - xy$,

$(8 - 8a) \cdot (1-b) = 6 \Rightarrow \mathbf{a \Delta b = ?}$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

Çözüm / Solution

Yanıt / Answer **C**

$x = a, y = b \Rightarrow a \Delta b = a + b - ab$

$$(8 - 8a)(1- b) = 6$$

$$8 - 8b - 8a + 8ab = 6$$

$$- 8b - 8a + 8ab = -2$$

$$8a + 8b - 8ab = 2$$

$$8(a + b - ab) = 2$$

$$a + b - ab = \frac{2}{8} = \frac{1}{4}$$

Örnek / Example 5

$3^a \Delta 3^b = a + b - 2a \cdot b \Rightarrow \frac{1}{9} \Delta 81 = ?$

A) 16 B) 18 C) 20 D) 22 E) 24

Çözüm / Solution Yanıt / Answer **B**

$\frac{1}{9} \Delta 81 = 3^{-2} \Delta 3^4$

$a = -2 , b = 4 \Rightarrow$

$3^{-2} \Delta 3^4 = -2 + 4 - 2 \cdot 4 (-2)$

$= -2 + 4 + 16$

$= 18$

Örnek / Example 6

$\sqrt{a} \Delta \sqrt{b} = \frac{a+b-1}{4} \Rightarrow 3 \Delta 4 = ?$

A) 12 B) 10 C) 8 D) 6 E) 4

Çözüm / Solution Yanıt / Answer **D**

$\sqrt{a} = 3 \Rightarrow a = 9$

$\sqrt{b} = 4 \Rightarrow b = 16$

$\Rightarrow \sqrt{9} \Delta \sqrt{16} = \frac{9+16-1}{4}$

$3 \Delta 4 = \frac{24}{4} = 6$

Özellik / Property

★ **Kapalılık Özelliği**

A kümesi üzerinde tanımlanan bir Δ işlemi için,

∀x, y ∈ A için x Δ y ∈ A ise, A kümesi Δ işlemine göre **kapalıdır** denir.

Closure Property

an operation Δ defin edonset A is closed if,

∀x, y ∈ A, then x Δ y ∈ A.

★ **Değişme Özelliği**

A kümesi üzerinde tanımlanan bir Δ işlemi için,

∀x,y ∈ A için x Δ y = y Δ x ise Δ işlemi **değişme özelliğine** sahiptir denir.

Commutative Property

An operation Δ, defined on set A is commutative if for

∀x,y ∈ A,x Δ y = y Δ x.

★ **Birleşme Özelliği**

A kümesi üzerinde tanımlanan bir Δ işlemi için, ∀x, y, z ∈ A için (xΔy)Δz = xΔ(yΔz) ise Δ işleminin **birleşme özelliğine** sahiptir denir.

Associative Property

An operation Δ defined on set A is associative if

for ∀x, y, z ∈ A, (x Δ y) Δ z = x Δ (y Δ z).

★ **Dağılma Özelliği**

A kümesi üzerinde tanımlanan Δ ve □ işlemlerinde

∀x, y, z ∈ A için x Δ (y □ z) = (x Δ y) □ (x Δ z) ise, Δ işleminin □ işlemi üzerinde **dağılma özelliği vardır** denir.

Distributive Property

Let Δ and ■ be are two operations defined on set A, if

for ∀x,y ∈ A, x Δ (y ■ z) = (x Δ y) ■ (x Δ z), Δ is distributive over ■.

★ **Birim (Etkisiz) Eleman**

A kümesi üzerinde tanımlanan bir Δ işleminde, ∀ x ∈ A için x Δ e = e Δ x = x olacak biçimde e ∈A varsa e ∈A ya Δ işlemine göre **etkisiz elemandır** denir.

Identity Element

Let Δ be an operation on set A, if for ∀ x ∈ A, x Δ e = e Δ x = x, e ∈ A is identity element of the operation Δ.

★ **Ters Eleman**

A kümesi üzerinde tanımlanan Δ işleminde, A'nın Δ işlemine göre birim elemanı e ise, ∀x ∈ A için x Δ y = y Δ x = e koşulunu sağlayan y ∈ A varsa bu y elemanına Δ işlemine göre x'in tersi denir ve x^{-1} = y biçiminde gösterilir.

Inverse Element

For an operation Δ defined on set A, if identity element for the set A under the operation Δ is e and for ∀x ∈ A there exist y ∈ A satisfying x Δ y = y Δ x = e, y is said to be the inverse of x under Δ and denoted as x^{-1} = y.

★ **Yutan Eleman**

A kümesi üzerinde bir Δ işlemi verildiğinde, ∀x,y ∈ A için x Δ y = y Δ x = y ise y sayısına Δ işleminin **yutan elemanı** denir.

Null Element

Let Δ be an operation defined on set A. If for ∀x,y ∈ A, x Δ y = y then y is said to be the null element.

ÖNEMLİ / IMPORTANT

- *Bir işlemin birim elemanı varsa bir tanedir.*
- *The indentity element of an operation is unique, if it exists.*

ÖNEMLİ / IMPORTANT

- *Bir Δ işleminin birim elemanı yoksa elemanların tersi yoktur (bulunamaz.) Birim eleman varsa, Δ işlemine göre ters eleman olabilir.*
- *For any operation Δ, if Δ has not identity element, then there isn't any inverse. (can't be found) If there exist an identity element, there may be some inverse element under the operation Δ.*

Örnek / Example 7

$x \square y = x + y - 7$,

$x \square e = x \Rightarrow e = ?$

A) 3 B) 5 C) 7 D) 9 E) 11

Çözüm / Solution Yanıt / Answer **C**

$x \square e = e \square x = x$

$x \square e = x + e - 7 = x \Rightarrow e = 7$

$\square$ işleminin birim elemanı 7'dir.

The identity element of the operation $\square$ is 7.

Örnek / Example 8

$x \Delta y = x + y - 2xy$,

$x \Delta e = x \Rightarrow e = ?$

A) -1 B) 0 C) 1 D) 2 E) 3

Çözüm / Solution Yanıt / Answer **B**

$x \Delta e = x$

$x + e - 2xe = x$

$e(1 - 2x) = 0$

$1 - 2x \neq 0 \Rightarrow e = 0$

Örnek / Example 9

$a * b = \frac{3ab}{3a+b}$,

$\frac{1}{3} * \frac{4}{x} = \frac{1}{5} * 3 \Rightarrow x = ?$

A) – 2 B) 1 C) 2 D) 3 E) 4

Çözüm / Solution

Yanıt / Answer **E**

$\frac{1}{3} * \frac{4}{x} = \frac{1}{5} * 3$

$\frac{3 \cdot \frac{1}{3} \cdot \frac{4}{x}}{3 \cdot \frac{1}{3} + \frac{4}{x}} = \frac{3 \cdot \frac{1}{5} \cdot 3}{3 \cdot \frac{1}{5} + 3}$

$\frac{\frac{4}{x}}{\frac{x+4}{x}} = \frac{\frac{9}{5}}{\frac{18}{5}} \Rightarrow \frac{4}{x+4} = \frac{1}{2}$

$x + 4 = 8 \Rightarrow x = 4$

Örnek / Example 10

$x \Delta y = 2x + 2y - xy - 2$,

$x \Delta e = x \Rightarrow e = ?$

A) – 1 B) 0 C) 1 D) 2 E) 3

Çözüm / Solution

Yanıt / Answer **C**

$x \Delta e = x$

$2x + 2e - ex - 2 = x$

$x(1 - e) - 2(1 - e) = 0$

$(1 - e)(x - 2) = 0$

$x - 2 \neq 0 \Rightarrow 1 - e = 0 \Rightarrow e = 1$

Örnek / Example 11

$x \circ y = x + y - 4$,

$3^{-1} = y \Rightarrow y = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

Çözüm / Solution

Yanıt / Answer **D**

$x \circ e = x$

$x + e - 4 = x$

$e = 4$

$3 \circ 3^{-1} = e$

$3 \circ y = 4$

$3 + y - 4 = 4$

$y = 5$

Örnek / Example 12

$x \Delta y = \dfrac{xy + 2(x+y)}{2}$

$2^{-1} = a \Rightarrow a = ?$

A) -1 B) 0 C) 1 D) 2 E) 3

Çözüm / Solution

Yanıt / Answer **A**

$x \Delta e = x \Rightarrow xe + 2x + 2e = 2x$

$e(x + 2) = 0$

$x \neq -2 \Rightarrow e = 0$

$2 \Delta a = e = 0$

$\dfrac{2a + 2(2+a)}{2} = 0 \Rightarrow 2a + 4 + 2a = 0$

$4a + 4 = 0$

$4a = -4$

$a = -1$

Örnek / Example 13

$x \Delta y = x + y - 3xy$,

$x \Delta y = y \Rightarrow y = ?$

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{4}$ E) $\frac{1}{5}$

Çözüm / Solution

Yanıt / Answer **C**

$x \Delta y = y \Delta x = y$

$x + y - 3xy = y$

$x - 3xy = 0$

$x(1 - 3y) = 0$

$y = \frac{1}{3}$

Örnek / Example 14

$x \star y = 4x - 3y$,

$1 \star y = y \Rightarrow y = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution

Yanıt / Answer **A**

$1 \star y = y$

$4 \cdot 1 - 3 \cdot y = y$

$4y = 4$

$y = 1$

Örnek / Example 15

Δ	F	E	R	H	A	T
F	T	F	E	R	H	A
E	F	E	R	H	A	T
R	E	R	H	A	T	F
H	R	H	A	T	F	E
A	H	A	T	F	E	R
T	A	T	F	E	R	H

(F Δ E) Δ (R Δ T) = ?

A) F B) E C) R D) A E) T

Çözüm / Solution

Yanıt / Answer **E**

F Δ E = F,

R Δ T = F ⇒ (F Δ E) Δ (R Δ T) = F Δ F = T

Örnek / Example 16

✪	M	E	T	R	O
M	E	T	R	O	M
E	T	R	O	M	E
T	R	O	M	E	T
R	O	M	E	T	R
O	M	E	T	R	O

$(M ✪ R)^{-1} ✪ T^2 = ?$

A) M B) E C) T D) R E) O

Çözüm / Solution

Yanıt / Answer **A**

e = O ⇒ $(M ✪ R)^{-1} = O^{-1} = O$

$T^2 = T ✪ T = M$

$(M ✪ R)^{-1} ✪ T^2 = O ✪ M = M$

⊠	M	E	T	R	O
M	E	T	R	O	M
E	T	R	O	M	E
T	R	O	M	E	T
R	O	M	E	T	R
O	M	E	T	R	O

→ Birim eleman / Identity element

MODÜLER ARİTMETİK / MODULER SYSTEMS

a, b, m ∈ Z, m > 1 olmak üzere

β = {(a,b) : m, (a – b)} yi tam böler} bağıntısı bir denklik bağıntısıdır.

Her (a,b) ∈ β için

a ≡ b (mod m) biçiminde yazılır ve a sayısı modül m de b ye denktir denir.

a, b, m ∈ Z, and m > 1

The relation β = {(a,b) : m divides (a – b)} is an equation relation.

Each (a,b) ∈ β is written as a ≡ b (mod m) and read as a is equivalent to b in mod m.

$$\begin{array}{r|l} a & m \\ \hline - & k \\ \hline b & \end{array} \Rightarrow a \equiv b \pmod m$$

$$a \equiv m \cdot k + b,\ k \in Z$$

ÖNEMLİ / IMPORTANT

- *a sayısı m sayısına bölünürse 0, 1, 2, 3, ..., (m – 1) kalanlarını verir.*
- *If the number a is divided to m, the remainders will be 0, 1, 2, 3, ..., (m – 1).*

DENKLİK SINIFLARI / EQUIVALENCE CLASSES

Her tamsayı m ile bölündüğünde hangi kalanı veriyor ise o kalana denktir. m ye bölündüğünde aynı kalanı veren sayıların kümesine kalan sınıfların kümesi denir.

$Z_{/m} = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, ..., \overline{m-1}\}$ şeklinde gösterilir.

When dividing an integer to m, the integer is called equivalent to that remainder. Set of numbers with same remainder when divided to m, is called set of remainders class it is shown by $Z_{/m} = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, ..., \overline{m-1}\}$.

PEKİŞTİRME / REINFORCE

✓ 19 ≡ x (mod 3) ⇒ **x = ?**
..... [1]

✓ 231 ≡ x (mod 5) ⇒ **x = ?**
..... [1]

✓ 1144 ≡ x (mod 9) ⇒ **x = ?**
..... [1]

✓ 346 ≡ x (mod 10) ⇒ **x = ?**
..... [6]

Örnek / Example 17

1993 = x (mod 6)

1987 = y (mod 3) ⇒ **x + y = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution

Yanıt / Answer **B**

$$\begin{array}{r|l} 1993 & 6 \\ \hline - 1992 & 332 \\ \hline 1 \rightarrow x & \end{array} \qquad \begin{array}{r|l} 1987 & 3 \\ \hline - 1986 & 662 \\ \hline 1 \rightarrow y & \end{array}$$

x + y = 1 + 1 = 2

Örnek / Example 18

$Z_{/5}$,

$3^3 \cdot 2^4 + 3^2 \cdot 2^3 + 1 = ?$

A) 0 B) 1 C) 2 D) 3 E) 4

Çözüm / Solution

Yanıt / Answer **A**

$$\begin{array}{l} 3^3 \cdot 2^4 + 3^2 \cdot 2^3 + 1 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 2 \cdot 1 + 4 \cdot 3 + 1 \end{array} \begin{array}{l} = 2 + 12 + 1 \\ = 15 \\ = 0 \end{array}$$

Örnek / Example 19

$Z_{/7}$,

$\frac{(6x+5)^2}{x+2} = ?$

A) $x + 2$ B) $x - 2$ C) 2 D) $x^2 + 2$ E) x

Çözüm / Solution

Yanıt / Answer **A**

$$\frac{(6x+5)^2}{x+2} = \frac{36x^2+60x+25}{x+2}$$

$$= \frac{x^2+4x+4}{x+2}$$

$$= \frac{(x+2)^2}{x+2}$$

$$= x+2$$

Örnek / Example 20

$Z_{/5}$,

$\left(\frac{2}{3}\right)^3 + \frac{3}{4} + 1 = ?$

A) 4 B) 3 C) 2 D) 1 E) 0

Çözüm / Solution

Yanıt / Answer **C**

$Z_{/5}$ $\frac{2}{3} \equiv 4$, $Z_{/5}$ $\frac{3}{4} \equiv 2$

$4 + 2 + 1 = 7$

$Z_{/5}$ $7 \equiv 2$

Örnek / Example 21

$Z_{/7}$,
$2x + 5 = 3 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3 D) 5 E) 6

Çözüm / Solution

Yanıt / Answer **E**

$2x + 5 = 3$

$2x = -2$

$x = -1$

$Z_{/7}\ -1 = 6$

Örnek / Example 22

$Z_{/11}$,
$f(x) = \bar{5}x + \bar{8} \Rightarrow$ **$f(\bar{4})$ = ?**

A) 3 B) 5 C) 6 D) 7 E) 8

Çözüm / Solution

Yanıt / Answer **C**

$f(\bar{4}) = \bar{5} \cdot \bar{4} + \bar{8}$

$= \overline{28}$

$Z_{/11}\ 28 \equiv 6$

Örnek / Example 23

$Z_{/7}$,
$f(x) = \bar{6}x + \bar{4}$,
$g(x) = \bar{5}x + \bar{6}$
$\Rightarrow$ **(fog)(x) = ?**

A) $\bar{2}x + \bar{5}$ B) $\bar{2}x + \bar{4}$ C) $\bar{2}x + \bar{3}$
D) $\bar{2}x + \bar{1}$ E) $\bar{2}x$

Çözüm / Solution

Yanıt / Answer **A**

$(fog)(x) = f(g(x))$

$= f(\bar{5}x + \bar{6})$

$= \bar{6} \cdot (\bar{5}x + \bar{6}) + \bar{4}$

$= \overline{30}x + \overline{40}$ $[\ \overline{30} = \bar{2},\ \ \overline{40} = \bar{5}\ ]$

$= \bar{2}x + \bar{5}$

PEKİŞTİRME / REINFORCE

$Z_{/9}$

✓ $f(x) = \bar{3}x + \bar{2} \Rightarrow$ **$f(\bar{4})$ = ?**

..... [5]

✓ $f(x) = x + \bar{3} \Rightarrow$ **$f(\bar{1})$ = ?**

..... [4]

PEKİŞTİRME / REINFORCE

✓ $1918^{1913} \equiv x \pmod 5 \Rightarrow x = ?$

..... [3]

Örnek / Example 24

$2003^{2003} \equiv x \pmod 5 \Rightarrow x = ?$

A) 0 B) 1 C) 2 D) 3 E) 4

Çözüm / Solution Yanıt / Answer **C**

$2003 \equiv 3 \pmod 5$

$3^1 \equiv 3 \pmod 5$
$3^2 \equiv 4 \pmod 5$
$3^3 \equiv 2 \pmod 5$
$3^4 \equiv 1 \pmod 5$

$2003 \mid 5$ — $3 \rightarrow$ kalan / remainder

$\Rightarrow (2003)^{2003} \equiv 3^{2003} \equiv 3^3$

$\equiv 2 \pmod 5$

Örnek / Example 25

$(-5)^{17} \equiv x \pmod 7 \Rightarrow x = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

Çözüm / Solution Yanıt / Answer **C**

$(-5)^{17} \equiv x \pmod 7 \quad (7-5 = 2)$

$2^{17} \equiv x \pmod 7$

$2^1 \equiv 2 \pmod 7$
$2^2 \equiv 4 \pmod 7$
$2^3 \equiv 1 \pmod 7$

$17 \mid 3$ — $15 \mid 5$ — $2 \rightarrow$ kalan / remainder

$2^2 \equiv x \pmod 7 \Rightarrow x = 4$

BİR SAYININ KUVVETİNİN BİRLER BASAMAĞI
ONES PLACE of an EXPONENT of a NUMBER

Bir sayının birler basamağını bulma işlemi (mod10)'a göre kalanın bulunması ile eş anlamlıdır.

To find ones place of an exponential is same as to find the remainder when the number is divided by 10 (mod 10).

Örnek / Example 26

$3^{95} = ...x \Rightarrow x = ?$

A) 9 B) 8 C) 7 D) 5 E) 2

Çözüm / Solution

Yanıt / Answer **C**

$3^1 \equiv 3 \pmod{10}$
$3^2 \equiv 9 \pmod{10}$
$3^3 \equiv 7 \pmod{10}$
$3^4 \equiv 1 \pmod{10}$

$3^3 \equiv x \pmod{10}$

$95 \mid 4$
$-\quad\; 23$
$3 \rightarrow$ kalan / remainder

$x = 7$.

O halde, 3^{95} sayısının birler basamağı 7'dir.

Therefore, the ones place of the number 3^{95} is 7.

Özellik / Property

$x \equiv y \pmod{m}$

$u \equiv v \pmod{m}$, olmak üzere / is

- ★ $x + u = (y + v) \pmod{m}$
- ★ $x - u = (y - v) \pmod{m}$
- ★ $x \cdot u = y \cdot v \pmod{m}$
- ★ $x^n = y^n \pmod{m}$
- ★ $ax = ay \pmod{m} \quad a \neq 0$

Örnek / Example 27

$x \geq 0$

$3x \equiv 2 \pmod 5 \Rightarrow$ **min(x) = ?**

A) 0 B) 2 C) 4 D) 8 E) 12

Çözüm / Solution

Yanıt / Answer **C**

$x \geq 0$

3'ün $Z_{/5}$ çarpmaya göre tersi 2 olduğundan

Since 2 is the multiplicative inverse of 3 on $Z_{/5}$,

$2 \cdot 3x \equiv 2 \cdot 2 \pmod 5$

$x \equiv 4 \pmod 5$

$\Rightarrow x = 5k + 4$

$k = 0 \Rightarrow \min(x) = 4.$

Örnek / Example 28

$$f(x) = \begin{cases} x^2 + 1 & x \equiv 0 \pmod 3 \\ 2x - 1 & x \equiv 1 \pmod 3 \\ 3x + 1 & x \equiv 2 \pmod 3 \end{cases}$$

$\Rightarrow$ **f(6) + f(7) + f(11) = ?**

A) 80 B) 81 C) 82 D) 83 E) 84

Çözüm / Solution

Yanıt / Answer **E**

$6 \equiv 0 \pmod 3 \Rightarrow f(6) = 6^2 + 1 = 37$

$7 \equiv 1 \pmod 3 \Rightarrow f(7) = 2 \cdot 7 - 1 = 13$

$11 \equiv 2 \pmod 3 \Rightarrow f(11) = 3 \cdot 11 + 1 = 34$

$\Rightarrow f(6) + f(7) + f(11) = 37 + 13 + 34 = 84$

Örnek / Example 29

$Z_{/7}$,

$(\overline{4})^{-1} + (\overline{3})^3 = ?$

A) $\overline{1}$ B) $\overline{2}$ C) $\overline{3}$ D) $\overline{4}$ E) $\overline{5}$

Çözüm / Solution

Yanıt / Answer **A**

$4^{-1} = \frac{\overline{1}}{\overline{4}} = \overline{2}$ $[\overline{1} = \overline{8} \pmod 7]$

$(\overline{3})^3 = \overline{27} = \overline{6}$ $[\overline{27} = \overline{6} \pmod 3]$

$(\overline{4})^{-1} + (\overline{3})^3 = \overline{2} + \overline{6} = \overline{8} = \overline{1} \pmod 7$

Örnek / Example 30

$f: Z_{/7} \to Z_{/7}$,

$f(x) = \bar{2}x + \bar{3}$

$\Rightarrow f^{-1}(\bar{4}) = ?$

A) $\bar{0}$ B) $\bar{1}$ C) $\bar{2}$ D) $\bar{3}$ E) $\bar{4}$

Çözüm / Solution

Yanıt / Answer **E**

$f(x) = \bar{2}x + \bar{3} \Rightarrow f^{-1}(\bar{2}x + 3) = x$

$\bar{2}x + 3 = \bar{4}$

$\bar{2}x = \bar{1}$

$x = \frac{\bar{1}}{\bar{2}} = \frac{\bar{8}}{\bar{2}} = \bar{4}$

Örnek / Example 31

$f: Z_{/7} \to Z_{/7}$,

$g: Z_{/7} \to Z_{/7}$,

$\left.\begin{matrix} f(x) = \bar{6}x + \bar{3} \\ g(x) = \bar{4}x + \bar{2} \end{matrix}\right\} \Rightarrow f(x) \cdot g(x) = ?$

A) $x^2 + \bar{5}x + \bar{4}$ B) $\bar{3}x^2 + \bar{5}x + \bar{2}$ C) $\bar{3}x^2 + \bar{3}x + \bar{6}$

D) $\bar{2}x^2 + \bar{5}x + \bar{4}$ E) $\bar{2}x^2 + \bar{5}x + \bar{3}$

Çözüm / Solution

Yanıt / Answer **C**

$f(x) \cdot g(x) = (\bar{6}x + \bar{3}) \cdot (\bar{4}x + \bar{2})$

$= \bar{24}x^2 + \bar{12}x + \bar{12}x + \bar{6}$

$= \bar{3}x^2 + \bar{3}x + \bar{6}$

Örnek / Example 32

$f : Z_{/7} \to Z_{/7}$,

$g: Z_{/7} \to Z_{/7}$,

$\left.\begin{array}{l}(fog)(x) = \overline{3}x + \overline{2} \\ f(x) = \overline{4}x + \overline{3}\end{array}\right\} \Rightarrow \mathbf{g(x) = ?}$

A) $\overline{3}x + \overline{6}$

B) $\overline{2}x + \overline{5}$

C) $\overline{5}x + \overline{4}$

D) $\overline{6}x + \overline{5}$

E) $\overline{3}x + \overline{1}$

Çözüm / Solution

Yanıt / Answer **D**

$f^{-1}o\,(fog)(x) = f^{-1}(\overline{3}x + \overline{2}) \Rightarrow g(x) = f^{-1}(\overline{3}x + \overline{2})$

$f(x) = \overline{4}x + \overline{3} \Rightarrow f^{-1}(x) = \underset{(2)}{\frac{x - \overline{3}}{4}} = \overline{2}x - \overline{6}$

$\Rightarrow g(x) = \overline{2} \cdot (\overline{3}x + \overline{2}) - \overline{6}$

$= \overline{6}x + \overline{5}$

Örnek / Example 33

$f : Z_{/7} \to Z_{/7}$,

$f(x) = 2x + 3 \Rightarrow \mathbf{f^{-1}(x) + 3 \cdot f(x) = ?}$

A) $3x + 1$

B) $2x + 3$

C) $x + 3$

D) $3x + 4$

E) $3x + 2$

Çözüm / Solution

Yanıt / Answer **D**

$f(x) = 2x + 3 \Rightarrow f^{-1}(x) = \frac{x - 3}{2} = \frac{1}{2}x + \frac{-3}{2}$

$f^{-1}(x) = 4x + 2$

$f^{-1}(x) + 3 \cdot f(x) = 4x + 2 + 6x + 9$

$= 10x + 11$

$= 3x + 4$

Örnek / Example 34

Bir terzi bir elbiseyi 4 günde dikiyor ve 2 gün dinleniyor. İlk elbiseyi dikmeye cuma günü başlayan terzi 14. elbiseyi hangi gün bitirir?

A tailor sews up a dress in 4 days and rests for two days afterwaeds. If the tailor starts sewing the first dress in friday,finishes 14th dress in which day of the week?

A) Çarşamba / Wednesday
B) Perşembe / Thirsday
C) Cuma / Friday
D) Pazar / Sunday
E) salı / Tuesday

Çözüm / Solution

Yanıt / Answer **A**

$14 \cdot 6 = 84$

$84 - 2 = 82$ günde bitirir / finishes in 82 days

$$\begin{array}{r|l} 82 & 7 \\ -\ 7 & \hline 11 \\ \hline 12 & \\ -\ 7 & \\ \hline 5 & \end{array}$$

⇒ Çarşamba / Wednesday

Örnek / Example 35

4 günde bir nöbet tutan bir hemşire 52. nöbetini pazartesi tuttuğuna göre ilk nöbetini hangi gün tutmuştur?

A nurse who takes a shift of work every 4 days,if this nurse takes her 52th shift in Monday, in which day of the week she took her first?

A) Cumartesi / Saturday
B) Pazar / Sunday
C) Pazartesi / Monday
D) salı / Tuesday
E) Cuma / Friday

Çözüm / Solution

Yanıt / Answer **B**

$$\underbrace{1 \quad 2 \quad 3 \quad \cdots \quad 52}_{51}$$

$51 \cdot 4 = 204$

$$\begin{array}{r|l} 204 & 7 \\ -\ 203 & \hline 29 \\ \hline 1 & \end{array}$$

⇒ gün önce Pazar / 1 day before was Sunday

1. $\forall x, y \in R$,

$x \uparrow y = 3x + 2y + 1$,

$3 \uparrow (2 \uparrow 1) = 2 \uparrow a$

$\Rightarrow$ **a = ?**

A) 9 B) $\frac{21}{2}$ C) $\frac{23}{2}$ D) $\frac{19}{2}$ E) 1

2. $\forall x, y \in R$,

$a \square b = 2a - 3b$,

$(1 \square x) \square (-1) = 3$

$\Rightarrow$ **x = ?**

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) $\frac{3}{2}$ E) 2

3. $\forall x, y \in R$,

$\sqrt[3]{x} \Delta \sqrt[3]{y} = \frac{x+y+13}{4} \Rightarrow$ **$2 \Delta 3$ = ?**

A) 2 B) 4 C) 8 D) 12 E) 16

4. $a \Delta b = \begin{cases} 2a-b & , a < b \\ a^2+3b & , a = b \\ a+2b & , a > b \end{cases}$

$\Rightarrow$ **$(2 \Delta 3) \Delta (5 \Delta -2)$ = ?**

A) 2 B) 3 C) 4 D) 5 E) 6

5. $\forall x, y \in R$,

$x \Delta y = 2x + 2y + mxy + 8$,

$x \Delta (-4) = x$

$\Rightarrow$ **m = ?**

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) 2 D) 4 E) 8

6. **$(b^{-1} \Delta a^{-1}) \Delta (c \Delta d^{-1})$ = ?**

Δ	a	b	c	d	e
a	b	c	d	e	a
b	c	d	e	a	b
c	d	e	a	b	c
d	e	a	b	c	d
e	a	b	c	d	e

A) a B) b C) c D) d E) e

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7. $Z_{/5}$,

$$\frac{2^3+3^3+4^3}{2^2+3^2+4^2} = ?$$

A) 0
B) 1
C) 2
D) 3
E) 4

8. $Z_{/5}$,

$f(x) = \bar{3}x + \bar{2} \Rightarrow f^{-1}(x) = ?$

A) $\bar{2}x + \bar{1}$
B) $\bar{2}x + \bar{4}$
C) $x + \bar{3}$
D) $x + \bar{2}$
E) $\bar{2}x$

9. $13^{79} \equiv x \pmod 5 \Rightarrow x = ?$

A) 0
B) 1
C) 2
D) 3
E) 4

10. $x \in N$,

$1997^x \equiv 3 \pmod 5 \Rightarrow \min(x) = ?$

A) 1
B) 2
C) 3
D) 4
E) 5

11. $7^{8n+5} \equiv x \pmod 5 \Rightarrow x = ?$

A) 4
B) 3
C) 2
D) 1
E) 0

12. $77^{77} \equiv x \pmod{10} \Rightarrow x = ?$

A) 2
B) 3
C) 5
D) 6
E) 7

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
B	C	D	C	B	A	B	A	C	C	C	E

1. $2 \uparrow 1 = 3 \cdot 2 + 2 \cdot 1 + 1$

$= 6 + 2 + 1 = 9$

$3 \uparrow 9 = 3 \cdot 3 + 2 \cdot 9 + 1 = 9 + 18 + 1 = 28$

$2 \uparrow a = 3 \cdot 2 + 2a + 1 = 7 + 2a$

$28 = 7 + 2a \Rightarrow a = \frac{21}{2}$

Yanıt / The Answer is B

2. $(1 \square x) = 2 \cdot 1 - 3x = 2 - 3x$

$(2 - 3x) \square (-1) = 3$

$2(2-3x) - 3(-1) = 3$

$4 - 6x + 3 = 3$

$4 = 6x$

$x = \frac{2}{3}$

Yanıt / The Answer is C

3. $\left.\begin{matrix} \sqrt[3]{x} = 2 \Rightarrow x = 8 \\ \sqrt[3]{y} = 3 \Rightarrow y = 27 \end{matrix}\right\} \Rightarrow x = 8, y = 27$

$2 \Delta 3 = \frac{8 + 27 + 13}{4} = \frac{48}{4} = 12$

Yanıt / The Answer is D

4. $2 \Delta 3 = 2 \cdot 2 - 3 = 1 \quad (2 < 3)$

$5 \Delta -2 = 5 + 2(-2) = 1 \quad (5 > -2)$

$1 \Delta 1 = 1^2 + 3 \cdot 1 = 4 \quad (1 = 1)$

Yanıt / The Answer is C

5. $e = -4$

$x \Delta (-4) = x$

$2x + 2(-4) + mx(-4) + 8 = x$

$2x - 8 - 4mx + 8 - x = 0$

$x - 8 - 4mx + 8 = 0$

$x(1 - 4m) = 0$

$m = \frac{1}{4}$

Yanıt / The Answer is B

6. Tablodaki işlemin birim elemanı e'dir.

Identity element of the operation in table is e.

$a^{-1} = d, \quad b^{-1} = c, \quad d^{-1} = a$

$(b^{-1} \Delta a^{-1}) \Delta (c \Delta d^{-1})$

$= \underbrace{(c \Delta d)}_{b} \Delta \underbrace{(c \Delta a)}_{d}$

$= a$

Yanıt / The Answer is A

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7. $\frac{2^3+3^3+4^3}{2^2+3^2+4^2} = \frac{8+27+64}{4+9+16}$

$= \frac{3+2+4}{4+4+1}$

$= \frac{4}{4} = 1$

Yanıt / The Answer is B

8. $f(x) = \overline{3}x + \overline{2} \Rightarrow f^{-1}(x) = \frac{x-\overline{2}}{\overline{3}}$

$= \frac{\overline{1}}{\overline{3}}x - \frac{\overline{2}}{\overline{3}}$

$= \frac{\overline{6}}{\overline{3}}x + \frac{\overline{3}}{\overline{3}}$

$= \overline{2}x + \overline{1}$

Yanıt / The Answer is A

9. $\left.\begin{array}{l}(13)^1 \equiv 3 \pmod 5\\ (13)^2 \equiv 4 \pmod 5\\ (13)^3 \equiv 2 \pmod 5\\ (13)^4 \equiv 1 \pmod 5\end{array}\right\} \Rightarrow$ $79 \mid \underline{4}$, $\quad 19$

$3 \rightarrow$ kalan / remainder

$3^3 \equiv x \pmod 5 \Rightarrow x = 2$

Yanıt / The Answer is C

10. $x = 1 \Rightarrow 1997 \equiv 2 \pmod 5$

$x = 2 \Rightarrow 1997^2 \equiv 4 \pmod 5$

$x = 3 \Rightarrow 1997^3 \equiv 3 \pmod 5$

$\Rightarrow \min(x) = 3$

Yanıt / The Answer is C

11. $7^{8n+5} \equiv 2^{8n+5} \equiv x \pmod 5$

$\left.\begin{array}{l}2^1 \equiv 2 \pmod 5\\ 2^2 \equiv 4 \pmod 5\\ 2^3 \equiv 3 \pmod 5\\ 2^4 \equiv 1 \pmod 5\end{array}\right\} \Rightarrow (2^{4n})^2 \cdot 2^5 \equiv (1)^2 \cdot (2^4 \cdot 2) = 2 \Rightarrow x = 2$

Yanıt / The Answer is C

12. $77^{77} \equiv 7^{77} \equiv x \pmod{10}$

$\left.\begin{array}{l}7^1 \equiv 7 \pmod{10}\\ 7^2 \equiv 9 \pmod{10}\\ 7^3 \equiv 3 \pmod{10}\\ 7^4 \equiv 1 \pmod{10}\end{array}\right\} \Rightarrow$ $77 \mid \underline{4}$, $\quad 19$

$1 \rightarrow$ kalan / remainder

$7^1 \equiv x \pmod{10} \Rightarrow x = 7$

Yanıt / The Answer is E

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1. $a,b \in IR$, $a \Delta b = \begin{cases} 2a-b, & a>b \\ 2b-a, & a<b \\ a+b, & a=b \end{cases}$

$\Rightarrow \sqrt{(5 \Delta 3) \Delta (-2 \Delta 3)} = ?$

A) $\sqrt{13}$ B) 8 C) 7 D) $\sqrt{6}$ E) 3

[HARRAN ÜNİVERSİTESİ – YÖS 2020]

---

2. $x \bigcirc y = -2(y \Delta x) + 2y$

$y \Delta x = 3y - 2x$

$\Rightarrow 4 \bigcirc 3 = ?$

A) –2 B) 0 C) 2 D) 3 E) 4

[DUMLUPINAR ÜNİVERSİTESİ – YÖS 2018]

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3. $95 < x < 102$

$5^x \equiv 3 \pmod 7 \Rightarrow x = ?$

A) 97 B) 98 C) 99 D) 100 E) 101

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2017]

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4. I. $x \Delta y = \begin{cases} y-x, & |x|>|y| \\ x-y, & |x|=|y| \\ 2\cdot(x+y) & |x|<|y| \end{cases}$

II. $(1 \Delta (-1)) \Delta (3 \Delta 0) = ?$

I. Eşitlikte Δ işleminin görevi belirlenmiştir. Buna göre II. Eşitlikte soru işaretinin yerine aşağıdakilerden hangisi gelmelidir?

The operation Δ is established in equation I. According to this operation determine which of the following number does stand for the question mark in equation II?

A) –2 B) –1 C) 0 D) 1 E) 2

[SELÇUK ÜNİVERSİTESİ – YÖS 2017]

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5. $16^8 + 16^7$

sayısının 5 ile bölümünden kalan kaçtır?

What is the remainder when the number $16^8 + 16^7$ is divided by 5?

A) 1 B) 2 C) 3 D) 4 E) 5

[KARADENİZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

---

6. $k \in Z+$

$5\cdot(8)^{4k+2016} \equiv x \pmod 6 \Rightarrow x = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

---

7. $m \Delta n = 7m - 3n - 5$

$x \Delta x = x \Rightarrow x = ?$

A) $\frac{5}{3}$ B) $\frac{3}{2}$ C) $\frac{5}{4}$ D) $\frac{3}{4}$ E) $\frac{7}{2}$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2016]

---

8. $\left.\begin{array}{l} \frac{3}{x} \Delta \frac{y}{2} = 6x - y + 5 \\ 6 \Delta a = 14 \end{array}\right\} \Rightarrow a = ?$

A) $\frac{3}{2}$ B) $\frac{1}{2}$ C) –2 D) $-\frac{5}{2}$ E) –3

[İSTANBUL ÜNİVERSİTESİ – YÖS 2015]

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9. $a \Delta b = a(b - a)k$

$\Rightarrow 2 \Delta (3 \Delta 5) = ?$

A) $12k^2 - k$

B) $8k$

C) $3k^2 - 4$

D) $k(2k - 4)$

E) $4k(3k - 1)$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2015]

---

10. $3^{200} \equiv x \pmod 7$

$\Rightarrow x = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2015]

---

11. **3^{303} sayısının birler basamağındaki rakam kaçtır?**

Which of the following is the ones digit of the number 3^{303}?

A) 1 B) 3 C) 5 D) 7 E) 9

[AKDENİZ ÜNİVERSİTESİ – YÖS 2014]

---

12. $x + y \equiv 6 \pmod 7$

$3x - y \equiv 2 \pmod 7$

$x - y \equiv ? \pmod 7$

A) 2 B) 3 C) 4 D) 5 E) 6

[AKDENİZ ÜNİVERSİTESİ – YÖS 2013]

---

13. **I.** $a \otimes b = \begin{cases} a^3 + b^3, & a < b \\ 3ab - 15 & a \geq b \end{cases}$

II. $(-2) \otimes (3 \otimes 2) = ?$

I. Eşitlikte $\otimes$ işleminin görevi belirlenmiştir. Buna göre II. Eşitlikte soru işaretinin yerine aşağıdakilerden hangisi gelmelidir?

The operation $\otimes$ is established in equation I. According to this operation determine which of the following number does stand for the question mark in equation II?

A) 13 B) 14 C) 15 D) 17 E) 19

[SELÇUK ÜNİVERSİTESİ – YÖS 2013]

---

14. $17^{2013} \equiv x \pmod 3$

$\Rightarrow x = ?$

A) 4 B) 5 C) 3 D) 1 E) 2

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2012]

---

15. $m \otimes n = mn - 1$

$\Rightarrow 3^{2 \otimes 5} \otimes 3^{1 \otimes 4} = ?$

A) $3^5 - 2$ B) $3^{12} - 1$ C) 3 D) 27 E) $3^{27} - 5$

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2012]

---

16. $3^{2a} \bigtriangledown \frac{2^5}{b} = 6ab - 7$

$9 \bigtriangledown 8 = ?$

A) 5 B) 17 C) 23 D) 31 E) 47

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2011]

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	E	E	A	B	B	A	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	B	D	D	E	E	B	B

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1. $5 \Delta 3 = 2 \cdot 5 - 3 = 7$

$-2 \Delta 3 = 2 \cdot 3 - (-2) = 8$

$7 \Delta 8 = 2 \cdot 7 - 8 = 6$

$\sqrt{(5 \Delta 3) \Delta (-2 \Delta 3)} = \sqrt{6}$

Yanıt / The Answer is D

---

2. $4 \circ 3 = -2(3 \Delta 4) + 2 \cdot 3$

$3 \Delta 4 = 3 \cdot 3 - 2 \cdot 4$

$= 1$

$4 \circ 3 = -2 \cdot 1 + 2 \cdot 3$

$= -2 + 6$

$= 4$

Yanıt / The Answer is E

---

3. $5^1 \equiv 5$

$5^2 \equiv 4$

$5^3 \equiv 6$

$5^4 \equiv 2$

$5^5 \equiv 3$

$5^6 \equiv 1 \Rightarrow (5^6)^{16} \cdot 5^5 \equiv 3 (\text{mod} 7)$

$6 \cdot 16 + 5 = 101$

Yanıt / The Answer is E

---

4. $1 \Delta (-1) \quad |1| = |-1|$

$1 \Delta (-1) = 1 - (-1) = 2$

$3 \Delta 0 \quad |3| > |0|$

$3 \Delta 0 = 0 - 3 = -3$

$(1 \Delta (-1)) \Delta (3 \Delta 0)$

$2 \Delta -3 = |2| < |-3|$

$2 \Delta -3 = 2 \cdot (2 - 3)$

$= 2 \cdot -1$

$= -2$

Yanıt / The Answer is A

---

5. $16^8 + 16^7 \equiv x \, (\text{mod} 5)$

$16 \equiv 1 \, (\text{mod} 5) \Rightarrow 16^8 \equiv 1 (\text{mod} 5)$

$16 \equiv 1 \, (\text{mod} 5) \Rightarrow 16^7 \equiv 1 (\text{mod} 5)$

$\Rightarrow x = 2$

Yanıt / The Answer is B

---

6. $8^1 \equiv 2 \pmod 6$

$8^2 \equiv 4 \pmod 6$

$8^3 \equiv 2 \pmod 6$

$8^4 \equiv 4 \pmod 6$

$\vdots \qquad \vdots$

$8^{4k+2016} \equiv 4 \pmod 6$

$5 \cdot 4 \equiv x \pmod 6$

$2 \equiv x \pmod 6$

Yanıt / The Answer is B

---

7. $m \Delta n = 7m - 3n - 5$

$x \Delta x = 7 \cdot x - 3 \cdot x - 5 = x$

$4x - 5 = x$

$3x = 5$

$x = \frac{5}{3}$

Yanıt / The Answer is A

---

8. $\frac{3}{x} \Delta \frac{y}{2} = 6x - y + 5$

$6 \Delta a = 14$

$\frac{3}{x} = 6, \frac{y}{2} = a$

$6x = 3$

$x = \frac{1}{2}, y = 2a$

$6 \cdot \frac{1}{2} - 2a + 5 = 14$

$8 - 14 = 2a$

$-6 = 2a$

$-3 = a$

Yanıt / The Answer is E

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9. $a \Delta b = a(b-a)k$

$2 \Delta (3 \Delta 5) = 2 \Delta [3 \cdot (5-3) \cdot k]$

$= 2 \Delta [6k]$

$= 2 \cdot [6k-2] \cdot k$

$= 4 \cdot [3k-1] \cdot k$

$= 4k\,[3k-1]$

Yanıt / The Answer is E

---

10. $3^1 = 3$

$3^2 = 2$

$3^3 = 6$

$3^4 = 4$

$3^5 = 5$

$3^6 = 1$

$3^{200} = (3^6)^{33} \cdot 3^2 = 1 \cdot 2 = 2 \pmod 7$

Yanıt / The Answer is B

---

11. $3^1 \equiv 3 \pmod{10}$

$3^2 \equiv 9 \pmod{10}$

$3^3 \equiv 7 \pmod{10}$

$3^4 \equiv 1 \pmod{10}$

$3^5 \equiv 3 \pmod{10}$

$303 \mid 4$

$= \quad 75$

23

20

$= \quad$

$3 \rightarrow$ Kalan / Remainder

Yanıt / The Answer is D

---

12. $x + y = 6$

$+\ 3x - y = 2$

$4x = 8$

$x = 2$

$x + y = 6 \Rightarrow 2 + y = 6$

$y = 4$

$2 - 4 \equiv -2 \pmod 7$

$-2 + 7 \equiv 5 \pmod 7$

Yanıt / The answer is D

---

13. $3 \otimes 2 \rightarrow 3 \geq 2 \rightarrow 3 \cdot 3 \cdot 2 - 15 = 3$

$3 \otimes 2 = 3$

$-2 \otimes \underbrace{(3 \otimes 2)}_{3} \rightarrow -2 \otimes 3 \rightarrow -2 < 3$

$\underbrace{\qquad\qquad}_{15}$

$\rightarrow (-2)^3 + (3)^3$

$\rightarrow -8 + 27$

$\rightarrow 19$

Yanıt / The answer is E

---

14. $17 \equiv 2 \pmod 3$

$2^1 \equiv 2 \pmod 3$

$2^2 \equiv 1 \pmod 3$

$\vdots \qquad \vdots$

$2^{2013} \equiv x \pmod 3 \Rightarrow x = 2$

$2013 \mid 2$

$= \quad$

$1 \rightarrow$ Kalan / Remainder

Yanıt / The Answer is E

---

15. $2 \otimes 5 = 2 \cdot 5 - 1 = 9$

$1 \otimes 4 = 1 \cdot 4 - 1 = 3$

$\Rightarrow 3^{2 \otimes 5} \otimes 3^{1 \otimes 4} = 3^9 \cdot 3^3 - 1$

$= 3^{12} - 1$

Yanıt / The Answer is B

---

16. $3^{2a} = 9 = 3^2 \Rightarrow 2a = 2$

$a = 1$

$\frac{2^5}{b} = 8 \Rightarrow 32 = 8b$

$b = 4$

$9 \bigtriangledown 8 = 6 \cdot 1 \cdot 4 - 7 = 24 - 7 = 17$

Yanıt / The Answer is B

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1. $x \in Z$,

$a \circledast b = a \cdot b + \frac{b}{a}$, $x \circledast (2 \circledast 4) = 52 \Rightarrow \mathbf{x = ?}$

A) 5 B) 4 C) 3

D) 2 E) 1

2. $x \Delta y = \begin{cases} 3x - y & , x \cdot y > 0 \\ x - y & , x \cdot y < 0 \end{cases}$

$x \bigstar y = 2x + y \Rightarrow \mathbf{(-4 \bigstar 2) \Delta (-2 \Delta -3) = ?}$

A) −19 B) −16 C) −15

D) −12 E) −9

3. $x \Delta y = 3xy - 3x - 9y + 12$,

$x \Delta e = x \Rightarrow \mathbf{e = ?}$

A) 1 B) $\frac{3}{4}$ C) $\frac{4}{3}$

D) $-\frac{1}{3}$ E) $-\frac{3}{4}$

4. $x \blacktriangledown y = 3x - 4y + 1$

$\Rightarrow \mathbf{2 \blacktriangledown 3 = ?}$

A) −5 B) 0 C) 2

D) 4 E) 6

5. $(a, b) \square (c, d) = a^2 \cdot c + b \cdot d^2 \Rightarrow \mathbf{(2, 1) \square (3, 3) = ?}$

A) 19 B) 18 C) 20

D) 21 E) 22

6. $(x,y) \Delta (z,t) = x^3 \cdot z - y^3 \cdot t$

$\Rightarrow \mathbf{(-1,1) \Delta (-2,2) = ?}$

A) 0 B) 1 C) 2

D) 3 E) 4

7. $x \bigcirc y = x - y$

$x \bigstar y = x^2 + xy + y^2$

$x \bigtriangledown y = x^3 - y^3$

$\Rightarrow \mathbf{\frac{(4 \bigtriangledown 1)}{(4 \bigstar 1)} \cdot \frac{1}{(4 \bigcirc 1)} = ?}$

A) 5 B) 4 C) 3

D) 2 E) 1

8. $a \blacklozenge b = a \cdot b$

$a \square b = a^2 + b^2 - ab$

$a \Delta b = a + b$

$\Rightarrow \mathbf{(\sqrt[3]{3} \Delta 1) \blacklozenge (\sqrt[3]{3} \square 1) = ?}$

A) $\sqrt{3} + 1$ B) $\sqrt{3} - 1$ C) 3

D) $3\sqrt{5}$ E) 4

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9. $214^{16} \equiv x \pmod 3 \Rightarrow x = ?$

A) 5 B) 4 C) 3

D) 2 E) 1

10. $2^{81} \equiv x \pmod 4 \Rightarrow x = ?$

A) 0 B) 1 C) 2

D) 3 E) 4

11. $(2006)^{2006} \equiv x \pmod{10} \Rightarrow x = ?$

A) 0 B) 2 C) 4

D) 6 E) 8

12. $(2018)^{2018} \equiv x \pmod{10} \Rightarrow x = ?$

A) 8 B) 4 C) 2

D) 6 E) 0

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13. $Z_{/5} \Rightarrow \left(\frac{3}{4}\right)^3 = ?$

A) 4 B) 3 C) 2

D) 1 E) 0

14. $2^{138} + 3^{23} \equiv x \pmod 4 \Rightarrow x = ?$

A) 0 B) 1 C) 2

D) 3 E) 4

15. $4^{12} + 3^{80} \cdot 2^9 \equiv x \pmod 6 \Rightarrow x = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

16. $1 + 2 + 3 + ... + 100 \equiv x \pmod 2 \Rightarrow x = ?$

A) 1 B) 0 C) 2

D) 4 E) 5

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	C	C	A	D	A	E	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	A	D	B	B	D	D	B

1. $\frac{7}{x \star y} = \frac{1}{x} + y \Rightarrow 2 \star 3 = ?$

A) $\frac{14}{3}$ B) $\frac{14}{6}$ C) 2

D) $\frac{7}{2}$ E) $\frac{49}{2}$

2. $x \Delta y = x + y - 2\,(y \Delta x) \Rightarrow 2 \Delta 4 = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

3. $a \star 2b = \frac{a}{b} + a - b^2 \Rightarrow 32 \star 16 = ?$

A) –20 B) –24 C) –28

D) –32 E) –64

4. $a \blacktriangle b = a^{2b} + a^b \cdot b^a$,

$x \blacktriangle y = 6$

$y \blacktriangle x = 3$ $\Rightarrow x^y + y^x = ?$

A) 6 B) 5 C) 4

D) 3 E) 2

5. $xTy = \frac{x \cdot y}{x + y}$,

$1 + \frac{2}{1 + \frac{2Tm}{2}} = \frac{3}{2} \Rightarrow m = ?$

A) –2 B) –3 C) –4

D) –5 E) –6

6. $A = \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^{k-1}$

$A \equiv x \pmod 3 \Rightarrow x = ?$

A) 0 B) 2 C) 1

D) 4 E) 5

7. $A = 7^{11} + 6! + 3^{15} \equiv x \pmod{10} \Rightarrow x = ?$

A) 0 B) 1 C) 2

D) 4 E) 6

8. $Z_{/5}, x^2 + \bar{3} = \bar{2} \Rightarrow ÇK(SS) = ?$

A) $\{\bar{1}, \bar{2}\}$

B) $\{\bar{2}, \bar{3}\}$

C) $\{\bar{3}, \bar{4}\}$

D) $\{\bar{1}, \bar{3}\}$

E) $\{\bar{2}, \bar{4}\}$

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9. $1! + 2! + 3! + \ldots + 99! + 100! \equiv x \pmod 8 \Rightarrow$ **x = ?**

A) 0 B) 1 C) 2

D) 3 E) 4

10. $5^{121} + 7^{92} \equiv x \pmod{11} \Rightarrow$ **x = ?**

A) 3 B) 5 C) 6

D) 9 E) 10

11. $2 + 2^2 + 2^3 + 2^4 + \ldots + 2^{99} \equiv x \pmod{10} \Rightarrow$ **x = ?**

A) 0 B) 1 C) 2

D) 3 E) 4

12. $x \in \bar{Z}^-$,

$17 - 4x \equiv 5 \pmod{14} \Rightarrow$ **max(x) = ?**

A) –5 B) –4 C) –3

D) –2 E) –1

13. $A = 277^{77} \equiv x \pmod{10} \Rightarrow$ **x = ?**

A) 9 B) 8 C) 7

D) 4 E) 2

14. $1995^{1995} \equiv x \pmod 9 \Rightarrow$ **x = ?**

A) 3 B) 4 C) 5

D) 6 E) 0

15. $A = 11^{4k+8} \equiv x \pmod 8 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

16. Z_{77}, $f(x) = \bar{3}x + \bar{2} \Rightarrow$ **$f^{-1}(x)$ = ?**

A) $\bar{2}x + \bar{3}$ B) $\bar{2}x + \bar{5}$ C) $\bar{3}x + \bar{4}$

D) $\bar{5}x + \bar{4}$ E) $\bar{6}x + \bar{1}$

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YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	B	C	D	B	A	A	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	E	E	B	C	E	A	D

1. $\forall x, y \in R$,

$$x \Delta y = \begin{cases} 3x - y & x \cdot y > 0 \\ x - y & x \cdot y < 0 \end{cases}$$

$x \star y = 2x + y \Rightarrow$ **$(-4 \star 2) \Delta (-2 \Delta -3) = ?$**

A) -19 B) -16 C) -15

D) -12 E) -9

2. $a, b \in Z^+$

$a \oplus b = a \cdot b + a + b$, $x \oplus (3 \oplus 7) = 95 \Rightarrow$ **$x = ?$**

A) 5 B) 4 C) 3

D) 2 E) 1

3. $(84)^{-63} \equiv x \pmod 5 \Rightarrow$ **$\min(x) = ?$**

A) 0 B) 1 C) 2

D) 3 E) 4

4. $Z_{/5}$,

$f(x) = \overline{3}x + \overline{2} \Rightarrow$ **$f^{-1}(x) = ?$**

A) $\overline{2}x + \overline{3}$ B) $\overline{2}x + \overline{5}$ C) $\overline{3}x + \overline{4}$

D) $\overline{2}x + \overline{1}$ E) $\overline{6}x + \overline{1}$

5. $x \otimes y = (x - y)^{y-2}$,

$(2 \otimes 3) + (3 \otimes 2) = ?$

A) 4 B) 3 C) 2

D) 1 E) 0

6. $Z_{/5} \Rightarrow \left(\frac{3}{4}\right)^3 = ?$

A) 4 B) 3 C) 2

D) 1 E) 0

7. $0! + 1! + 2! + 3! + \ldots\ldots + 99! + 100! = A$

$A \equiv x \pmod 8 \Rightarrow$ **$x = ?$**

A) 0 B) 1 C) 2

D) 3 E) 4

8. $x \Delta y = x \cdot y - x - y + 3$

Δ işlemine göre hangi elemanın tersi yoktur?

According to operation Δ which one of the elements below doesn't have an inverse element?

A) -1 B) $-\frac{1}{2}$ C) 0

D) 1 E) 2

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $a \Delta b = 2a + 2b - ab - 2 \Rightarrow 3^{-1} = ?$

A) $\frac{3}{2}$ B) 3 C) $\frac{2}{5}$

D) -3 E) $\frac{3}{5}$

10. $(-5)^{-27} \equiv x \pmod 7 \Rightarrow x = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

11. $Z_{/7}$ de

$2x + 5y = 6$

$2x + 2y = 5 \Rightarrow$ **ÇK(SS) = ?**

A) {(3,2)} B) {(2,5)} C) {(1,5)}

D) {(0,3)} E) {(3,4)}

12. $x \Delta y = x + y + 2 - a$

$5^{-1} = 3 \Rightarrow a = ?$

A) 4 B) 7 C) 6

D) 5 E) 4

METROPOL YAYINLARI • METROPOL PUBLICATIONS

13.

Δ	İ	N	C	E
İ	N	C	E	İ
N	C	E	İ	N
C	E	İ	N	C
E	İ	N	C	E

$(İ \Delta N^{-1}) \Delta x = C \Rightarrow x = ?$

A) İ B) N C) C

D) E E) Ø

14.

Δ	A	L	İ	C	O
A	İ	C	O	A	L
L	C	O	A	L	İ
İ	O	A	L	İ	C
C	A	L	İ	C	O
O	L	İ	C	O	A

$(L \Delta İ)^{-1} \Delta (O^{-1} \Delta A^{-1}) = ?$

A) A B) L C) İ

D) C E) O

15. $5x + 4 \equiv 1 \pmod{11} \Rightarrow x = ?$

A) 5 B) 6 C) 7

D) 8 E) 9

16. $Z_{/5}$ de

$x^2 + 3x - 3 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) {1,3} B) {1,4} C) {2,4}

D) {3,2} E) {3,4}

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	D	E	D	E	B	C	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	A	C	C	D	D	B	E

BÖLÜM CHAPTER 15

TARİHSEL NOT / HISTORICAL NOTE

Sir David Roxbee Cox [1924 -]

Matematiksel istatik, olasılık ve yöneylem araştırması alanlarında çalışmış İngiliz istatistikçi. Deney tasarımı, ikili dataların analizi, stokastik süreçler, kalite kontolünde ve yöneylem araştırmasında yeni yöntemlerin geliştirilmesi, kuyruk ve yenilenme teorisine olan katkıları ile tanınır.

British statistician distinguished for his work in mathematical statistics and applied probability, especially in relation to industrial and operational research. His varied contributions include the design and analysis of statistical experiments, analysis of binary data and of point stochastic process and the development of new methods in quality control and operational research particularity in relation to queuing and renewal theory.

SAYMA İLKELERİ (PERMÜTASYON, KOMBİNASYON), BİNOM ve OLASILIK
COUNTING PRINCIPLES (PERMUTATIONS, COMBINATIONS), BINOMIAL & PROBABILITY

Bu bölüm, 27 çözümlü örnek, 12 çözümlü test sorusu, 4 çözümlü YÖS sorusu, 32 test sorusu içermektedir.

This chapter includes 27 examples with solutions, 12 tests questions with solutions, 4 YÖS questions with solutions and 32 test questions.

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SAYMA İLKELERİ (PERMÜTASYON, KOMBİNASYON), BİNOM ve OLASILIK
COUNTING PRINCIPLES (PERMUTATIONS, COMBINATIONS), BINOMIAL & PROBABILITY

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BÖLÜM
15
CHAPTER

SAYMA İLKELERİ (PERMÜTASYON, KOMBİNASYON), BİNOM ve OLASILIK

COUNTING PRINCIPLES (PERMUTATIONS, COMBINATIONS), BINOMIAL & PROBABILITY

SAYMA ve SAYMANIN İLKELERİ
COUNTING and PRINCIPLES of COUNTING

Tanım: Sayma, nesnelerin ve olayların sayılarla birebir eşlenmesidir. Saymanın temel ilkeleri, **toplama ve çarpmadır.**

Definition: Counting is matching the objects and the events with the numbers. The fundamental principles of counting are the principle of **addition** and the principle of **multiplication.**

TOPLAMA İLKESİ / THE PRINCIPLE of ADDITION

Birincisi n_1 yolla, ikincisi n_2 yolla yapılabilen iki işlem göz önüne alındığında, iki işlemden ancak birinden biri yapılabiliyorsa, bu işlemlerden biri **veya** diğeri $n_1 + n_2$ yolla yapılabilir. Bu ilke, sonlu sayıda işlemi içine alan durumlar için genelleştirilebilir.

Two different events which the 1st one can happen in n_1 ways and the 2nd one can happen in n_2 ways are taken in mind for an operation, if only one way of one of two events can happen in a single run, the operation itself can happen eigther by the first way or the second way in $n_1 + n_2$ ways. This principle can be generalized for infinite number of operations.

PEKİŞTİRME / REINFORCE

✓ Birbininden farklı 7 kitap ile 8 dergi arasından bir kitap veya bir dergi kaç farklı şekilde seçilebilir?

✓ From 7 different books and 8 different magazines, one book or magazine is going to be picked, in how many different ways can this task be done?

.....[15]

Örnek / Example 1

Bir torbada 4 tane kırmızı, 5 tane beyaz bilye olduğuna göre, **bu torbadan 1 tane kırmızı veya 1 tane beyaz bilye kaç farklı şekilde alınabilir?**

If there are 4 red and 5 white marbles in a bag. How many different ways can 1 red or 1 white marble be chosen from the bag?

A) 9 B) 10 C) 12 D) 16 E) 20

Çözüm / Solution

Yanıt / Answer **A**

4 tane kırmızı bilyeden 1 tane kırmızı bilye 4 farklı şekilde seçilebilir. 5 tane beyaz bilyeden 1 tane beyaz bilye 5 farklı şekilde seçilebilir.

4 tane kırmızı, 5 tane beyaz bilye arasından 1 tane kırmızı veya 1 tane beyaz bilye toplama ilkesine göre 4 + 5 = 9 farklı şekilde seçilir.

1 red marble can be chosen from 4 red marbles in 4 different ways, and 1 white marble can be chosen from 5 white marbles in 5 different ways.

Among the 4 red and 5 white marbles, 1 red or 1 white marble can be chosen in 4 + 5 = 9 different ways.

ÇARPMA İLKESİ / THE PRINCIPLE of MULTIPLICATION

n_1 farklı yolla ortaya çıkan bir olay göz önüne alındığında, her n_1 yolu için, bu yolu izleyen ikinci olay n_2 farklı yolla ortaya çıkarsa, tüm olayın farklı biçimde ortaya çıkma sayısı $\mathbf{n_1 \cdot n_2}$ olur. Eğer n_2 olayının her biri için bunu izleyen üçüncü olay n_3 yolla ortaya çıkarsa, tüm olayın ortaya çıkma sayısı $\mathbf{n_1 \cdot n_2 \cdot n_3}$ olur. Bu şekilde devam edilirse,

$\mathbf{n_1 \cdot n_2 \cdot n_3 \cdot \ldots\ldots n_k}$ yazılabilir. Buna, **çarpma ilkesi denir.**

The first event can occur in n_1 different ways. After the first event has occurred. The second event can occur in n_2 different ways and the third event can occur in n_3 different ways. The number of ways that the whole event can occur is $\mathbf{n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_k}$ ways. This is called the principle of multiplication.

n_1	n_2	n_3	n_4		n_{r-1}	n_r

$n_1 \cdot n_1 \cdot n_1 \cdot n_{1 \ldots} n_{r-1} \cdot n_r$

Örneğin; Bir okuldaki 2 matematik, 3 Türkçe, 4 tarih öğretmeni arasından 1 matematik, 1 Türkçe ve 1 tarih öğretmeni, $2 \cdot 3 \cdot 4 = 24$ farklı yolla seçilebilir.

For example; From 2 Mathematics, 3 Literature and 4 History teachers in one school, 1 Mathematics, 1 Literature and 1 History teachers can be chosen in $2 \cdot 3 \cdot 4 = 24$ different ways.

ÖNEMLİ / IMPORTANT

• Toplama yoluyla sayma yapılırken **"veya"** ile **"ya da"**, çarpma yoluyla sayma yapılırken ise **"ve"** bağlacının kullanıldığına dikkat edilmelidir.

• We are taken into consideration, when counting by addition rule, **"either"** and **"or"** is used; and when counting by multiplication rule, **"and"** is used.

Örnek / Example 2

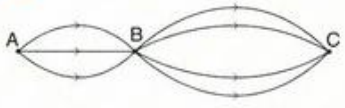
A kentinden B kentine giden 3, B kentinden C kentine giden 4 farklı yol vardır. **Bir kişi aynı yoldan birden fazla geçmemesi koşuluyla A dan C ye, B den geçmek koşuluyla kaç farklı gidiş – geliş yapabilir?**

There are 3 ways from the City A to the City B, and 4 ways from B to C, if the person will not pass the same path more than once. In how many different ways can he go from A to C with passing through the City Band then return?

A) 72 B) 108 C) 144 D) 288 E) 324

Çözüm / Solution

Yanıt / Answer **A**



Bu kişinin, A dan B ye gideceği her yol için B den C ye dört seçeneği vardır. Bu nedenle;

A dan C ye, $3 \cdot 4 = 12$ farklı yoldan gidilebilir.

C den A ya ise, $3 \cdot 2 = 6$ farklı yoldan gidebilir.

$3 \cdot 4 \cdot 3 \cdot 2 = 72$ gidiş – geliş yapabilir.

There are 3 ways from A to B and 4 ways from B to C.

For that reason,

$3 \cdot 4 = 12$ ways to go from A to C and $3 \cdot 2 = 6$ ways from C to A. There are $3 \cdot 4 \cdot 3 \cdot 2 = 72$ ways to go from A to C and from C to A.

PEKİŞTİRME / REINFORCE

✓ {0, 1, 2, 3, 4, 5} kümesinin elemanlarını kullanarak rakamları tekrarsız ve 5 ile bölünebilen 4 basamaklı kaç farklı sayı yazılabilir?

✓ How many different 4 digit numbers with no repeating digits which are divisible by 5 can be written using elements of the set {0, 1, 2, 3, 4, 5}?

.....[108]

Etkinlikler / Activities

A = {1, 2, 3, 4} kümesinin elemanlarıyla 3 basamaklı
3 digit numbers with elements of set a = {1, 2, 3, 4}

a. Kaç farklı sayı yazılabilir?

How many different numbers can be written?

Çözüm / Solution:

Yüzler, onlar ve birler basamağının her birine 4 farklı rakamdan biri gelecektir.
O halde, 4·4·4 = 64 farklı sayı yazılabilir.
4 will be placed into the ones, tens and hundreds places and 4·4·4 = 64 different numbers can be written.

Yüzler Basamağı Hundreds Place	Onlar Basamağı Tens Place	Birler Basamağı Ones Place
4	4	4

b. Rakamları tekrarsız kaç farklı sayı yazılabilir?

How many different numbers can be written with no repetition allowed?

Çözüm / Solution:

Yüzler basamağına 4 farklı rakamdan biri,
onlar basamağına 3 farklı rakamdan biri,
birler basamağına 2 farklı rakamdan biri gelebilir.
O halde, 4 · 3 · 2 = 24 farklı sayı yazılabilir.
4 different numbers is placed in hudreds place, 3 different numbers in tens digits and 2 different numbers can be written.
Therefore, 4·3·2 = 24 different numbers can be written.

Yüzler Basamağı Hundreds Place	Onlar Basamağı Tens Place	Birler Basamağı Ones Place
4	3	2

c. Rakamları tekrarsız kaç farklı çift sayı yazılabilir?

How many different even numbers can be written with no repetition allowed?

Çözüm / Solution:

Bir sayının çift olabilmesi için birler basamağının çift olması gerekir.
O halde, birler basamağına 2 veya 4 rakamlarından biri gelmelidir.
Bu iki sayıdan biri birler basamağına yazıldığında geriye kalan 3 sayıdan biri yüzler, 2 sayıdan biri onlar basamağına yazılabilir.
O halde, 3 · 2 · 2 = 12 farklı sayı yazılabilir.
If the number is an even, then ones place must be an even. The numbers 2 or 4 will be choosen.
The hundreds place is to be 3 tens place is 2 and ones place is to be 2.
Therefore, 3·2·2 = 12 different numbers can be written.

Yüzler Basamağı Hundreds Place	Onlar Basamağı Tens Place	Birler Basamağı Ones Place
3	2	2

{2, 4}

d. 300 den büyük kaç farklı sayı yazılabilir?

How many different numbers greater than 300 can be written?

Çözüm / Solution:

Sayının 300 den büyük olması için yüzler basamağına 3 veya 4 rakamlarından biri gelmelidir.

Yüzler Basamağı Hundreds Place	Onlar Basamağı Tens Place	Birler Basamağı Ones Place
2	4	4

Rakamların tekrarsız koşulu olmadığından onlar ve birler basamağına 4 rakamdan {3, 4} biri gelebilir. Bu nedenle, $2 \cdot 4 \cdot 4 = 32$ farklı sayı yazılabilir.

If the numbers greater than 300, in hundreds place is to be 3 or 4.
There is no repetition, tens and ones places are to be 4 in four numbers in the set.
Therefore, 2·4·4 = 32 different numbers can be written.

e. 300 den büyük rakamları tekrarsız kaç farklı sayı yazılabilir?

How many different numbers greater than 300 can be written with no repetition allowed?

Çözüm / Solution:

Yüzler basamağına 2 farklı sayı gelebilir. Fakat rakamlar tekrarsız olacağından onlar basamağına 3, birler basamağına 2 farklı sayı gelebilir. O halde, $2 \cdot 3 \cdot 2 = 12$ farklı sayı yazılabilir.

Yüzler Basamağı Hundreds Place	Onlar Basamağı Tens Place	Birler Basamağı Ones Place
2	3	2

There is no repetition, tens place is to be 3 ones place is to be 2 and hundreds place is to be 2.
Therefore, 2·3·2 = 12 different numbers can be written.

PERMÜTASYON / PERMUTATIONS

Tanım: n farklı elemanın bir elemanı birinci, diğeri ikinci bir diğeri de üçüncü olacak şekilde sıralanarak devam etmesine **permütasyon** denir.

n farklı nesnenin (her biri bir kez kullanılması koşuluyla) r tanesi ($r \le n$) sıralanırsa, elde edilecek farklı sıralanışların sayısı P(n, r) ile gösterilirse,

Definition: A permutation of n different elements is an ordering of the elements such that one element is first, one is second, one is third and so on.

The number of permutations (arrangements, orders) of n elements taken r at a time. It is denoted by P(n, r).

$$P(n,r)=\frac{n!}{(n-r)!}$$

yazılır. Burada, P (n, r), n elemanlı bir kümenin r li permütasyonlarının sayısını göstermektedir.

Here, P(n,r) denotes the number of permutations of n elements r at a time.

PEKİŞTİRME / REINFORCE

✓ **P(5, 2) = ?**

.....[20]

✓ **P(6, 4) = ?**

..... [360]

✓ **P(7, 0) = ?**

..... [1]

✓ **P(9, 1) = ?**

..... [9]

ÖNEMLİ / IMPORTANT

- P(n, n) = n!
- P(n, 1) = n

ÖNEMLİ / IMPORTANT

- Permütasyon tanımına göre birbirinden farklı diziliş olayları permütasyonla çözülebilir. Her problem çarpma kuralı ile de çözülebilir.
- According to the definition of permutation it can be used in determining the number of ways that elements can be arranged (in order). Every problem can be solved by the multiplication rule.

Etkinlikler / Activities

a. $P(4,2)=\frac{4!}{(4-2)!}=\frac{4!}{2!}=\frac{4\cdot 3\cdot 2!}{2!}=12$

b. $P(3,1)=\frac{3!}{(3-1)!}=\frac{3!}{2!}=3$

c. $P(6,2)=\frac{6!}{(6-2)!}=\frac{6!}{4!}=30$

d. $P(10,3)=\frac{10!}{(10-3)!}=\frac{10!}{7!}=720$

Örnek / Example 3

$P(n, 3) = 2^{-1} \cdot P(n, 4) \Rightarrow$ **n = ?**

A) 5 B) 6 C) 7 D) 8 E) 9

Çözüm / Solution

Yanıt / Answer **A**

$P(n, 3) = 2^{-1} \cdot P(n, 4) \Rightarrow$

$$\frac{n!}{(n-3)!}=2^{-1}\cdot\frac{n!}{(n-4)!}$$

$$\frac{1}{(n-3)}=\frac{1}{2} \Rightarrow n-3=2 \Rightarrow n=5$$

Örnek / Example 4

8 atletin katıldığı bir final koşusunda Altın, Gümüş, Bronz madalyaları için ilk üç sıra kaç farklı şekilde belirlenebilir?

8 athletes enter a final running contest, for the gold, silver and bronze medals. in how many different ways can these athletes become first, second and third?

A) 300 B) 336 C) 360 D) 400 E) 420

Çözüm / Solution

Yanıt / Answer **B**

$$P(8,3) = \frac{8!}{(8-3)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 336$$

PEKİŞTİRME / REINFORCE

✓ **Bir yarışta 10 at kaç farklı şekilde birinci, ikinci ve üçüncü gelebilir?**

✓ in a race, there are 10 horses. in how many ways can horses become in first, second and third places?

.....[720]

Örnek / Example 5

3 erkek ve 4 kız yan yana dizilmiş 7 sandalyeye, kızlar ve erkekler kendi aralarında yan yana kaç değişik biçimde oturabilirler?

In how many ways can 3 boys and 4 girls be seated in a row, if boys and girls are to occupy adjacent seats?

A) 144 B) 216 C) 288 D) 324 E) 408

Çözüm / Solution

Yanıt / Answer **C**

Burada dağılımın iki şekli vardır.

I. K K K K E E E,

II. E E E K K K K.

Her iki durumda da kızlar P(4, 4) = 4! = 24, erkekler P(3, 3) = 3! = 6 şekilde oturabilirler.

Böylece hepsi 2! · 4! · 3! = 288 değişik şekilde oturabilirler.

There are 2 kinds of distribution.

I. K K K K E E E,

II. E E E K K K K.

Girls can be seat in p(4, 4) = 4! = 24 ways and girls can sit in P(3, 3) = 3! = 6 ways, and finaly boys and girls can be seat in 2!·4!·3! = 288 different ways.

PEKİŞTİRME / REINFORCE

✓ **A = {1, 2, 3, 4, 5} kümesinin elemanlarını kullanarak rakamları tekrarsız ve üç basamaklı kaç farklı sayı yazılabilir?**

✓ How many different 3 digit numbers can be written using the elements of set A = {1, 2, 3, 4, 5} with no repeatation allowed?

.....[60]

TEKRARLI (YİNELEMELİ) PERMÜTASYON
REPEATING PERMUTATIONS

n tane nesnenin n_1 tanesi bir türden, n_2 tanesi ikinci türden, ... n_k tanesi de k ninci türden olmak üzere n nesnenin n tanesi sıralandığında, $\frac{n!}{n_1! \cdot n_2! \ldots n_k!}$ tane farklı düzen elde edilebilir.

The number of arrangements of n things taken all at a time, of which n_1 are from first type, n_2 others are from second type, ..., and n_k are from type k, can be arranged in $\frac{n!}{n_1! \cdot n_2! \ldots n_k!}$ different ways.

PEKİŞTİRME / REINFORCE

✓ 99988559 sayısının rakamları kullanılarak 8 basamaklı kaç farklı sayı yazılabilir?

✓ How many different 8 digit numbers can be written by using digits of number 99988559?

.....**[420]**

Etkinlikler / Activities

ELEMENT sözcüğünün harfleri ile anlamlı veya anlamsız:

with the letters of the word ELEMENT, meaningfull or meaningless:

a. Kaç sözcük yazılabilir?

How many words can be written?

$$\frac{7!}{3! \cdot 1! \cdot 1! \cdot 1! \cdot 1!} = 840$$

b. E ile başlayıp E ile biten anlamlı veya anlamsız kaç sözcük yazılabilir?

How many words can be written beginning with E and ending with E?

E lerden 2 tanesinden birini başa, diğeri sona yazılırsa geriye kalan 5 harfle 5! = 120 farklı sayı yazılabilir.

There are 2 E's on the letters of the word. One E is written at the beginning of letter and the other E is written to the end of letter 5 letters can be written in 5! = 120 different ways.

DAİRESEL PERMÜTASYON / CIRCULAR PERMUTATIONS

n tane farklı nesnenin bir daire etrafında farklı sıralanmasında nesnelerden birisi bir yere yerleştirilir ve diğer (n–1) nesnenin sıralanışı,

$(n-1)\cdot(n-2)\cdot \ldots \cdot 2\cdot 1 = (n-1)!$ olur. Bu da, n nesnenin dairesel sıralanış sayısını verir.

The number of ways of arranging n different objects around a circle object is $(n-1)!$.

in other words, it is written by $(n-1)\cdot(n-2)\cdot \ldots \cdot 2\cdot 1 = (n-1)!$. It is the number of ways of arranging n different objects around a circle object.

Örnek / Example 6

6 kişilik bir aile yuvarlak bir masa etrafında kaç değişik biçimde oturabilirler?

In how many different ways can a family of six people be seated around a table?

A) 24 B) 48 C) 64 D) 78 E) 120

Çözüm / Solution

Yanıt / Answer **E**

$n = 6, \quad n - 1 = 5$

$(n - 1)! = 5! = 120$

Örnek / Example 7

6 kişilik bir aile anne ve baba yan yana olmak üzere, yuvarlak bir masa etrafında kaç değişik biçimde oturabilirler?

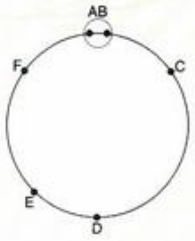
In how many ways can 6 persons of a family be seated at a round table if mother and father always sit together?

A) 24 B) 48 C) 96 D) 120 E) 240

Çözüm / Solution

Yanıt / Answer **B**

Anne ve baba, yan yana 2 farklı şekilde oturabilir.
Anne ve baba, yan yana oldukları için ikisi bir kişi gibi düşünülürse, aile 5 kişi kabul edilebilir.
Mother and father can sit together in two different ways, it is assumed that the family have 5 person.



$2! \cdot (n-1)! = 2! \cdot (5-1)!$

$= 2 \cdot 4!$

$= 48$

ÖNEMLİ / IMPORTANT

• n elemanlı bir kümenin r elemanlı ($n \geq r$) alt kümelerinden her birine n'in r'li kombinasyonu denir.

• Each one of r element subsets of a set with n elements ($n \geq r$) is called a combination of r elements from n elements.

KOMBİNASYON / COMBINATIONS

Tanım: n tane farklı nesne verilmiş ise, bu nesnelerden herhangi $r \leq n$ (r pozitif bir tamsayı) tanesi seçilip alınır ve bu r nesne herhangi bir sırada yazılırsa, buna r ninci mertebeden **kombinasyon** denir.

Permütasyon, sıralama problemlerinin çözümünü içerir ve sıralama önemlidir. Kümenin elemanlarından alt küme seçme problemleri kombinasyon problemi olup sıralama önemli değildir.

A = {a, b, c} kümesinin 2 li permütasyonları ve kombinasyonları aşağıda gösterilmiştir.

Definition: The number of combinations of a set of n different objects taken r at a time $r \leq n$, (is a positive integer).

In the permutations order is important but in the combinations order is not important.

Let us consider the set {a, b, c} and the permutations and combinations taken two at a time.

Farklı üç nesnenin 6 tane 2 li permütasyonu varken, 2 li kombinasyonları 3 tanedir.

3 different objects have 6 permutations with 2 elements while they have 3 combinations with 2 elements.

Permütasyonlar Permutations	Kombinasyonlar Combinations
ab	{a, b}
ba	{a, c}
ac	{b, c}
ca	
bc	
cb	

n elemanlı bir kümenin r elemanlı ($n \geq r$) kombinasyonlarının sayısı:

The number of combinations with n different objects taken r at a time ($n \geq r$) is:

$$C(n,r) = \binom{n}{r} = \frac{n!}{r!\cdot(n-r)!} = \frac{P(n,r)}{r!} \text{ dır.}$$

Özellik / Property

★ $C(n,0) = \binom{n}{0} = 1$

★ $C(n,1) = \binom{n}{1} = \binom{n}{n-1} = n$

★ $C(n, 0) + C(n, 1) + C(n, 2) + \ldots + C(n, n) = 2^n$

★ $C(n, r) = C(n, k) \Rightarrow$ i. $n = r + k$ ii. $r = k$

Örnek / Example 8

$C(3n, 2) - 8 \cdot C(n, n-2) = 42$ ise **n kaçtır?**

If $C(3n, 2) - 8 \cdot C(n, n-2) = 42$, then what is the value of n ?

A) 7 B) 6 C) 5 D) 4 E) 3

Çözüm / Solution

Yanıt / Answer **A**

$$\frac{(3n)!}{(3n-2)!\cdot 2!} - \frac{8\cdot n!}{2!\cdot(n-2)!} = 42 \Rightarrow \frac{3n(3n-1)}{2} - \frac{8\cdot n\cdot(n-1)}{2}$$

$$\frac{9n^2 - 3n - 8n^2 + 8n}{2} = 42$$

$$n^2 + 5n - 84 = 0$$

$$(n-7)\cdot(n+12) = 0$$

$$n - 7 = 0 \Rightarrow n = 7$$

Örnek / Example 9

Bir okulda öğrenci birliği için her yıl 4 üye seçiliyor.

Buna göre, 10 uygun öğrenci arasından birliğe kaç türlü üye seçilebilir?

In a school a comitee of 4 students gets picked every 4 years.

In how many diffferent ways can 4 students be selected out of 10 students for a committe in school?

A) 180 B) 210 C) 240 D) 360 E) 400

Çözüm / Solution

Yanıt / Answer **B**

Sıralama işlemi olmadığından problem, bir kombinasyon problemidir. n = 10 elemanlı bir kümenin 4 elemanlı alt kümelerinin sayısı bulunmalıdır.

There is no ordering, for that reason problem deals with combination problems. Number of subsets with 4 elements of a 10 element set should be found.

$$C(10,4) = \frac{10!}{4!\cdot(10-4)!} = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2} = 210$$

Örnek / Example 10

Bir sınıfta 6 öğrenci vardır. Her sınava 2 öğrenci girmesi koşulu ile bu 6 öğrenci 3 ayrı sınava kaç farklı yolla girebilir?

There is 6 students in a class. In how many different ways can 6 students take 3 different examinations if 2 of them is to be included in one examination?

A) 60 B) 90 C) 120 D) 150 E) 180

Çözüm / Solution

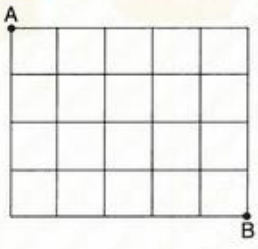
Yanıt / Answer **B**

2 öğrencinin birinci sınava girmesi için, $\binom{6}{2} = 15$ yol, bundan sonra 2 öğrencinin ikinci sınava girmesi için, $\binom{4}{2} = 6$ değişik yol vardır. Geri kalan 2 öğrenci de üçüncü sınava girer.

Böylece öğrenciler toplam olarak $15 \cdot 6 = 90$ yolla sınava girebilirler.

Since 2 always included in the examinations we must select 2 out of 6, hence the number of ways $\binom{6}{2} = 15$ for the first examination for the 2nd examination, we must select 2 out of 4, hence the number of ways $\binom{4}{2} = 6$ by this way, the students take the examinations totally $15 \cdot 6 = 90$ ways.

Örnek / Example 11



Şekildeki dikdörtgende kare şekilinde yollar verilmiştir. A dan B ye kaç farklı biçimde gidilebilir?

In the rectangular shape city block above,by using only right or down movements over line segments, in how many different ways can a person go from point A to point B?

A) 90 B) 96 C) 100 D) 118 E) 126

Çözüm / Solution

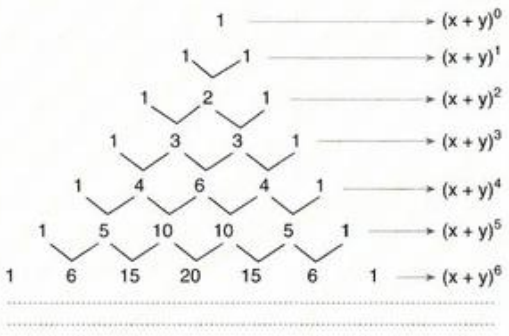
Yanıt / Answer **E**

sağa 5 yol

aşağı 4 yol

toplam 9 yol $\Rightarrow \binom{9}{5} = \frac{9!}{5!\cdot 4!} = 126$

BİNOM AÇILIMI ve PASCAL ÜÇGENİ
BINOMIAL EXPANSION and PASCAL'S TRIANGLE



$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \ldots + \binom{n}{n}y^n$$

ÖNEMLİ / IMPORTANT

- Pascal üçgeninin her bir satırındaki ilk ve son sayı 1'dir. Her satırdaki diğer sayılar, üzerinde yer alan iki sayının toplamından elde edilir.
- The first and last number in each row of Pascal's triangle is 1. Every other number in each row is formed by adding the two numbers immediately above the number.

Özellik / Property

1. $(x + y)^n$ nin açılımında, n + 1 tane terim vardır.
   There are (n + 1) terms in the expansion of $(x + y)^n$.

2. $(x + y)^n$ açılımında, her terimdeki x ve y nin üslerinin toplamı n'ye eşittir.
   In each terms the sum of the power of x and y is equal to n in the expansion of $(x + y)^n$.

3. Baştan (r + 1). terim, $\boxed{\binom{n}{r}x^{n-r}y^r}$ dir.
   The $(r + 1)^{th}$ term from beginning is $\boxed{\binom{n}{r}x^{n-r}y^r}$.

ÖNEMLİ / IMPORTANT

- Her bir açılımda, x ve y simetriktir. x'in kuvveti 1 azalırken, y'nin kuvveti 1 artar.
- In each expansion, x and y have symmetrical. The powers of x decrease by 1 in successive terms, whereas the powers of y increase by 1.

PEKİŞTİRME / REINFORCE

✓ $(x - 2)^6 = \ldots + Ax^5 + \ldots \Rightarrow A = ?$

..... [–12]

Örnek / Example 12

$(x + 5)^6 = \ldots + Ax^4 + \ldots \Rightarrow A = ?$

A) 125 B) 250 C) 325 D) 375 E) 425

Çözüm / Solution

Yanıt / Answer **D**

$$Ax^4 = \binom{6}{r}\cdot(x)^{6-r}\cdot(5)^r$$

$\Rightarrow 6 - r = 4$

$r = 2$

$$A = \binom{6}{2}\cdot 5^2 = \frac{6\cdot 5}{2}\cdot 25 = 375$$

PEKİŞTİRME / REINFORCE

✓ $(x^4 + y^3)^n = \ldots + Ax^8y^6 + \ldots$
$\Rightarrow$ **A + n = ?**

..... [10]

Özellik / Property

p, q, r $\in$ N,

$p + q + r = n \Rightarrow$

$$(ax + by + cz)^n = (ax)^p \cdot (by)^q \cdot (cz)^r \cdot \frac{n!}{p!\cdot q!\cdot r!}$$

Örnek / Example 13

$(x + 2y + z)^5 = \ldots + Ax^3yz + \ldots \Rightarrow$ **A = ?**

A) 10 B) 20 C) 30 D) 40 E) 50

Çözüm / Solution Yanıt / Answer **D**

Çözüm / Solution 1

$p = 3$

$q = 1$

$r = 1$

$n = p + q + r = 3 + 1 + 1 = 5$

$$A = \frac{5!}{3!\cdot 1!\cdot 1!}\cdot (2)^1 = 40$$

Çözüm / Solution 2

$2y + z = t \Rightarrow (x + t)^5 = \ldots + Kx^3t^2 + \ldots$

$$Kx^3t^2 = \binom{5}{r}x^{5-r}t^r$$

$r = 2$

$$K = \binom{5}{2} = 10$$

$t = 2y + z \Rightarrow (2y + z)^2 = \ldots + Ly^1z^1 + \ldots$

$$Lyz = \binom{2}{r}(2y)^{2-r}(z)^r$$

$$r = 1 \Rightarrow L = \binom{2}{1}\cdot 2^1 = 4$$

$A = K\cdot L = 10\cdot 4 = 40$

PEKİŞTİRME / REINFORCE

✓ $\left(x^2 + \frac{y}{3}\right)^5 = \ldots + Ax^8y + \ldots \Rightarrow$ **A = ?**

..... [5/3]

Örnek / Example 14

$(2a - b - c)^8 = \ldots + Mab^6c + \ldots \Rightarrow$ **M = ?**

A) −128 B) −112 C) −96 D) 96 E) 112

Çözüm / Solution Yanıt / Answer **B**

$$\frac{8!}{1!\cdot 6!\cdot 1!}\cdot (2a)^1\cdot (-b)^6\cdot (-c)^1 = 56\cdot 2\cdot (-1)\cdot 1\cdot ab^6c$$

$$= -112ab^6c \Rightarrow M = -112$$

Örnek / Example 15

$\left(x^2 - \frac{2}{x}\right)^8 = \ldots + Ax^7 + \ldots \Rightarrow$ **A = ?**

A) –448 B) –312 C) 112 D) 312 E) 448

Çözüm / Solution

Yanıt / Answer **A**

$(x^2)^{8-r} \cdot (-2x^{-1})^r = x^7$

$16 - 2r - r = 7$

$r = 3$

$\binom{8}{3} \cdot (-2)^3 = 56 \cdot (-8) = -448$

Örnek / Example 16

$\left(x - \frac{2}{y}\right)^7 = Ax^7 - Bx^6 \cdot y^{-1} + Cx^5 y^{-2} - \ldots \Rightarrow$ **C = ?**

A) –64 B) –48 C) 48 D) 64 E) 84

Çözüm / Solution

Yanıt / Answer **E**

$\binom{7}{r} \cdot (x)^{7-r} \cdot (-2y^{-1})^r = C \cdot x^5 \cdot y^{-2}$

$7 - r = 5 \Rightarrow r = 2$

$\binom{7}{2} \cdot x^5 \cdot 2^2 \cdot y^{-2} \Rightarrow C = \frac{7!}{2! \cdot 8!} \cdot 4 = 84$

Örnek / Example 17

$\left(\sqrt{x} - \frac{1}{\sqrt[3]{x}}\right)^{10} = \ldots + A + \ldots \Rightarrow$ **A = ?**

A) –210 B) –120 C) 60 D) 120 E) 210

Çözüm / Solution

Yanıt / Answer **E**

$\binom{10}{r} \cdot \left(x^{\frac{1}{2}}\right)^{10-r} \cdot \left(-x^{-\frac{1}{3}}\right)^r = A$

$\Rightarrow \frac{10-r}{2} - \frac{r}{3} = 0$

$30 - 5r = 0 \Rightarrow 5r = 30$

$r = 6$

$A = \frac{10!}{6! \cdot 4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210$

OLASILIK / PROBABILITY

Tanım: Olabilecek sonuçların belli fakat hangisinin olacağı kesin olmayan olayların matematiksel sayılarla ifade edilmesine **olasılık** denir.

1. **Deney:** Sonucun gözlenmesi için yapılan her işe **olasılıkta deney** denir.
2. **Çıktı :** Yapılan deneyin sonuçlarından her birine **çıktı** denir.
3. **Örnek Uzay:** Tüm çıktıların oluşturduğu kümeye **örnek uzay** denir ve E ile gösterilir.
4. **Olay:** Bir örnek uzayın her alt kümesine **olay** denir.
5. **Olanaksız (İmkansız), Kesin Olaylar:** E örnek uzayının bir alt kümesi A olsun. Eğer $A \neq \emptyset$ ise olanaksız, $A = E$ ise kesin olaydır.
   Örneğin;
   a. Bir zarın atılması deneyinde gelen sayının 6 dan büyük olması olayı olanaksız olaydır.
   b. Bir zarın atılması deneyinde gelen sayının 6 dan küçük veya eşit gelmesi olayı kesin olaydır.
6. **Ayrık Olaylar:** A ve B kümeleri, E örnek uzayın birer alt kümesi olmak üzere, $A \cap B = \emptyset$ ise A ve B olaylarına **ayrık olaylar** denir. Örneğin; bir zarın atılması deneyinde gelen sayının 2 den küçük gelmesi olayı ile 4 ten büyük gelmesi olayları ayrık olaylardır.

Definition: The numeric expression of the results that might happen but not certain which one will happen is called probability.

1. **Experiment:** Any happening to observe the result is called an experiment.
2. **Output:** Every result of the experiment is called output.
3. **Sample Space:** The set that consists of all the outputs is called sample space. It represents as E.
4. **Event:** Every subset of a sample space is called event.
5. **Impossible Event, Certain Event:** A is a subset of E. If $A = \emptyset$, then A is an impossible event. If $A = E$, then A is a certain event.
   For example;
   a. The event of getting a number greater than 6 on the experiment of rolling a dice is an impossible event.
   b. The event of getting a number equal to 6 or less than 6 on the experiment of rolling a dice is a certain event.
6. **Mutually Exclusive Events:** A and B are the subsets of E. If $A \cap B = \emptyset$, then the event A and the event B are called mutually exclusive events. For instance the event of getting a number less than 2 and the event of getting a number more than 4 are independent events when a dice is rolled.

ÖNEMLİ / IMPORTANT

- Bir madeni paranın havaya atılması olayında, paranın üst yüzünün yazı veya tura geleceği bilinir, fakat hangi oranda yazı, hangi oranda tura geleceği olasılık hesaplamalarıyla bulunabilir.
- For instance, when a coin is tossed, we know the surface of the coin is either head or tail but we use probability calculations to find out what ratio of the event is head and what ratio of the event is tail.

ÖNEMLİ / IMPORTANT

- Bir zarın atılması deneyinde çıktılar 1, 2, 3, 4, 5, 6 bir paranın iki kere havaya atılması deneyinde çıktılar YY, YT, TY, TT dır (T: Tura, Y: Yazı).
- The outputs of the experiment of rolling a dice are 1, 2, 3, 4, 5, 6. The outputs of the experiment of tossing a coin for two times are YY, YT, TY, TT. (Y = tail, T: head)

ÖNEMLİ / IMPORTANT

- İki madeni paranın atılması olayında örnek uzay, E = {(YY), (YT), (TY), (TT)} dır.
- Bir zarın atılması deneyinde altı mümkün sonuç olup E = {1, 2, 3, 4, 5, 6} dır.
- The sample space of the event of tossing two coins is E = {(YY), (YT), (TY), (TT)}.
- The sample space of the event of rolling a dice is E = {1, 2, 3, 4, 5, 6}.

Etkinlikler / Activities

Bir zar atılması deneyinde,

a. Örnek uzayını bulunuz.

b. Çift gelmesi olayını bulunuz.

c. Tek gelmesi olayını bulunuz.

d. 7 gelmesi olayını bulunuz.

e. 4 ten küçük gelmesi olayını bulunuz.

f. 2 den büyük gelmesi olayını bulunuz.

g. 2 gelmemesi olayını bulunuz.

On an experiment of rolling a dice,

a. Find the sample space.

b. Find the event of getting an even integer.

c. Find the event of getting an odd integer.

d. Find the event of getting 7.

e. Find the event of getting a number less than 4.

f. Find the event of getting a number greater than 2.

g. Find the event of not getting 2.

Çözümler:

a. Örnek uzayının elemanları, E = {1, 2, 3, 4, 5, 6} dır.

b. Çift gelmesi olayı Ç = {2, 4, 6} dır.

c. Tek gelmesi olayı T = {1, 3, 5} dir. Ç ve T kümelerinin kesişimi boş küme olduğundan, ayrık olaylardır.

d. 7 gelmesi olayı imkansız olaydır.

e. 4 ten küçük gelmesi olayı A = {1, 2, 3} tür.

f. 2 den büyük gelmesi olayı B = {3, 4, 5, 6} dır.

g. 2 gelmemesi olayı A = {2} kümesinin tümleyeni A′ = {1, 3, 4, 5, 6} dir.

Solutions:

a. The sample space is E = {1, 2, 3, 4, 5, 6} .

b. The event of getting an even integer is E = {2, 4, 6}.

c. The event of getting an odd integer is O = {1, 3, 5}. Since E∩O = Ø.

d. The event of getting 7 is an impossible event.

e. The event of getting a number less than 4 is A = {1, 2, 3}.

f. The event of getting a number greater than 2 is B = {3, 4, 5, 6}.

g. The complement of the set A = {2}, the event of getting 2, is A′ = {1, 3, 4, 5, 6}.

ÖNEMLİ / IMPORTANT

• E örnek uzayında A ve B kümeleri için küme işlemleri geçerlidir.

A∩B, A ve B olayı

A∪B, A veya B olayı

A′ , A olayının tümleyenidir.

A∪A′ = E

• The operations of sets are obtained for the sets A and B in the sample space E.

The event of A and B A∩B,

The event of A or B is A∪B,

The complement of A′ is A.

A∪A′ = E

ÖNEMLİ / IMPORTANT

- **Bir olayın olma olasılığı 0 dan küçük, 1 den büyük olamaz.**

i. $P(A) + P(A') = 1$

ii. Kesin olay için **P(E) = 1**
İmkansız olay için **P(Ø) = 0** dır.

iii. $A, B \in S$ için $A \cap B = Ø$ (A ve B ayrık olaylar) ise **$P(A \cup B) = P(A) + P(B)$** dir.

- **The probability of an event can not be less than 0 and greater than 1.**

i. $P(A) + P(A') = 1$

ii. For a certain event; **P(E) = 1**
For impossible event; **P(Ø) = 0**

iii. For $A, B \in S$, if $A \cap B = Ø$ (A an B are independent events) then
$P(A \cup B) = P(A) + P(B)$.

OLASILIK FONKSİYONU / THE PROBABILITY FUNCTION

Tanım: E örnek uzayının tüm alt kümelerinin oluşturduğu kümeye S denirse,

P: S → [0, 1] şeklinde tanımlanan fonksiyona **olasılık fonksiyonu** denir ve $A \in S$ olmak üzere, P(A) sayısına da **A olayının olasılığı** denir.

Olasılıkla ilgili, aşağıdaki aksiyomlar verilebilir:

i. Olasılık fonksiyonunda görüntü kümesi [0, 1] olduğundan,
$A \in S$ için $0 \le P(A) \le 1$ dir.

Definition: If S is a set that consists of all the subsets of E, the function P that defined from S to [0, 1] is called the probability function. If $A \in S$, P(A) is the probability of A.

An axioms is given below about the probability:

i. Since the range of the probability function is [0, 1],
for $A \in S$, $0 \le P(A) \le 1$.

PEKİŞTİRME / REINFORCE

✓ Örnek uzayı E = {A, B, C, D} olan bir olasılık fonksiyonunda $A \cap B = Ø$ dir.
$P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$ olduğuna göre, A veya B olayının olma olasılığı kaçtır?

✓ The sample space of a probability function is E = {A, B, C, D} and $A \cap B = Ø$. If $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$, what is the probability of the event A or B ?

.....[5/6]

Örnek / Example 21

A, B ve C kişilerinin katıldığı bir yarışmada A nın birinci olma olasılığı $\frac{1}{5}$, B nin birinci olma olasılığı $\frac{1}{3}$ olduğuna göre, **yarışta C nin birinci olma olasılığı kaçtır?**

in a contest between 3 persons, A, B and C, the probability of getting in first for person $\frac{1}{5}$ the probability of getting in first for person B is $\frac{1}{3}$. What is the probability of getting in first for person C?

A) $\frac{3}{5}$ B) $\frac{7}{15}$ C) $\frac{8}{15}$ D) $\frac{11}{15}$ E) 1

Çözüm / Solution

Yanıt / Answer **B**

Örnek uzay / the sample space is, E = {A, B, C} $P(A) = \frac{1}{5}$, $P(B) = \frac{1}{3}$ tür.

Olasılık fonksiyonundan / According to the probability function, P(A) + P(B) + P(C) = 1

$$\frac{1}{5} + \frac{1}{3} + P(C) = 1 \Rightarrow P(C) = 1 - \left(\frac{8}{15}\right)$$

$$P(C) = \frac{7}{15}$$

Özellik / Property

Olasılık Fonksiyonunun Özellikleri / Properties of the Probabilty Function

1. $A \cap B \neq \emptyset$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ dir.
2. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(P(A \cap B \cap C)$
3. A olayının tümleyeni A' olmak üzere / If the complement of A is A', then,
   $P(A) = 1 - P(A')$ dır.
4. $A \subset B$ ise / then $P(A) \leq P(B)$ dir.

Etkinlikler / Activities

A ve B olayları E örnek uzayının iki alt kümesi olmak üzere,
A and B are two subsets of the sample space E.

$P(A) = \frac{2}{3}$, $P(B) = \frac{1}{2}$ ve $P(A \cup B) = \frac{5}{6}$ ise

a. **P(A') ve P(B') kaçtır /** What is the value of P(A') and P(B')**?**

Çözüm / Solution**:**

$P(A) = 1 - P(A')$

$\frac{2}{3} = 1 - P(A') \Rightarrow P(A') = 1 - \frac{2}{3} = \frac{1}{3}$

$P(B) = 1 - P(B') \Rightarrow \frac{1}{2} = 1 - P(B') \Rightarrow P(B') = 1 - \frac{1}{2} = \frac{1}{2}$

b. **P(A∩B) kaçtır /** What is the value of $P(A \cap B)$ **?**

Çözüm / Solution**:**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{5}{6} = \frac{2}{3} + \frac{1}{2} - P(A \cap B) \Rightarrow P(A \cap B) = \frac{7}{6} - \frac{5}{6} = \frac{1}{3}$

c. **P(A'∩B') kaçtır /** What is the value of $P(A' \cap B')$ **?**

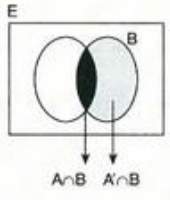
Çözüm / Solution**:**

$A' \cap B' = (A \cup B)'$ (De Morgen Kuralı)

$P(A' \cap B') = P(A \cup B)' \Rightarrow P(A \cup B)' = 1 - P(A \cup B) = 1 - \frac{5}{6} = \frac{1}{6}$

d. **P(A'∩B) kaçtır /** What is the value of $P(A' \cap B)$**?**

Çözüm / Solution**:**



$\Rightarrow P(B) = P(A \cap B) + P(A' \cap B)$ [(A∩B) ve (A'∩B) ayrık olaylar]
[(A∩B) and (A'∩B) are independent events.]

$\frac{1}{2} = \frac{1}{3} + P(A' \cap B)$

$\frac{1}{6} = P(A' \cap B)$

Örnek / Example 22

Can, içinde 8 kırmızı, 6 beyaz top bulunan bir torbadan elinde bir beyaz top olana kadar art arda top çekiyor. Can, en çok kaç top çekmiştir? (Çekilen toplar tekrar torbaya atılmıyor.)

Can picks marbles from a pouch that contains 8 red and 6 white marbles, concecutively until he has 1 white marble. What is the maximum number of marbles he has at the end? (He doesn't put the marbles back in the pouch.)

A) 6 B) 7 C) 9 D) 10 E) 11

Çözüm / Solution Yanıt / Answer **C**

Önce bütün kırmızı topları birer birer çektiğinden, 8. çekişte torbada sadece beyaz toplar kalır. 9. çekişte beyaz top gelir. Yanıt 9'dur.

Since the questions asks the maximum number of marbles he has we think that he picks all the red marbles one by one. After the eight picking there would be only white marble in the pouch. At the nineth time, he picks white marble. The answer is nine.

ÖNEMLİ / IMPORTANT

- $E = (A_1, A_2, ..., A_n\}$ örnek uzayında $P(A_1) = P(A_2) = ... = P(A_n)$ ise E eş olumlu bir örnek uzaydır.
- If $P(A_1) = P(A_2) = ... = P(A_n)$, then $E = (A_1, A_2, ..., A_n\}$ is an equally likely sample space.

EŞ OLUMLU ÖRNEK UZAYDA OLASILIK HESAPLARI
PROBABILITY CALCULATIONS in EQUALLY LIKELY SAMPLE SPACES

Tanım: Bir deneyde bütün sonuçların olabilme olasılığı eşit ise bu şekildeki örnek uzaylara **eş olumlu örnek uzay** denir. Örneğin; bir zar atılması olayında 6 sayının da gelme olasılığı aynıdır. O halde, zar atma deneyinde örnek uzay bir eş olumlu örnek uzaydır.

Eş olumlu bir örnek uzayında, $A \subset E$ olmak üzere,

$$P(T) = \frac{s(T)}{s(E)} \begin{matrix} \text{istenilen sonuç sayısı} \\ \text{Tüm olası sonuç sayısı} \end{matrix}$$

Örneğin; bir madeni paranın atılması deneyinde,

$E = \{Y, T\}$ olup tura gelme olasılığı,

$P(T) = \frac{s(T)}{s(E)} = \frac{1}{2}$ dir.

Aynı şekilde bir zar atma deneyinde 2 gelme olasılığı $\frac{1}{6}$ dır.

Definition: If the possibilities of all the events are equal then this kind of space are called equally likely sample space. For instance the probability of each number, from 1 to 6 inclusively, is equal, when a dice is tossed. Because of that, for the experiment of rolling a dice the sample is an equally likely sample space.

$$P(T) = \frac{s(T)}{s(E)} \begin{matrix} \text{number of desired results} \\ \text{number of all results} \end{matrix}$$

For instance the sample space of the experiment of tossing a coin is $E = \{Y, T\}$. The probability of getting a head is

$P(T) = \frac{s(T)}{s(E)} = \frac{1}{2}$

With the same logic, the probability of getting 2 when a dice is rolled is $\frac{1}{6}$.

Örnek / Example 23

Bir zar atma deneyinde üst yüze gelen sayının 4 ten büyük olma olasılığı kaçtır?

What is the probability of getting 4 when a dice is rolled?

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

Çözüm / Solution Yanıt / Answer **B**

E = {1, 2, 3, 4, 5, 6} ise s(E) = 6 dır.

Zarın 4 ten büyük gelme olayı, C = {5, 6} olduğundan, s(C) = 2 dir.

$$P(C)=\frac{s(C)}{s(E)}=\frac{2}{6}=\frac{1}{3}$$

If E = {1, 2, 3, 4, 5, 6}, then s(E) = 6.

The event of getting a number greater than 4 is C = {5, 6}, s(C) = 2.

$$P(C)=\frac{s(C)}{s(E)}=\frac{2}{6}=\frac{1}{3}$$

PEKİŞTİRME / REINFORCE

✓ Bir zarın atılması deneyinde üst yüze gelen sayının 2 den büyük 6 dan küçük olması olasılığı kaçtır?

✓ What is the probability of getting a number greater than 2 and less than 6 when a dice is rolled?

[1/2]

KOŞULLU OLASILIK (BAĞIMLI OLAYLAR) CONDITIONAL PROBABILITY (DEPENDENT EVENTS)

Tanım: Bir olayın gerçekleşmesi başka bir olayın gerçekleşmesine bağlı ise bu iki olaya **bağımlı olaylar** denir.

Bir örnek uzayının iki olayı A ve B olup A nın gerçekleşmesi, B olayının gerçekleşmesine bağlı ise A olayının, B olayına bağlı koşullu olasılığı P(A\B) ile gösterilir.

Definition: Two events are called dependent events if the occurrance of one has effect on the occurrance of the other.

A and B are two events of a sample space and if the occurrance of A is depended on occurrance of B, then the conditional probability of A conditioned to B is shown as P(A\B).

$$P(A \setminus B)=\frac{P(A \cap B)}{P(B)}$$

PEKİŞTİRME / REINFORCE

✓ Bir çift zar atıldığında, üst yüze gelen sayıların toplamı 8 olduğu bilindiğine göre bu sayılardan birinin 2 olma olasılığı kaçtır?

✓ When two dice is rolled, the sum of the numbers on the surfaces is 8. What is the probability of one of tne numbers is 2?

..... $\left[\frac{2}{5}\right]$

Örnek / Example 24

Bir zar atma deneyinde üst yüze gelen sayının çift olduğu bilindiğine göre 4 olma olasılığı kaçtır?

In the event of rolling a dice, what is the probability that the number 4 coming up if it is known that the number that came up is an even number?

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

Çözüm / Solution Yanıt / Answer **B**

Çift gelme olayına Ç denirse, Ç = {2, 4, 6} olup, s(Ç) = 3 olur.

4 gelme olayına A denirse, A = {4} ve s(A) = 1 olur.

$A \cap Ç = \{4\}$

$s(A \cap Ç) = 1$

If the event of getting an even integer is Ç = {2, 4, 6}, then s(Ç) = 3.

If the event of getting 4 is A = {4}, then s(A) = 1.

$A \cap Ç = \{4\}$

$s(A \cap Ç) = 1$

$$\Rightarrow P(A \setminus Ç) = \frac{s(A \cap Ç)}{s(Ç)} = \frac{1}{3}$$

BAĞIMSIZ OLAYLAR / INDEPENDENT EVENTS

Tanım: A ve B, E örnek uzayının iki olayı olmak üzere bu iki olaydan birinin olması diğerinin olmasını etkilemiyorsa A ve B olaylarına **bağımsız olaylar** denir.

Definition: Two events are called as independent events if the occurance of one has no effect on the occurrance of the other.

$$P(A) \neq 0,\ P(B) \neq 0 \Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

Örnek / Example 25

Bir zar ile madeni para aynı anda havaya atılıyor. Paranın yazı, zarın üst yüzündeki sayının 3 ten büyük olması olasılığı kaçtır?

A dice is rolled and a coin is tossed at the same time. What is the possibility of getting a tail and getting a number greater than 3?

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

Çözüm / Solution Yanıt / Answer **C**

Bir zar ile paranın atılması olayları bağımsız olaylar olup $P(A \cap B) = P(A) \cdot P(B)$ dir.

Zarın 3 ten büyük gelmesi olayına A denirse,

Since the event of tossing a coin and the event of rolling a dice are independent

$P(A \cap B) = P(A) \cdot P(B)$.

If the event of getting a number greater than 3 is A, then

$P(A) = \frac{3}{6} = \frac{1}{2}$ olur.

Paranın yazı gelmesi olayına B denirse,

If the event of getting a tail is B, then

$P(B) = \frac{1}{2}$ olur.

$P(A \cap B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Örnek / Example 26

Bir torbada kırmızı ve beyaz olmak üzere 15 tane bilye vardır. Torbadan rasgele seçilen iki bilyenin birinin kırmızı, diğerinin beyaz olma olasılığı $\frac{8}{15}$ ise bu torbada kaç tane kırmızı bilye vardır?

There are a total of 15 marbles in colors of red and white in a bag, when taking two marbles randomly from this bag, the probability of one of them being a red and the other being a white marble is $\frac{8}{15}$, accordingly how many red marbles are there in this bag?

A) 5 B) 6 C) 7 D) 8 E) 10

Çözüm / Solution

Yanıt / Answer **C**

Kırmızı bilye sayısı / number of red murbles = x
beyaz bilye sayısı / number of white murbles = 15 – x

$$\Rightarrow P(K)=\frac{\binom{x}{1}\cdot\binom{15-x}{1}}{\binom{15}{2}}=\frac{8}{15}$$

$$\frac{x\cdot(15-x)}{15\cdot 7}=\frac{8}{15}$$

$$x\cdot(15-x)=56$$

$$x=7$$

Örnek / Example 27

$P(A)=\frac{1}{2}$, $P(B)=\frac{5}{6}$

$P(A\cap B)=\frac{1}{3} \Rightarrow \mathbf{P(A\cap B')=?}$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

Çözüm / Solution

Yanıt / Answer **E**

$$P(A\cap B')=P(A)-P(A\cap B)$$

$$=\frac{1}{2}-\frac{1}{3}=\frac{3-2}{6}=\frac{1}{6}$$

1. **6 kişi, belli iki kişinin yanyana oturmaması koşulu ile bir sıraya kaç değişik şekilde oturabilirler?**

If 2 certain persons can not be seated side by side, in how many different ways can 6 people be seated on a bench?

A) 720
B) 600
C) 480
D) 420
E) 360

2. **ISIYAYAR sözcüğünün harflerinden kaç tane 8 harfli anlamlı ya da anlamsız sözcük yazılabilir?**

How many 8 lettered words can be written by using the letters of the word ISIYAYAR?

A) 5040
B) 4808
C) 3080
D) 3040
E) 2160

3. **Bir sınıfta 9 erkek, 6 kız öğrenci vardır. Bir öğretmen 6 kişilik bir yarışma ekibi oluşturacaktır. Buna göre, bu gruplardan kaç tanesinde yalnızca iki kız öğrenci bulunur?**

There are 9 male and 6 female students in a classroom. A teacher wants to choose a competition group of 6 people. In how many of these groups have only 2 female students?

A) 1890
B) 1840
C) 1780
D) 1620
E) 1580

4. **Bir servis aracında 8 evli çift bulunmaktadır. Bu 16 kişi arasından 5 kişi seçilecektir. Bu 5 kişi içerisinde evli çift olmayacağına göre, kaç değişik seçim yapılabilir?**

There are 8 married couple in a shuttle. 5 people will be chosen from this 16 people. If there will be no married couple in the group, how many different choises can be made?

A) 1840
B) 1806
C) 1804
D) 1792
E) 1644

5. **Aralarında 1 Matematik ve 1 Türkçe kitaplarının da bulunduğu bir grup farklı kitap bir rafa sıralanacaktır. Buna göre, Matematik ve Türkçe kitaplarının yanyana gelmemesi koşuluyla 6!·5 farklı şekilde sıralanabilen bu kitapların sayısı kaçtır?**

A group of books which contains Turkish and Math books will be put in order. When Turkish and Math books can not stand adjacent, they can be put together in 6!·5 ways. What is the number of these books?

A) 4
B) 5
C) 6
D) 7
E) 8

6. **"SELİM" kelimesinin harflerini birer kez kullanarak yazılabilen 3 harfli kelimelerden kaç tanesinde M harfi bulunur?**

How many 3 lettered words constituted by using the letters of "SELİM", contains M? (Each letter of "SELİM" can be used only once in of these words.)

A) 24
B) 30
C) 32
D) 36
E) 54

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7. **3 erkek, 6 kız bir yuvarlak masa etrafında 2 erkek arasına 2 kız gelmesi koşuluyla kaç farklı şekilde oturabilir?**

in how many ways 3 men and 6 women can be seated around a round table if two women must be seated between each two men?

A) 2880
B) 1440
C) 720
D) 360
E) 180

---

8. $(3x^2 + 1)^8 = \ldots + Ax^4 + \ldots \Rightarrow$ **A = ?**

A) 212
B) 236
C) 252
D) 302
E) 316

---

9. $(x + y + z)^8 = \ldots + Ax^2y^3z^3 + \ldots \Rightarrow$ **A = ?**

A) 440
B) 460
C) 480
D) 520
E) 560

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10. **GELENLER sözcüğündeki her harf bir karta yazılıp bir torbaya atılıyor. Aynı anda torbadan üç kart çekilerek yan yana konulduğunda GEL sözcüğünün yazılması olasılığı kaçtır?**

Each letter of the word "GELENLER" is written on a card and they all are put in a pouch. If 3 cards are picked at the same time and placed side by side, what is the probability of writing the word "GEL"?

A) $\frac{3}{56}$ B) $\frac{2}{63}$ C) $\frac{3}{64}$ D) $\frac{1}{42}$ E) $\frac{1}{56}$

---

11. **Bir torbada eşit sayıda, aynı büyüklükte kırmızı ve siyah top vardır. Bu torbadan, yerine geri konulmamak üzere art arda çekilen iki bilyenin de siyah renkte olma olasılığı 13/53 ise ilk durumda torbada kaç bilye vardır?**

There are equally sized red and black balls in a pouch. When two balls are picked in a row without putting back to the pouch, the probability of getting both balls black is 13/53. How many balls were in the pouch at the beginning?

A) 27 B) 36 C) 42 D) 54 E) 72

---

12. **Bir kutuda 8 sarı, 5 kırmızı bilye vardır. Kutudan rastgele alınan iki bilyenin her ikisinin de kırmızı olma olasılığı kaçtır?**

There are 8 yellow are 5 red marbles in a pouch. What is the probability of getting two red marbles when 2 marbles are picked randomly from the punch?

A) $\frac{2}{13}$ B) $\frac{3}{19}$ C) $\frac{4}{29}$ D) $\frac{5}{39}$ E) $\frac{6}{43}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
C	A	A	D	D	D	B	C	E	E	D	D

1. 6 kişi koşulsuz olarak bir sıraya 6! =720 değişik biçimde oturabilir. Belli iki kişi A ve B olsun. Bu iki kişi kendi aralarında AB, BA olmak üzere 2! = 2 değişik biçimde oturabilir. Belli iki kişiyi bir tek kişi gibi düşünecek olursak $2! \cdot 5! = 2 \cdot 120 = 240$ değişik oturma biçiminde belli iki kişi yanyana bulunur.

$720 - 240 = 480$ değişik oturuş biçiminde ise belli iki kişi yanyana değildir.

6 person can be seated on a bench inconditionally in $P(6, 6) = 6! = 720$ different ways. 2 different person A and B can be seated near each other in $2! = 2$ different ways. If we consider these two certain person as one, in $5! = 2! \cdot 5! = 2 \cdot 120 = 240$ different way, those 2 person would sit next to each other.

in $720 - 240 = 480$ different ways these 6 people can sit whitout these two people sitting next to eachother.

Yanıt / The answer is C

---

2. $\frac{8!}{2! \cdot 2! \cdot 2! \cdot 1! \cdot 1!} = 5040$

Yanıt / The answer is A

---

3. 6 kişilik grupta 2 kız, 4 erkek öğrenci olmalıdır. O halde grup sayısı, $\binom{6}{2} \cdot \binom{9}{4} = 15 \cdot 126 = 1890$ dır.

in a group of 6, there must be 2 girls and 4 boys.

therefore, $\binom{6}{2}\binom{9}{4} = 15 \cdot 124 = 1890$

Yanıt / The answer is A

---

4. 8 çift arasından 5 çift $\binom{8}{5} = \frac{8 \cdot 7 \cdot 6}{6} = 56$ farklı şekilde seçilir. Her çift arasından, 1 kişi seçmenin 2, 5 çift arasından 5 kişi seçmenin $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$ yolu vardır. Buna göre bu 16 kişi arasından evli olmayan 5 kişi $56 \cdot 32 = 1792$ değişik yolla seçilebilir.

5 couples within 8 couples can be chosen in $\binom{8}{5} = \frac{8 \cdot 7 \cdot 6}{6} = 56$ different ways. One person within a couple can be chosen in 2 different ways. Then there are $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$ different ways to choose a person from 5 couple. Accordingly in $56 \cdot 32 = 1792$ ways can 5 people which are not couple can by chosen from these 16 people.

Yanıt / The answer is D

---

5. Kitapların sayısına n denirse, kitaplar rafa koşulsuz n! kadar farklı sırada sıralanabilir.

Matematik ve Türkçe kitapları yanyana olması koşuluyla (Türkçe ve Matematik kitapları bir kitap sayılır.)

$(n - 1)! \cdot 2$ farklı şekilde sıralanabilir.

O halde, kitaplar Matematik ve Türkçe kitaplarının yanyana gelmemesi koşuluyla $n! - (n-1)! \cdot 2$ farklı şekilde sıralanabilir.

If there are n number of books, they can be placed on a shelf in n! different ways individually with the condition of Turkish and Math books are placed side-by-side, they can be placed $(n - 1)! \cdot 2$ different ways.

$n! - (n-1)! \cdot 2 = 6! \cdot 5$

$n \cdot (n - 1)! - (n - 1)! \cdot 2 = 6! \cdot 5$

$\Rightarrow n - 1 = 6$

$n = 7$

Yanıt / The answer is D

---

6. 3 harfli kelimede M bulunacağı için M harfinin yanına 4 harften ikisi gelmeli, $C(4,2) = \frac{4!}{(4-2)! \cdot 2!} = 6$

Bu üç harfi kendi arasında, $P(3, 3) = 3! = 6$ farklı şekilde sıralanabilir. Çarpma ilkesinden, $6 \cdot 6 = 36$ farklı sayı yazılabilir.

$C(4,2) = \frac{4!}{(4-2)! \cdot 2!} = 6$ (Since the three lettered word contains M, the other two letter must be chosen from the rest of the letters of "SELİM".)

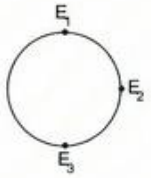
3 letters can be placed in $P(3, 3) = 3! = 6$ different ways.

so $6 \cdot 6 = 36$ different words can be written.

Yanıt / The answer is D

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7.



3 erkek, $(3-1)! = 2! = 2$ farklı şekilde oturabilir. Erkeklerin arasına kızları ikişer ikişer yerleştirdikten sonra kendi aralarında yer değiştirebileceklerinden 2 ile çarpılır.

$[C(6, 2)\cdot 2]\cdot[C(4, 2)\cdot 2]\cdot[C(2, 2)\cdot 2] = (15\cdot 2)\cdot(6\cdot 2)\cdot(2)$

$= 720$

Çarpma ilkesinden, $2\cdot 720 = 1440$ bulunur.

3 men can be seated in $(3-1)! = 2 = 2$ different ways around a round table. After each two women placed between two men, they can change place with each other, that's why we multiply the result by 2.

$[C(6, 2)\cdot 2]\cdot[C(4, 2)\cdot 2]\cdot[C(2, 2)\cdot 2] = (15\cdot 2)\cdot(6\cdot 2)\cdot(2)$

$= 720$

from multiplication rule, $2\cdot 720 = 1440$

Yanıt / The answer is B

---

8. $\binom{8}{r}\cdot(3x^2)^{8-r}\cdot 1 = A\cdot x^4$

$16 - 2r = 4$

$2r = 12$

$r = 6$

$\binom{8}{6}\cdot(3x^2)^2\cdot(1)^6 = \frac{8!}{6!\cdot 2!} = \frac{8\cdot 7}{2}\cdot 9 = 252$

Yanıt / The answer is C

---

9. $A = \frac{8!}{2!\cdot 3!\cdot 3!}\cdot(1)^2(1)^3(1)^3$

$= \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3!}{2\cdot 3\cdot 2\cdot 3!}$

$= 560$

Yanıt / The answer is E

---

10. $\binom{8}{3} = \frac{8!}{5!\cdot 3!} = 56 \Rightarrow \frac{1}{56}$

Bu çekilişlerin birinden GEL sözcüğü yazılabilir. O halde, istenen olasılık 1/56 dır.

Since only one picking could write the word GEL, the probability of the event is 1/56.

Yanıt / The answer is E

---

11. Torbada n tane kırmızı, n tane siyah top varsa, örnek uzaydaki nokta sayısı,

If there are n black and n red balls in a pouch, then the sample space is

$\binom{2n}{2} = \frac{(2n)!}{2!\cdot(2n-2)!} = \frac{2n(2n-1)}{2} = n(2n-1)$

$\binom{n}{2} = \frac{n(n-1)}{2} = \frac{n^2-n}{2}$

A = {İki bilyenin siyah olması}

A = {getting two black balls}

$P(A) = \frac{\frac{n^2-n}{2}}{n(2n-1)} = \frac{n-1}{4n-2} = \frac{13}{53}$

$53n - 53 = 52n - 26$

$n = 27 \cdot 2n = 54$

Yanıt / The answer is D

---

12. 1. bilyenin kırmızı olma olasılığı : 5/13

2. bilyenin kırmızı olma olasılığı: 4/12

2 bilyenin her ikisinin de kırmızı olma olasılığı,

$\frac{5}{13}\cdot\frac{4}{12} = \frac{5}{39}$ olur.

The probability of being red of the first marble: 5/13

The probability of being red of the second marble: 4/12

The probability of being red of both marbles:

$\frac{5}{13}\cdot\frac{4}{12} = \frac{5}{39}$

Yanıt / The answer is D

METROPOL YAYINLARI • METROPOL PUBLICATIONS

1. $\binom{10}{2} + \binom{10}{4} + \ldots + \binom{10}{10} = x$

$\binom{10}{3} + \binom{10}{5} + \ldots + \binom{10}{9} = y$

ise / if $x + y - 1 = ?$

A) 1013 B) 1012 C) 1011

D) 1010 E) 1009

[ONDOKUZ MAYIS ÜNİVERSİTESİ – YÖS 2020]

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2. $n \in Z^+$

$(x + y)^n = \ldots + 10x^{n-3} \cdot y^3 + \ldots \Rightarrow$ **n = ?**

A) 3 B) 4 C) 5 D) 6 E) 7

[YILDIZ TEKNİK ÜNİVERSİTESİ – YÖS 2017]

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3. $A = 3x^3 + 2x^2 + ax - 1$

$B = 2x^3 - (2a + 1)x^2 + 4x + 1$

$A \cdot B = \ldots + 2x^4 + \ldots$

$\Rightarrow$ **a = ?**

A) –2 B) –1 C) 2

D) 4 E) 5

[İSTANBUL ÜNİVERSİTESİ – YÖS 2014]

4. $a = 2013$

$b = 2015$

$\Rightarrow a^5 - 5a^4 \cdot b + 10a^3 \cdot b^2 - 10a^2 \cdot b^3 + 5a \cdot b^4 - b^5 = ?$

A) –32 B) –21 C) 0

D) 16 E) 20

[MARMARA ÜNİVERSİTESİ – YÖS 2013]

---

5. $\left(x - \frac{2}{x}\right)^5 = Ax^5 + Bx^3 + Cx + Dx^{-1} + Ex^{-3} + Fx^{-5}$

$\Rightarrow$ **C = ?**

A) –36 B) –20 C) 24

D) 40 E) 64

[YÖS 2009]

METROPOL YAYINLARI • METROPOL PUBLICATIONS

YANITLAR / ANSWERS

1	2	3	4	5
B	C	D	A	D

1. $x + y = \binom{10}{2} + \binom{10}{3} + \ldots + \binom{10}{10}$

$x + y + 11 = \binom{10}{0} + \binom{10}{1} + \ldots \binom{10}{10}$

$x + y + 11 = 2^{10}$

$x + y - 1 = 2^{10} - 12 = 1012$

Yanıt / The Answer is B

---

2. $(x + y)^n = \ldots 10x^{n-3}\cdot y^3 + \ldots$

$\binom{n}{r}\cdot x^{n-r}\cdot y^r \Rightarrow r = 3$

$\binom{n}{3} = 10 \Rightarrow n = 5$

Yanıt / The Answer is C

---

3. $3x^3\cdot 4x = 12x^4$

$2x^2 - (2a + 1)x^2 = -2(2a + 1)x^4$

$ax\cdot 2x^3 = 2ax^4$

$12x^4 - 2(2a + 1)x^4 + 2ax^4 = 2x^4$

$12 - 4a - 2 + 2a = 2$

$8 = 2a \Rightarrow a = 4$

Yanıt / The Answer is D

METROPOL YAYINLARI • METROPOL PUBLICATIONS

4. $a^5 - 5a^4\cdot b + 10a^3\cdot b^2 - 10a^2\cdot b^3 + 5a\cdot b^4 - b^5 = (a - b)^5$

$= (2013 - 2015)^5$

$= (-2)^5$

$= -32$

Yanıt / The Answer is A

---

5. $C\cdot x = \binom{5}{3}\cdot x^3\cdot\left(\frac{-2}{x}\right)^2$

$C\cdot x = \frac{5!}{(5-3)!\cdot 3!}\cdot x^3\cdot\frac{4}{x^2}$

$C\cdot x = 40x \Rightarrow C = 40$

Yanıt / The Answer is D

1. $\frac{(5n-4)!}{(5n-5)!} = 46 \Rightarrow$ **P(n, 3) = ?**

A) 360 B) 480 C) 580 D) 640 E) 720

2. $n \geq 3$,
$P(n, 3) = 6n^2 + 3n - 9 \Rightarrow$ **P(n, 2) = ?**

A) 36 B) 42 C) 54 D) 72 E) 84

3. $P(4n, 2) - 32C(n, n-2) = 96 \Rightarrow$ **C(n, 4) = ?**

A) 84 B) 70 C) 64 D) 56 E) 48

4. **Aynı ebatta ve aynı kumaştan yapılmış 8 filamadan 3 ü kırmızı, 4 ü mavi, 1 i ise beyazdır. Bu 8 filama iki bina arasına gerilmiş bir ipe kaç değişik şekilde asılabilir?**
8 banners, 3 red, 4 blue and 1 white, which is equally sized and produced from same cloth will be hung between two buildings. In how many different ways they can be hung?

A) 144 B) 280 C) 288 D) 324 E) 432

5. **"ARAPKİR" sözcüğündeki harflerle anlamlı ya da anlamsız 7 harfli kaç sözcük yazılabilir?**
How many different 7 letter words, meaningful or meaningless, can be written by using the letters of the word "ARAPKİR" ?

A) 1260 B) 1400 C) 1560 D) 1640 E) 1720

6. **1278934 sayısını oluşturan rakamlardan, rakamları farklı üç basamaklı kaç sayı yazılabilir?**
How many different 3 digit numbers, which have different digits, can be written by using the digits of the number 1278934?

A) 840 B) 420 C) 210 D) 105 E) 42

7. **6 kişilik bir aile anne ve baba yanyana oturmak üzere, yuvarlak bir masa etrafına kaç değişik biçimde oturabilirler?**
How many different ways are probable for a family of 6 people to be seated around a round table when father and mother must be seated side-by-side?

A) 240 B) 180 C) 120 D) 80 E) 48

8. $(x-1)^9 = \ldots + a \cdot x^2 \ldots \Rightarrow$ **a = ?**

A) –36 B) –18 C) 0 D) 18 E) 36

METROPOL YAYINLARI • METROPOL PUBLICATIONS

9. $(x+3)^7 = \ldots + a{\cdot}x^5 \ldots \Rightarrow$ **a = ?**

A) 189 B) 199 C) 209 D) 219 E) 229

10. $(4x^2+y)^3 = Ax^6 + Bx^4y + Cx^2y^2 + Dy^3 \Rightarrow$ **A + B + C + D = ?**

A) 32 B) 64 C) 75 D) 100 E) 125

11. $\left(x^2+\frac{1}{x}\right)^6 = x^{12} + \ldots + A + \ldots \Rightarrow$ **A = ?**

A) 10 B) 15 C) 18 D) 20 E) 24

12. **A ve B** $P(A)=\frac{3}{8}$, $P(B)=\frac{1}{2}$ **ve** $P(A\cap B)=\frac{1}{4}$ **olan olaylar ise** $P(A'\cap B')$ **kaçtır?**

If A and B are any events $P(A)=\frac{3}{8}$, $P(B)=\frac{1}{2}$ ve $P(A\cap B)=\frac{1}{4}$ then what is the value of $P(A'\cap B')$?

A) $\frac{1}{2}$ B) $\frac{3}{8}$ C) $\frac{3}{4}$ D) $\frac{5}{8}$ E) $\frac{1}{4}$

13. A ve B olayları aynı örnek uzaya aittir.

A and B are events in the same sample space.

$P(A\cap B)=\frac{1}{2}$ ve(and) $P(B')=\frac{1}{4} \Rightarrow$ **P(A∩B') = ?**

A) $\frac{1}{3}$ B) $\frac{1}{6}$ C) $\frac{1}{2}$ D) $\frac{3}{4}$ E) $\frac{4}{5}$

14. **Bir yüzme yarışında, A, B ve C yarışmacıları vardır. Bu yarışmacılardan A nın kazanma olasılığı B ile aynı, C nin ise 3 katıdır. Buna göre, B ya da C nin kazanma olasılığı kaçtır?**

There are 3 competitiors, A, B and C, in a competition. The probability of coming in first for A is equal to B, and three times that of C. What is the probability of coming in first for B or C?

A) $\frac{1}{7}$ B) $\frac{2}{7}$ C) $\frac{6}{7}$ D) $\frac{5}{7}$ E) $\frac{4}{7}$

15. **4 ü bozuk olan 9 nesneden 2 si rastgele çekiliyor. Buna göre, çekilen iki nesnenin sağlam nesne olma olasılığı kaçtır?**

When picking 2 objects from 9 objects of which 4 of them are defective. What is the probability of picking two non-defective units?

A) $\frac{5}{9}$ B) $\frac{5}{12}$ C) $\frac{5}{18}$ D) $\frac{1}{6}$ E) $\frac{2}{3}$

16. **Bir torbada 5 beyaz, 4 siyah bilye vardır. Bu torbadan 3 bilye rastgele çekiliyor. Çekilen bu üç bilyeden 2 sinin beyaz, 1 inin siyah olma olasılığı kaçtır?**

There are 5 white and 4 black marbles in a pouch. 3 of them are picked randomly. What is the probability of picking 2 white and 1 black marbles?

A) $\frac{10}{21}$ B) $\frac{5}{14}$ C) $\frac{3}{7}$ D) $\frac{2}{7}$ E) $\frac{5}{21}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	D	B	B	A	C	E	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	E	B	B	B	E	C	A

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1. **Altı kişilik bir aile yuvarlak masa etrafına yan yana olmayacak şekilde kaç farklı şekilde oturabilirler?**

A family of 6 people in how many different ways can sit around a round table with parents sitting side by side?

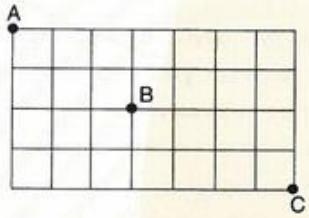
A) 72 B) 48 C) 40 D) 36 E) 24

2. **1,2,3,4,5 ve 6 rakamları kullanılarak 2000 den büyük 5000 den küçük dört basamaklı rakamları farklı kaç sayı yazılabilir?**

How many 4 digit numbers with no repeating digits bigger than 2000 and less than 5000 can be written using digits 1,2,3,4,5, and 6?

A) 120 B) 136 C) 144 D) 172 E) 180

3.

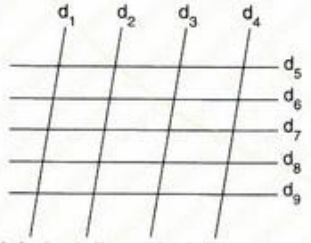


Şekilde görülen yollardan A den C ye B dan geçmek koşuluyla kaç farklı biçimde gidilebilir?

How many different ways are from A to B, by passing through the point C?

A) 25 B) 50 C) 100 D) 150 E) 200

4.

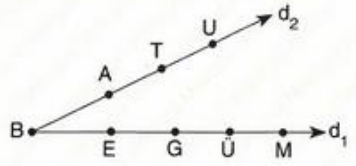


Birbirine paralel 4 doğru birbirine paralel 5 doğru ile kesilirse kaç tane paralelkenar oluşur?

From crossing 4 parallel lines with another 5 parallel lines,how many parallelograms are made?

A) 30 B) 40 C) 60 D) 80 E) 90

5.

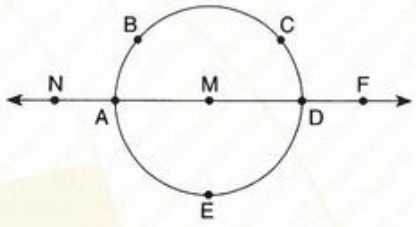


Şekilde verilen 8 noktadan kaç farklı üçgen oluşturulur?

How many different triangles can be drown from the 8 points in the figure above?

A) 40 B) 42 C) 48 D) 56 E) 60

6.

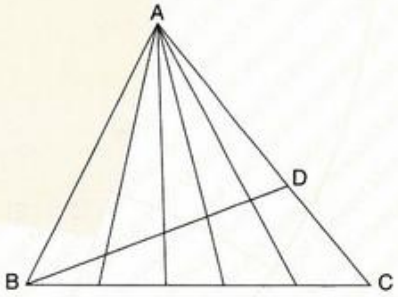


Şekildeki 8 noktadan en çok kaç doğru geçer?

What is the maximum number of straight lines that can be drown using 8 points given in figure above?

A) 16 B) 18 C) 19 D) 24 E) 28

7.



Şekilde kaç tane üçgen vardır?

How many triangles are in the figure above?

A) 33 B) 35 C) 36 D) 42 E) 48

8. **Aynı düzlem üzerinde bulunan 5 farklı kare en fazla kaç noktada kesişir?**

What is the maximum number of intersect points of 5 different squares that are on the same plane ?

A) 40 B) 50 C) 60 D) 70 E) 80

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9. $(x^2 - y^3)^5 = x^{10} - \ldots + Ax^6y^6 + \ldots \Rightarrow$ **A = ?**

A) 10 B) 12 C) 15 D) 18 E) 20

10. $(x - y)^n = \ldots + a{\cdot}x^2{\cdot}y^5 \ldots \Rightarrow$ **a + n = ?**

A) –35 B) –28 C) –21 D) –14 E) –7

11. $\left(x - \frac{2}{x}\right)^8 = \ldots + a{\cdot}x^6 + \ldots \Rightarrow$ **a = ?**

A) –18 B) –16 C) –14 D) –12 E) –10

12. $\left(x - \frac{3}{x}\right)^5 = \ldots + a{\cdot}x + \ldots \Rightarrow$ **a = ?**

A) 80 B) 85 C) 90 D) 95 E) 100

13. $\left(x - \frac{2}{\sqrt{x}}\right)^7 = x^7 \ldots + Ax^4 + \Rightarrow$ **A = ?4**

A) –84 B) –64 C) 64 D) 84 E) 114

14. $\left(\sqrt{x} + \frac{x}{3}\right)^6 = \ldots + Ax^4 + \ldots \Rightarrow$ **A = ?**

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1

D) $\frac{4}{3}$ E) $\frac{5}{3}$

15. Bir torbada aynı büyüklükte 5 kırmızı, 4 sarı ve 3 yeşil top vardır. Bu torbadan, yerine geri konulmamak üzere art arda çekilen iki topun da sarı olma olasılığı kaçtır?

There are 5 red, 4 yellow, and 3 green marbles with equal dimentions in a bag, if two marbles are randomely taken out from the bag withut putting them back, what is the probability of both of them being yellow marbles?

A) $\frac{1}{12}$ B) $\frac{1}{11}$ C) $\frac{1}{10}$ D) $\frac{1}{9}$ E) $\frac{1}{8}$

16. Bir madeni para ve bir zar aynı anda atılıyor. Madeni paranın yazı, zarın üst yüzeyindeki sayının asal sayı olma olasılığı kaçtır?

A coin and a dice are thrown at the same time. what is the probability of the coin coming up tails and the dice coming up a prime number?

A) $\frac{1}{4}$ B) $\frac{1}{5}$ C) $\frac{1}{6}$ D) $\frac{1}{7}$ E) $\frac{1}{8}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	E	D	C	B	C	B	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	D	B	C	D	E	B	A

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